10 Capital Budgeting Decision Methods

"Everything is worth what its purchaser will pay for it."

—*Publilius Syrus*

Capital Budgeting at DaimlerChrysler

DaimlerChrysler AG (DCAG) announced in 2000 its plans to invest $600 million to expand its Tuscaloosa, Alabama, manufacturing site to double production for the next generation Mercedes-Benz M-Class sport utility vehicle. The expansion was expected to create up to 2,000 new jobs and to increase production capacity from 80,000 units to 160,000 units. This capital budgeting project was a major investment by DaimlerChrysler AG in its Mercedes-Benz U.S. International, Inc., (MBUSI) operation in Tuscaloosa, AL.

DaimlerChrysler was predicted to become the fifth largest employer in Alabama. The total capital investment made by DaimlerChrysler since 1994 in Alabama would be more than $1.3 billion after completion of this expansion.

Is this a value adding investment for DaimlerChrysler? It surely thought so or it would not have given the go-ahead for the project. How did company executives evaluate this major investment before giving final approval? This chapter will help you to understand this important process.

Source: http://www.daimlerchrysler.com/dccom/0-5-7153-1-9756-1-0-0-0-0-0-8-7145-0-0-0-0-0-0-1.htm, May 25, 2006.

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Chapter Overview

We now look at the decision methods that analysts use to determine whether to approve a given investment project and how they account for a project's risk. Investment projects such as DaimlerChrysler's expansion project can fuel a firm's success, so effective project selection often determines a firm's value. The decision methods for choosing acceptable investment projects, then, are some of the most important tools financial managers use.

We begin by looking at the capital budgeting process and four capital budgeting decision methods: *payback*, *net present value*, *internal rate of return,* and *modified internal rate of return*. Then we discuss how to select projects when firms limit their budget for capital projects, a practice called capital rationing. Finally, we look at how firms measure and compensate for the risk of capital budgeting projects.

The Capital Budgeting Process

All businesses budget for capital projects—an airline that considers purchasing new planes, a production studio that decides to buy new film cameras, or a pharmaceutical company that invests in research and development for a new drug. **Capital budgeting** is the process of evaluating proposed large, long-term investment projects. The projects may be the purchase of fixed assets, investments in research and development, advertising, or intellectual property. For instance, Microsoft's development of Windows XP™ software was a capital budgeting project.

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Long-term projects may tie up cash resources, time, and additional assets. They can also affect a firm's performance for years to come. As a result, careful capital budgeting is vital to ensure that the proposed investment will add value to the firm.

Before we discuss the specific decision methods for selecting investment projects, we briefly examine capital budgeting basics: capital budgeting decision practices, types of capital budgeting projects, the cash flows associated with such projects, and the stages of the capital budgeting process.

Decision Practices

Financial managers apply two decision practices when selecting capital budgeting projects: *accept/reject* and *ranking*. The accept/reject decision focuses on the question of whether the proposed project would add value to the firm or earn a rate of return that is acceptable to the company. The ranking decision lists competing projects in order of desirability to choose the best one.

The accept/reject decision determines whether a project is acceptable in light of the firm's financial objectives. That is, if a project meets the firm's basic risk and return requirements, it will be accepted. If not, it will be rejected.

Ranking compares projects to a standard measure and orders the projects based on how well they meet the measure. If, for instance, the standard is how quickly the project pays off the initial investment, then the project that pays off the investment most rapidly would be ranked first. The project that paid off most slowly would be ranked last.

Types of Projects

Firms invest in two categories of projects: independent projects and mutually exclusive projects. **Independent projects** do not compete with each other. A firm may accept none, one, some, or all from among a group of independent projects. Say, for example, that a firm is able to raise funds for all worthwhile projects it identifies. The firm is considering two new projects—a new telephone system and a warehouse. The telephone system and the warehouse would be independent projects. Accepting one does not preclude accepting the other.

In contrast, **mutually exclusive** projects compete against each other. The best project from among a group of acceptable mutually exclusive projects is selected. For example, if a company needed only one copier, one proposal to buy a Xerox™ copier and a second proposal to buy a Toshiba™ copier would be two projects that are mutually exclusive.

Capital Budgeting Cash Flows

Although decision makers examine accounting information such as sales and expenses, capital budgeting decision makers base their decisions on relevant cash flows associated with a project. The relevant cash flows are **incremental cash flows**—cash flows that will occur if an investment is undertaken but that won't occur if it isn't.

For example, imagine that you are an animator who helps create children's films. Your firm is considering whether to buy a computer to help you design your characters more quickly. The computer would enable you to create two characters in the time it now takes you to draw one, thereby increasing your productivity.

The incremental cash flows associated with the animation project are (1) the initial investment in the computer and (2) the *additional* cash the firm will receive because you will double your animation output for the life of the computer. These cash flows

will occur if the new computer is purchased and will not occur if it is not purchased. These are the cash flows, then, that affect the capital budgeting decision.

Estimating these incremental cash flows, the subject of Chapter 11, is a major component of capital budgeting decisions, as we see in the next section.

Stages in the Capital Budgeting Process

The capital budgeting process has four major stages:

1. Finding projects

2. Estimating the incremental cash flows associated with projects

3. Evaluating and selecting projects

4. Implementing and monitoring projects

This chapter focuses on stage 3—how to evaluate and choose investment projects. We assume that the firm has found projects in which to invest and has estimated the projects' cash flows effectively.

Capital Budgeting Decision Methods

The four formal capital budgeting decision methods we will examine are payback, net present value, internal rate of return, and modified internal rate of return. Let's begin with the payback method.

The Payback Method

One of the simplest capital budgeting decision methods is the payback method. To use it, analysts find a project's **payback period**—the number of time periods it will take before the cash inflows of a proposed project equal the amount of the initial project investment (a cash outflow). In the Daimler Chrysler example discussed at the beginning of this chapter, the analysts would determine how many years it would take to recoup the initial $600 million investment.

How to Calculate the Payback Period To calculate the payback period, simply add up a project's projected positive cash flows, one period at a time, until the sum equals the amount of the project's initial investment. That number of time periods it takes for the positive cash flows to equal the amount of the initial investment is the payback period.

To illustrate, imagine that you work for a firm called the AddVenture Corporation. AddVenture is considering two investment proposals, Projects X and Y. The initial investment for both projects is $5,000. The finance department estimates that the projects will generate the following cash flows:

> **Take Note**
> Capital budgeting techniques are usually used only for projects with large cash outlays. Small investment decisions are usually made by the "seat of the pants." For instance, if your office supply of pencils is running low, you order more. You know that the cost of buying pencils is justified without undergoing capital budgeting analysis.

Cash Flows

	For Initial Investment	End of Year 1	End of Year 2	End of Year 3	End of Year 4
Project X	–$5,000	$2,000	$3,000	$ 500	$ 0
Project Y	–$5,000	$2,000	$2,000	$1,000	$2,000

By analyzing the cash flows, we see that Project X has a payback period of two years because the project's initial investment of $5,000 (a cash flow of –$5,000) will be recouped (by offsetting positive cash flows) at the end of year 2. In comparison, Project Y has a payback of three years.

Payback Method Decision Rule To apply the payback decision method, firms must first decide what payback time period is acceptable for long-term projects. In our case, if the firm sets two years as its required payback period, it will accept Project X but reject Project Y. If, however, the firm allows a three-year payback period, then both projects will be accepted if they are independent.

Problems with the Payback Method The payback method is used in practice because of its simplicity, but it has a big disadvantage in that it does not consider cash flows that occur after the payback period. For example, suppose Project Y's cash flows in year 4 were $10 million instead of $2,000. It wouldn't make any difference. Project Y would still be rejected under the payback method if the company's policy were to demand a payback of no more than two years. Failing to look beyond the payback period can lead to poor business decisions, as shown.

Another deficiency of the payback method is that it does not consider the time value of money. For instance, compare the cash flows of Projects A and B:

<div align="center">

Cash Flows

	For Initial Investment	End of Year 1	End of Year 2
Project A	–$5,000	$3,000	$2,000
Project B	–$5,000	$2,000	$3,000

</div>

Assuming the required payback period is two years, we see that both projects are equally preferable—they both pay back the initial investment in two years. However, time value of money analysis indicates that if both projects are equally risky, then Project A is better than Project B because the firm will get more money sooner. Nevertheless, timing of cash flows makes no difference in the payback computation.

Because the payback method does not factor in the time value of money, nor the cash flows after the payback period, its usefulness is limited. Although it does provide a rough measure of a project's liquidity and can help to supplement other techniques, financial managers should not rely on it as a primary decision method.

The Net Present Value (NPV) Method

The project selection method that is most consistent with the goal of owner wealth maximization is the net present value method. The **net present value (NPV)** of a capital budgeting project is *the dollar amount of the change in the value of the firm as a result of undertaking the project*. The change in firm value may be positive, negative, or zero, depending on the NPV value.

If a project has an NPV of zero, it means that the firm's overall value will not change if the new project is adopted. Why? Because the new project is expected to generate exactly the firm's required rate of return—no more and no less. A positive NPV means that the firm's value will increase if the project is adopted because the new project's

estimated return exceeds the firm's required rate of return. Conversely, a negative NPV means the firm's value will decrease if the new project is adopted because the new project's estimated return is less than what the firm requires.

Calculating NPV To calculate the net present value of a proposed project, we add the present value of a project's projected cash flows and then subtract the amount of the initial investment.[1] The result is a dollar figure that represents the change in the firm's value if the project is undertaken.

Formula for Net Present Value (NPV), Algebraic Method

$$NPV = \left[\frac{CF_1}{(1 + k)^1}\right] + \left[\frac{CF_2}{(1 + k)^2}\right] + \ldots + \left[\frac{CF_n}{(1 + k)^n}\right] - \text{Initial Investment}$$

(10-1a)

where: CF = Cash flow at the indicated times

k = Discount rate, or required rate of return for the project

n = Life of the project measured in the number of time periods

To use financial tables to solve present value problems, write the NPV formula as follows:

Formula for Net Present Value (NPV), Table Method

$$NPV = CF_1(PVIF_{k, 1}) + CF_2(PVIF_{k, 2}) + \ldots + CF_n(PVIF_{k, n}) - \text{Initial Investment}$$

(10-1b)

where: PVIF = Present Value Interest Factor

k = Discount rate, or required rate of return for the project

n = Life of the project measured in the number of time periods

To use the TI BAII Plus financial calculator to solve for NPV, switch the calculator to the spreadsheet mode, enter the cash flow values in sequence, then the discount rate, depicted as I on the TI BAII Plus calculator, then compute the NPV.

For simplicity, we use only the algebraic equation and the financial calculator to calculate NPV. If you prefer using financial tables, simply replace Equation 10-1a with Equation 10-1b.[2]

To show how to calculate NPV, let's solve for the NPVs of our two earlier projects, Projects X and Y. First, note the following cash flows of Projects X and Y:

Cash Flows

	For Initial Investment	End of Year 1	End of Year 2	End of Year 3	End of Year 4
Project X	–$5,000	$2,000	$3,000	$ 500	$ 0
Project Y	–$5,000	$2,000	$2,000	$1,000	$2,000

[1]We use many of the time value of money techniques learned in Chapter 8 to calculate NPV.

[2]If the future cash flows are in the form of an annuity, Equations 10-1a and 10-1b can be modified to take advantage of the present value of annuity formulas discussed in Chapter 8, Equations 8-4a and 8-4b.

Assume the required rate of return for Projects X and Y is 10 percent. Now we have all the data we need to solve for the NPV of Projects X and Y.

Applying Equation 10-1a and assuming a discount rate of 10 percent, the NPV of Project X follows:

$$\text{NPV} = \left[\frac{\$2,000}{(1 + .10)^1}\right] + \left[\frac{\$3,000}{(1 + .10)^2}\right] + \left[\frac{\$500_n}{(1 + .10)^3}\right] - \$5,000$$

$$= \frac{\$2,000}{1.1} + \frac{\$3,000}{1.21} + \frac{\$500}{1.331} - \$5,000$$

$$= \$1,818.1818 + \$2,479.3388 + \$375.6574 - \$5,000$$

$$= -\$326.82$$

We may also find Project X's NPV with the financial calculator at a 10 percent discount rate as follows:

TI BAII PLUS Financial Calculator Solutions
Project X NPV

Keystrokes	Display
CF	CF_0 = old contents
2nd CLR Work	CF_0 = 0.00
5000 +/- ENTER	CF_0 = –5,000.00
↓ 2000 ENTER	C01 = 2,000.00
↓	F01 = 1.00
↓ 3000 ENTER	C02 = 3,000.00
↓	F02 = 1.00
↓ 500 ENTER	C03 = 500.00
↓	F03 = 1.00
NPV	I = 0.00
10 ENTER	I = 10.00
↓ CPT	NPV = –326.82

The preceding calculations show that at a 10 percent discount rate, an initial cash outlay of $5,000, and cash inflows of $2,000, $3,000, and $500 at the end of years 1, 2, and 3, the NPV for Project X is –326.82.

To find the NPV of Project Y, we apply Equation 10-1a as follows:

$$\text{NPV} = \left[\frac{\$2,000}{(1 + .10)^1}\right] + \left[\frac{\$2,000}{(1 + .10)^2}\right] + \left[\frac{\$1,000}{(1 + .10)^3}\right] + \left[\frac{\$2,000}{(1 + .10)^4}\right] - \$5,000$$

$$= \frac{\$2,000}{1.1} + \frac{\$2,000}{1.21} + \frac{\$1,000}{1.331} + \frac{\$2,000}{1.4641} - \$5,000$$

$$= \$1,818.1818 + \$1,652.8926 + \$751.3148 + \$1,366.0269 - \$5,000$$

$$= \$588.42$$

Using the financial calculator, we solve for Project Y's NPV at a 10 percent discount rate as follows:

TI BAII PLUS Financial Calculator Solutions
Project Y NPV

Keystrokes	Display
CF	CF_0 = old contents
2nd CLR Work	CF_0 = 0.00
5000 +/- ENTER	CF_0 = –5,000.00
↓ 2000 ENTER	C01 = 2,000.00
↓ 2 ENTER	F01 = 2.00
↓ 1000 ENTER	C02 = 1,000.00
↓	F02 = 1.00
↓ 2000 ENTER	C03 = 2,00.00
↓	F03 = 1.00
NPV	I = 0.00
10 ENTER	I = 10.00
↓ CPT	NPV = 588.42

Our calculations show that with a required rate of return of 10 percent, an initial cash outlay of $5,000, and positive cash flows in years 1 through 4 of $2,000, $2,000, $1,000, and $2,000, respectively, the NPV for Project Y is $588.42. If we compare Project X's NPV of –326.82 to Project Y's NPV of $588.42, we see that Project Y would add value to the business and Project X would decrease the firm's value.

NPV Decision Rules NPV is used in two ways: (1) to determine if independent projects should be accepted or rejected (the accept/reject decision); and (2) to compare acceptable mutually exclusive projects (the ranking decision). The rules for these two decisions are

● NPV Accept/Reject Decision—A firm should accept all independent projects having NPVs greater than or equal to zero. Projects with positive NPVs will add to the value of the firm if adopted. Projects with NPVs of zero will not alter the firm's value but (just) meet the firm's requirements. Projects with NPVs less than zero should be rejected because they will reduce the firm's value if adopted. Applying this decision rule, Project X would be rejected and Project Y would be accepted.

- NPV Ranking Decision—The project from the mutually exclusive list with the highest positive NPV should be ranked first, the next highest should be ranked second, and so on. Under this decision rule, if the two projects in our previous example were mutually exclusive, Project Y would be ranked first and Project X second. (Not only is Project X second in rank here, but it is unacceptable because it has a negative NPV.)

The NPV Profile The k value is the cost of funds used for the project. It is the discount rate used in the NPV calculation because the cost of funds for a given project is that project's required rate of return. The relationship between the NPV of a project and k is inverse—the higher the k, the lower the NPV, and the lower the k, the higher the NPV.[3]

Because a project's NPV varies inversely with k, financial managers need to know how much the value of NPV will change in response to a change in k. If k is incorrectly specified, what appears to be a positive NPV could in fact be a negative NPV and vice versa—a negative NPV could turn out to be positive. Mutually exclusive project rankings could also change if an incorrect k value is used in the NPV computations.[4]

To see how sensitive a project's NPV value is to changes in k, analysts often create an NPV profile. The NPV profile is a graph that shows how a project's NPV changes when different discount rate values are used in the NPV computation.

Building an NPV profile is straightforward. First, the NPV of the project is calculated at a number of different discount rates. Then the results are plotted on the graph, with k values on one axis and the resulting NPV values on the other. If more than one project is included on the graph, the process is repeated for each project until all are depicted. To illustrate, we will build an NPV profile of Projects X and Y. We will plot Project X and then Project Y on the graph.

To begin, we first calculate the NPV of Project X with a number of different discount rates. The different k values may be chosen arbitrarily. For our purposes, let's use 0 percent, 5 percent, 10 percent, 15 percent, and 20 percent. The results of Project X's NPV calculations follow:

Discount Rate	Project X NPV
0%	$ 500.00
5%	$ 57.77
10%	–$ 326.82
15%	–$ 663.68
20%	–$ 960.65

> **Take Note**
> The NPV profile line is curved, not straight. The curve is steepest at low discount rates and gets more shallow at higher rates. This shape occurs because discounting is the inverse of the exponential compounding phenomenon, as described in Chapter 8.

Now Project X's NPV values may be plotted on the NPV profile graph. Figure 10-1 shows the results.

When the data points are connected in Figure 10-1, we see how the NPV of Project X varies with the discount rate changes. The graph shows that with a k of about 5.7 percent, the value of the project's NPV is zero. At that discount rate, then, Project X would provide the firm's required rate of return, no more and no less.

Next, we add project Y to the NPV profile graph. We calculate the NPV of Project Y at a number of different discount rates, 0 percent, 5 percent, 10 percent, 15 percent, and 20 percent. The results follow:

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[3]We are assuming here that the project is a typical one, meaning that it has an initial negative cash flow, the initial investment, followed by all positive cash flows. It is possible that if a project has negative cash flows in the future the relationship between NPV and k might not be inverse.

[4]The estimation of the cost of funds used for capital budgeting projects was covered in Chapter 9.
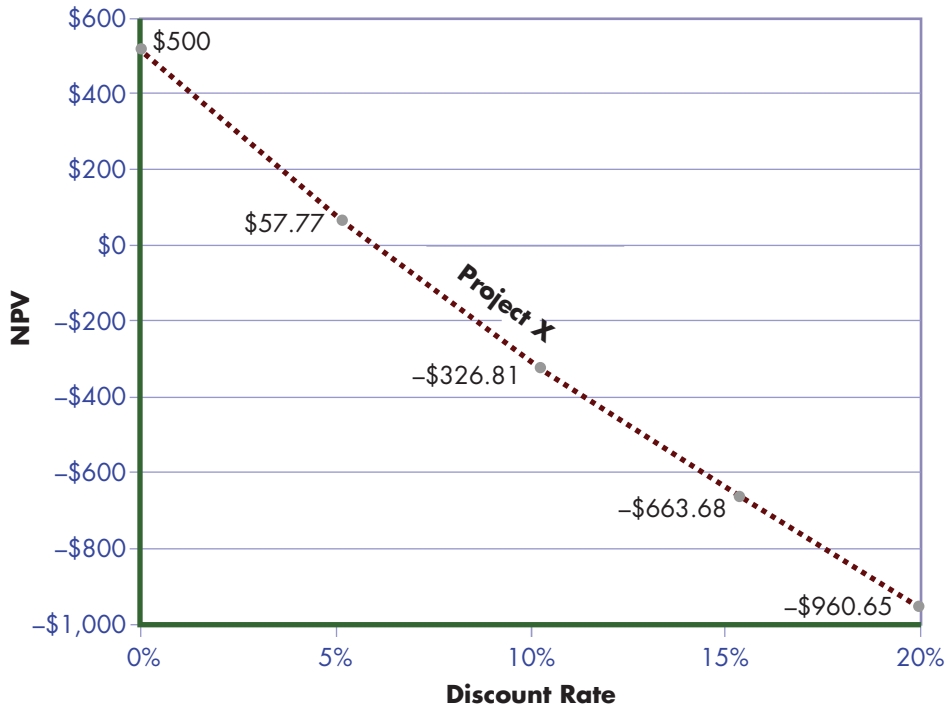
Figure 10-1
NPV Profile of Project X

The NPV profile shows how the NPV of project X varies inversely with the discount rate, k. Project X's NPV is highest ($500) when the discount rate is zero. Its NPV is lowest (–$960.65) when the discount rate is 20 percent.

Discount Rate	Project Y NPV
0%	$2,000.00
5%	$1,228.06
10%	$588.42
15%	$52.44
20%	–$401.23

Figure 10-2 shows these NPV values plotted on the NPV profile graph.

Notice in Figure 10-2 that Project Y's NPV profile falls off more steeply than Project X's. This indicates that Project Y is more sensitive to changes in the discount rate than Project X. Project X's NPV becomes negative and, thus, the project is unacceptable when the discount rate rises above about 6 percent. Project Y's NPV becomes negative and, thus, the project is unacceptable when the discount rate rises above about 16 percent.

Interactive Module
Go to the Interactive Spreadsheets you downloaded for chapter 10. Follow the instructions there. See how NPV varies with the discount rate used. Observe the graph that shows how NPV and IRR relate to each. Enter your own values and see how the graphs change.

Problems with the NPV Method Although the NPV method ensures that a firm will choose projects that add value to a firm, it suffers from two practical problems. First, it is difficult to explain NPV to people who are not formally trained in finance. Few nonfinance people understand phrases such as "the present value of future cash flows" or "the change in a firm's value given its required rate of return." As a result, many financial managers have difficulty using NPV analysis persuasively.

A second problem is that the NPV method results are in dollars, not percentages. Many owners and managers prefer to work with percentages because percentages can be easily compared with other alternatives; Project 1 has a 10 percent rate of return compared with Project 2's 12 percent rate of return. The next method we discuss, the internal rate of return, provides results in percentages.
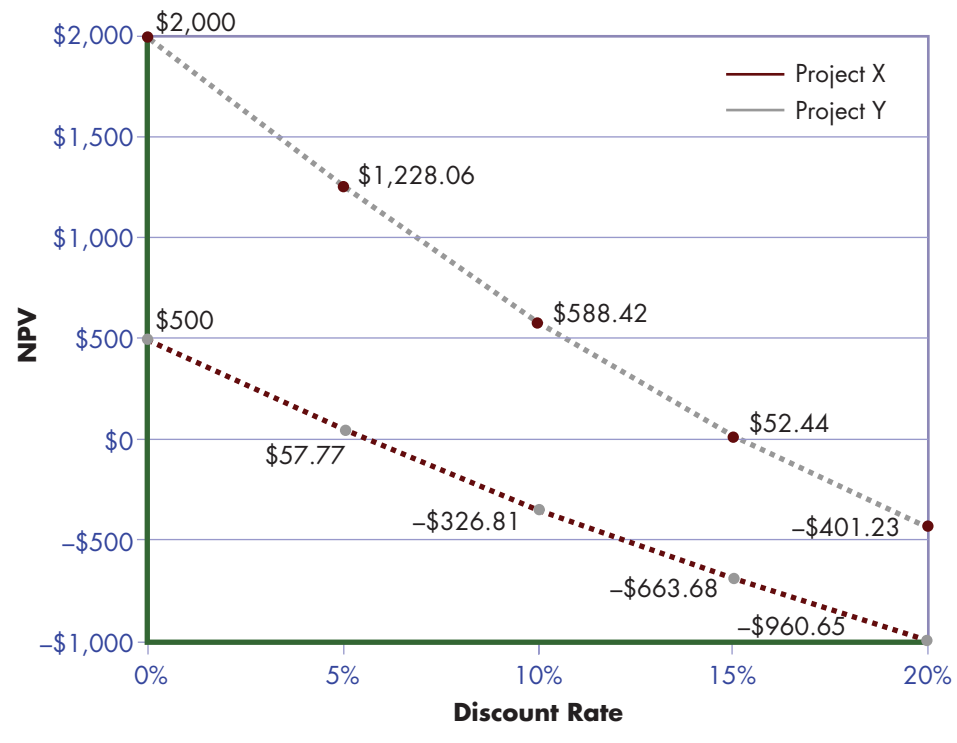
Figure 10-2 NPV Profile of Projects X and Y

The NPV file shows how the NPVs of two capital budgeting projects, Projects X and Y, vary inversely with the discount rate, k.

The Internal Rate of Return (IRR) Method

The **Internal Rate of Return (IRR)** is the estimated rate of return for a proposed project, given the project's incremental cash flows. Just like the NPV method, the IRR method considers all cash flows for a project and adjusts for the time value of money. However, the IRR results are expressed as a percentage, not a dollar figure.

When capital budgeting decisions are made using the IRR method, the IRR of the proposed project is compared with the rate of return management requires for the project. The required rate of return is often referred to as the **hurdle rate**. If the project's IRR is greater than or equal to the hurdle rate (jumps over the hurdle), the project is accepted.

Calculating Internal Rate of Return: Trial-and-Error Method If the present value of a project's incremental cash flows were computed using management's required rate of return as the discount rate and the result exactly equaled the cost of the project, then the NPV of the project would be zero. When NPV equals zero, the required rate of return, or discount rate used in the NPV calculation, is the projected rate of return, IRR. To calculate IRR, then, we reorder the terms in Equation 10-1a to solve for a discount rate, k, that results in an NPV of zero.

The formula for finding IRR, Equation 10-2, follows:

<div align="center">Formula for IRR   (10-2)</div>

$$NPV = 0 = \frac{CF_1}{(1 + k)^1} + \frac{CF_2}{(1 + k)^2} + \ldots + \frac{CF_n}{(1 + k)^n} - \text{Initial Investment}$$

To find the IRR of a project using Equation 10-2, fill in the cash flows, the n values, and the initial investment figure. Then choose different values for k (all the other values are known) until the result equals zero. The IRR is the k value that causes the left-hand side of the equation, the NPV, to equal zero.

To illustrate the process, let's calculate the IRR for Project X. Recall that the cash flows associated with Project X were as follows:

Cash Flows

	For Initial Investment	Year 1	Year 2	Year 3	Year 4
Project X	−$5,000	$2,000	$3,000	$500	$0

First, we insert Project X's cash flows and the times they occur into Equation 10-2.

$$NPV = 0 = \left[\frac{\$2,000}{(1+k)^1}\right] + \left[\frac{\$3,000}{(1+k)^2}\right] + \left[\frac{\$500}{(1+k)^3}\right] - \$5,000$$

Next, we try various discount rates until we find the value of k that results in an NPV of zero. Let's begin with a discount rate of 5 percent.

$$0 = \left[\frac{\$2,000}{(1+.05)^1}\right] + \left[\frac{\$3,000}{(1+.05)^2}\right] + \left[\frac{\$500}{(1+.05)^3}\right] - \$5,000$$

$$= \left[\frac{\$2,000}{1.05}\right] + \left[\frac{\$3,000}{1.1025}\right] + \left[\frac{\$500}{1.157625}\right] - \$5,000$$

$$= \$1,904.76 + \$2,721.09 + \$431.92 - \$5,000$$

$$= \$57.77$$

This is close, but not quite zero. Let's try a second time, using a discount rate of 6 percent.

$$0 = \left[\frac{\$2,000}{(1+.06)^1}\right] + \left[\frac{\$3,000}{(1+.06)^2}\right] + \left[\frac{\$500}{(1+.06)^3}\right] - \$5,000$$

$$= \left[\frac{\$2,000}{1.06}\right] + \left[\frac{\$3,000}{1.1236}\right] + \left[\frac{\$500}{1.191016}\right] - \$5,000$$

$$= \$1,886.79 + \$2,669.99 + \$419.81 - \$5,000$$

$$= -\$23.41$$

This is close enough for our purposes. We conclude the IRR for Project X is slightly less than 6 percent.

Although calculating IRR by trial and error is time-consuming, the guesses do not have to be made entirely in the dark. Remember that the discount rate and NPV are inversely related. When an IRR guess is too high, the resulting NPV value will be too low. When an IRR guess is too low, the NPV will be too high.

Take Note
We could estimate the IRR more accurately through more trial and error. However, carrying the IRR computation to several decimal points may give a false sense of accuracy, as in the case in which the cash flow estimates are in error.

Calculating Internal Rate of Return: Financial Calculator Finding solutions to IRR problems with a financial calculator is simple. After clearing previous values, enter the initial cash outlay and other cash flows and compute the IRR value.

The TI BAII PLUS financial calculator keystrokes for finding the IRR for Project X are shown next.

<div align="center">

TI BAII PLUS Financial Calculator Solution IRR

</div>

Keystrokes	Display
CF	CF_0 = old contents
2nd CLR Work	CF_0 = 0.00
5000 +/- ENTER	−5,000.00
↓ 2000 ENTER	C01 = 2,000.00
↓	F01 = 1.00
↓ 3000 ENTER	C02 = 3,000.00
↓	F02 = 1.00
↓ 500 ENTER	C03 = 500.00
↓	F03 = 1.00
IRR	IRR = 0.00
CPT	IRR = 5.71

We see that with an initial cash outflow of $5,000 and Project X's estimated cash inflows in years 1 through 3, the IRR is 5.71 percent.

IRR and the NPV Profile Notice in Figure 10-2 that the point where Project X's NPV profile crosses the zero line is, in fact, the IRR (5.71 percent). This is no accident. When the required rate of return, or discount rate, equals the expected rate of return, IRR, then NPV equals zero. Project Y's NPV profile crosses the zero line just below 16 percent. If you use your financial calculator, you will find that the Project Y IRR is 15.54 percent. If an NPV profile graph is available, you can always find a project's IRR by locating the point where a project's NPV profile crosses the zero line.

IRR Decision Rule When evaluating proposed projects with the IRR method, those that have IRRs equal to or greater than the required rate of return (hurdle rate) set by management are accepted, and those projects with IRRs that are less than the required rate of return are rejected. Acceptable, mutually exclusive projects are ranked from highest to lowest IRR.

> **Take Note**
> If you are using a financial calculator other than the TI BAII PLUS, the calculator procedure will be similar but the keystrokes will differ. Be sure to check your calculator's instruction manual.

Benefits of the IRR Method The IRR method for selecting capital budgeting projects is popular among financial practitioners for three primary reasons:

1. IRR focuses on all cash flows associated with the project.

2. IRR adjusts for the time value of money.

3. IRR describes projects in terms of the rate of return they earn, which makes it easy to compare them with other investments and the firm's hurdle rate.

Problems with the IRR Method The IRR method has several problems, however. First, because the IRR is a percentage number, it does not show how much the value of the firm will change if the project is selected. If a project is quite small, for instance, it may have a high IRR but a small effect on the value of the firm. (Consider a project that requires a $10 investment and returns $100 a year later. The project's IRR is 900 percent, but the effect on the value of the firm if the project is adopted is negligible.)

If the primary goal for the firm is to maximize its value, then knowing the rate of return of a project is not the primary concern. What is most important is the amount by which the firm's value will change if the project is adopted, which is measured by NPV.

To drive this point home, ask yourself whether you would rather earn a 100 percent rate of return on $5 (which would be $5) or a 50 percent rate of return on $1,000 (which would be $500). As you can see, it is not the rate of return that is important but the dollar value. Why? Dollars, not percentages, comprise a firm's cash flows. NPV tells financial analysts how much value in dollars will be created. IRR does not.

A second problem with the IRR method is that, in rare cases, a project may have more than one IRR, or no IRR. This is shown in detail in Appendix 10A.

The IRR can be a useful tool in evaluating capital budgeting projects. As with any tool, however, knowing its limitations will enhance decision making.

Conflicting Rankings between the NPV and IRR Methods

As long as proposed capital budgeting projects are independent, both the NPV and IRR methods will produce the same accept/reject indication. That is, a project that has a positive NPV will also have an IRR that is greater than the discount rate. As a result, the project will be acceptable based on both the NPV and IRR values. However, when mutually exclusive projects are considered and ranked, a conflict occasionally arises. For instance, one project may have a higher NPV than another project, but a lower IRR.

To illustrate how conflicts between NPV and IRR can occur, imagine that the AddVenture Company owns a piece of land that can be used in different ways. On the one hand, this land has minerals beneath it that could be mined, so AddVenture could invest in mining equipment and reap the benefits of retrieving and selling the minerals. On the other hand, the land has perfect soil conditions for growing grapes that could be used to make wine, so the company could use it to support a vineyard.

Clearly, these two uses are mutually exclusive. The mine cannot be dug if there is a vineyard, and the vineyard cannot be planted if the mine is dug. The acceptance of one of the projects means that the other must be rejected.

Now let's suppose that AddVenture's finance department has estimated the cash flows associated with each use of the land. The estimates are presented next:

Time	Cash Flows for the Mining Project	Cash Flows for the Vineyard Project
t_0	($736,369)	($736,369)
t_1	$500,000	$ 0
t_2	$300,000	$ 0
t_3	$100,000	$ 0
t_4	$ 20,000	$ 50,000
t_5	$ 5,000	$200,000
t_6	$ 5,000	$500,000
t_7	$ 5,000	$500,000
t_8	$ 5,000	$500,000

Note that although the initial outlays for the two projects are the same, the incremental cash flows associated with the projects differ in amount and timing. The Mining Project generates its greatest positive cash flows early in the life of the project, whereas the Vineyard Project generates its greatest positive cash flows later. The differences in the projects' cash flow timing have considerable effects on the NPV and IRR for each venture.

Assume AddVenture's required rate of return for long-term projects is 10 percent. The NPV and IRR results for each project given its cash flows are summarized as follows:

	NPV	IRR
Mining Project	$ 65,727.39	16.05%
Vineyard Project	$194,035.65	14.00%

These figures were obtained with our TI BAII PLUS calculator, in the same manner as shown earlier in the chapter.

The NPV and IRR results show that the Vineyard Project has a higher NPV than the Mining Project ($194,035.65 versus $65,727.39), but the Mining Project has a higher IRR than the Vineyard Project (16.05 percent versus 14.00 percent). AddVenture is faced with a conflict between NPV and IRR results. Because the projects are mutually exclusive, the firm can only accept one.

In cases of conflict among mutually exclusive projects, the one with the highest NPV should be chosen because NPV indicates the dollar amount of value that will be added to the firm if the project is undertaken. In our example, then, AddVenture should choose the Vineyard Project (the one with the higher NPV) if its primary financial goal is to maximize firm value.

The Modified Internal Rate of Return (MIRR) Method

One problem with the IRR method that was not mentioned earlier is that the only way you can actually receive the IRR indicated by a project is if you reinvest the intervening cash flows in the project at the IRR. If the intervening cash flows are reinvested at any rate lower than the IRR, you will not end up with the IRR indicated at the end of the project.

For example, assume you have a project in which you invest $100 now and expect to receive four payments of $50 at the end of each of the next four years:

t₀	**t₁**	**t**	**t₃**	**t₄**
–100	+50	+50	+50	+50

The IRR of this investment, calculated using either the trial-and-error or financial-calculator methods, is 34.9 percent. On the surface, this sounds like a fabulous investment. But, as they say, don't count your chickens before they hatch. If you can't reinvest each $50 payment at the IRR of 34.9 percent, forget it; you won't end up with an overall return of 34.9 percent. To see why, consider what would happen if, for example, you simply put each $50 payment in your pocket. At the end of the fourth year you would have $200 in your pocket. Now, using Equation 8-6, calculate the average annual rate of return necessary to produce $200 in four years with a beginning investment of $100:

Actual return on the investment per Equation 8-6

$$k = \left[\frac{200}{100}\right]^{\frac{1}{4}} - 1$$

$$= .189, \text{ or } 18.9\% \text{ (not } 34.9\%)$$

Despite the fact that the IRR of the investment was 34.9 percent, you only ended up with an 18.9 percent annual return. The only way you could have obtained an annual return of 34.9 percent was to have reinvested each of the $50 cash flows at 34.9 percent. This is called the *IRR reinvestment assumption.*

To get around the IRR reinvestment assumption, a variation on the IRR procedure has been developed, called the **Modified Internal Rate of Return (MIRR)** method. The MIRR method calls for assuming that the intervening cash flows from a project are reinvested at a rate of return equal to the cost of capital. To find the MIRR, first calculate how much you would end up with at the end of a project, assuming the intervening cash flows were invested at the cost of capital. The result is called the project's *terminal value.* Next, calculate the annual rate of return it would take to produce that end amount from the beginning investment. That rate is the MIRR.

Here is the MIRR calculation for the project in our example, in which $100 is invested at time zero, followed by inflows of $50 at the end of each of the next four years. Let us assume for this example that the cost of capital is 10 percent.

Step 1: Calculate the project's terminal value:

Time	Cash	FV of Cash Flow at t₄ if reinvested @ 10% Cost of Capital Flow (per Equation 8-a)
t₁	$50	FV = $50 x (1 + .10)³ = $ 66.55
t₂	$50	FV = $50 x (1 + .10)² = $ 60.50
t₃	$50	FV = $50 x (1 + .10)¹ = $ 55.00
t₄	$50	FV = $50 x (1 + .10)⁰ = $ 50.00
		Project's Terminal Value: $232.05

Step 2: Calculate the annual rate of return that will produce the terminal value from the initial investment:

Project's Terminal Value: $232.05

PV of Initial Investment: $100

per Equation 8-6:

$$\text{MIRR} = \left[\frac{232.05}{100}\right]^{\frac{1}{4}} - 1$$

$$= .234, \text{ or } 23.4\%$$

This is a much more realistic expectation than the 34.9 percent return from the project indicated by the IRR method. Remember, any time you consider investing in a project, you will not actually receive the IRR unless you can reinvest the project's intervening cash flows at the IRR. Calculate the MIRR to produce a more realistic indication of the actual project outcome.

In this section we looked at four capital budgeting decision methods: the payback method, the net present value method, the internal rate of return method, and the modified internal rate of return method. We also investigated how to resolve conflicts between NPV and IRR decision methods. Next, we turn to a discussion of capital rationing.

Capital Rationing

Our discussion so far has shown that all independent projects with NPVs greater than or equal to zero should be accepted. All such projects that are adopted will add value to the firm. To act consistently with the goal of shareholder wealth maximization, then, it seems that if a firm locates $200 billion worth of independent positive NPV projects, it should accept all the projects. In practice, however, many firms place dollar limits on the total amount of capital budgeting projects they will undertake. They may wish to limit spending on new projects to keep a ceiling on business size. This practice of setting dollar limits on capital budgeting projects is known as **capital rationing**.

If capital rationing is imposed, then financial managers should seek the combination of projects that maximizes the value of the firm within the capital budget limit. For example, suppose a firm called BeLimited does not want its capital budget to exceed $200,000. Seven project proposals, Proposals A to G, are available, as shown in Table 10-1.

Note that all the projects in Table 10-1 have positive net present values, so they are all acceptable. However, BeLimited cannot adopt them all because that would require the expenditure of more than $200,000, its self-imposed capital budget limit.

Under capital rationing, BeLimited's managers try different combinations of projects seeking the combination that gives the highest NPV without exceeding the $200,000 limit. For example, the combination of Projects B, C, E, F, and G costs $200,000 and yields a total NPV of $23,700. A better combination is that of Projects A, B, C, D, and F, which costs $200,000 and has a total NPV of $30,600. In fact, this combination has the highest total NPV, given the $200,000 capital budget limit (try a few other combinations to see for yourself), so that combination is the one BeLimited should choose.

In this section, we explored capital rationing. In the following section, we will examine how risk affects capital budgeting decisions.

Table 10-1   BeLimited Project Proposals

Project	Initial Cash Outlay	Net Present Value
A	$20,000	$8,000
B	$50,000	$7,200
C	$40,000	$6,500
D	$60,000	$5,100
E	$50,000	$4,200
F	$30,000	$3,800
G	$30,000	$2,000

Risk and Capital Budgeting

Suppose you are a financial manager for AddVenture Corporation. You are considering two capital budgeting projects, Project X (discussed throughout the chapter) and Project X2. Both projects have identical future cash flow values and timing, and both have the same NPV, if discounted at the same rate, k, and the same IRR. The two projects appear equally desirable.

Now suppose you discover that Project X's incremental cash flows are absolutely certain, but Project X2's incremental cash flows, which have the same expected value, are highly uncertain. These cash flows could be higher or lower than forecast. In other words, Project X is a sure thing and Project X2 appears to be quite risky. As a risk-averse person, you would prefer Project X over Project X2.

Financial managers are not indifferent to risk. Indeed, the riskier a project, the less desirable it is to the firm. To incorporate risk into the NPV and IRR evaluation techniques, we use the standard deviation and coefficient of variation (CV) measures discussed in Chapter 7. We begin by examining how to measure the risk of a capital budgeting project. Then we explain how to include the risk measurement in the NPV and IRR evaluation.

Measuring Risk in Capital Budgeting

Financial managers may measure three types of risk in the capital budgeting process. The first type is *project-specific risk,* or the risk of a specific new project. The second type is *firm risk,* which is the impact of adding a new project to the existing projects of the firm. The third is *market (or beta) risk,* which is the effect of a new project on the stockholders' nondiversifiable risk.

In this chapter we focus on measuring firm risk. To measure firm risk, we compare the coefficient of variation (CV) of the firm's portfolio before and after a project is adopted. The before-and-after difference between the two CVs serves as the project's risk measure from a firm perspective. If the CAPM approach (discussed in Chapter 7) is preferred, a financial manager could estimate the risk of a project by calculating the project's beta instead of the change in firm risk.

Computing Changes in the Coefficient of Variation Let's reconsider our earlier example, Project X. Recall that the IRR of Project X was 5.71 percent. Now suppose that after careful analysis, we determine this IRR is actually the most likely value from a

range of possible values, each with some probability of occurrence. We find the expected value and standard deviation of Project X's IRR distribution using Equations 7-1 and 7-2, respectively. We find that the expected value of the IRR distribution is 5.71 percent and the standard deviation is 2.89 percent.

To see how adding Project X to the firm's existing portfolio changes the portfolio's coefficient of variation (CV), we follow a five-step procedure.

Step 1: **Find the CV of the Existing Portfolio.** Suppose the expected rate of return and standard deviation of possible returns of AddVenture's existing portfolio are 6 percent and 2 percent, respectively. Given this information, calculate the CV of the firm's existing portfolio using Equation 7-3:

$$CV = \frac{\text{Standard Deviation}}{\text{Mean, or Expected Value}}$$

$$= \frac{.02}{.06}$$

$$= .3333, \text{ or } 33.33\%$$

Step 2: **Find the Expected Rate of Return of the New Portfolio (the Existing Portfolio Plus the Proposed Project).** Assume that the investment in Project X represents 10 percent of the value of the portfolio. In other words, the new portfolio after adding Project X will consist of 10 percent Project X and 90 percent of the rest of the firm's assets. With these figures, solve for the expected rate of return of the new portfolio using Equation 7-4. In the calculations, Asset A represents Project X and Asset B represents the firm's existing portfolio.

$$E(R_p) = (w_a \times E(R_a)) + (w_b \times E(R_b))$$

$$= (.10 \times .0571) + (.90 \times .06)$$

$$= .00571 + .05400$$

$$= .05971, \text{ or } 5.971\%$$

$$\textbf{\textit{s}}_p = \sqrt{w_a^2 \textbf{\textit{s}}_a^2 \ + \ w_b^2 \textbf{\textit{s}}_b^2 \ + \ 2w_a w_b r_{a,b} \textbf{\textit{s}}_a \textbf{\textit{s}}_b}$$

$$= \sqrt{(.10^2)(0.0289^2) \ + \ (.90^2)(0.02^2) \ + \ (2)(.10)(.90)(0.0)(.0289)(.02)}$$

$$= \sqrt{(.01)(.000835) \ + \ (.81)(.0004) \ + \ 0}$$

$$= \sqrt{.00000835 \ + \ .000324}$$

$$= \sqrt{.000332352}$$

$$= .0182, \text{ or } 1.82\%$$

Step 3: **Find the Standard Deviation of the New Portfolio (the Existing Portfolio Plus the Proposed Project).** Calculate the standard deviation of the new portfolio using Equation 7-5. Assume the degree of correlation (r) between project X (Asset A) and the firm's existing portfolio (Asset B) is zero. Put another way, there is no relationship between the returns from Project X and the returns from the firm's existing portfolio.

Step 4: **Find the CV of the New Portfolio (the Existing Portfolio Plus the Proposed Project).** To solve for the CV of the new portfolio with Project X included, we use Equation 7-3, as follows:

$$CV = \frac{\text{Standard Deviation}}{\text{Mean, or Expected value}}$$

$$= \frac{.0182}{.05971}$$

$$= .3048, \text{ or } 30.48\%$$

Step 5: **Compare the CV of the Portfolio with and without the Proposed Project.** To evaluate the effect of Project X on the risk of the AddVenture portfolio, we compare the CV of the old portfolio (without Project X) to the CV of the new portfolio (with Project X). The coefficients follow:

CV of the Portfolio without Project X	CV of the Portfolio with Project X	Change in CV
33.33%	30.48%	–2.85

The CV of the firm's portfolio dropped from 33.33 percent to 30.48 percent with the addition of Project X. This 2.85 percentage point decrease in CV is the measure of risk in Project X from the firm's perspective.

Adjusting for Risk

Most business owners and financial managers are risk averse, so they want the capital budgeting process to reflect risk concerns. Next, we discuss how to adjust for risk in the capital budgeting process—risk-adjusted discount rates. The discount rate is the cost of capital discussed in Chapter 9. A project's risk-adjusted discount rate is the cost of capital for that specific project.

Risk-Adjusted Discount Rates (RADRs) One way to factor risk into the capital budgeting process is to adjust the rate of return demanded for high- and low-risk projects. Risk-adjusted discount rates (RADRs), then, are discount rates that differ according to their effect on a firm's risk. The higher the risk, the higher the RADR, and the lower the risk, the lower the RADR. A project with a normal risk level would have an RADR equal to the actual discount rate.

To find the RADR for a capital budgeting project, we first prepare a risk adjustment table like the one in Table 10-2. We assume in Table 10-2 that the coefficient of variation, CV, is the risk measure to be adjusted for. The CV is based on the probability distribution of the IRR values.

Table 10-2 shows how the discount rates for capital budgeting projects might be adjusted for varying degrees of risk. The discount rate of a project that decreased the CV of the firm's portfolio of assets by 2.5 percentage points, for example, would be classified as a low-risk project. That project would be evaluated using a discount rate two percentage points lower than that used on average projects.

This has the effect of making Project X a more desirable project. That is, financial managers would calculate the project's NPV using an average discount rate that is two percentage points lower, which would increase the project's NPV. If the firm uses the

Table 10-2   Risk-Adjustment Table

If Adoption of the Proposed Capital Budgeting Project Would Change the CV of the Firm's Portfolio IRR by the Following Amount:	Then Assign the Project to the Following Risk Category:	And Adjust the Discount Rate for the Project as Follows:
More than a 1 percent increase	High risk	+2%
Between a 1 percent decrease and a 1 percent increase	Average risk	0
More than a 1 percent decrease	Low risk	–2%

IRR method, financial managers would compare the project's IRR to a hurdle rate two percentage points lower than average, which would make it more likely that the firm would adopt the project.

RADRs are an important part of the capital budgeting process because they incorporate the risk–return relationship. All other things being equal, the more a project increases risk, the less likely it is that a firm will accept the project; the more a project decreases risk, the more likely it is that a firm will accept the project.

What's Next

In this chapter we explored the capital budgeting process and decision methods. We examined four capital budgeting techniques—the payback method, net present value method, internal rate of return method, and modified internal rate of return method. We also discussed capital rationing and how to measure and adjust for risk when making capital budgeting decisions.

In the next chapter, we'll investigate how to estimate incremental cash flows in capital budgeting.

Summary

1. Explain the capital budgeting process.

Capital budgeting is the process of evaluating proposed investment projects. Capital budgeting projects may be independent—the projects do not compete with each other—or mutually exclusive—accepting one project precludes the acceptance of the other(s) in that group.

Financial managers apply two decision practices when selecting capital budgeting projects: accept/reject decisions and ranking decisions. The accept/reject decision is the determination of which independent projects are acceptable in light of the firm's financial objectives. Ranking is a process of comparing projects to a standard measure and ordering

the mutually exclusive projects based on how well they meet the measure. The capital budgeting process is concerned only with incremental cash flows; that is, those cash flows that will occur if an investment is undertaken but won't occur if it isn't.

2.   Calculate the payback period, net present value, internal rate of return, and modified internal rate of return for a proposed capital budgeting project.

The payback period is defined as the number of time periods it will take before the cash inflows from a proposed capital budgeting project will equal the amount of the initial investment. To find a project's payback period, add all the project's positive cash flows, one period at a time, until the sum equals the amount of the initial cash outlay for the project. The number of time periods it takes to recoup the initial cash outlay is the payback period. A project is acceptable if it pays back the initial investment within a time frame set by firm management.

The net present value (NPV) of a proposed capital budgeting project is the dollar amount of the change in the value of the firm that will occur if the project is undertaken. To calculate NPV, total the present values of all the projected incremental cash flows associated with a project and subtract the amount of the initial cash outlay. A project with an NPV greater than or equal to zero is acceptable. An NPV profile—a graph that shows a project's NPV at many different discount rates (required rates of return)— shows how sensitive a project's NPV is to changes in the discount rate.

The internal rate of return (IRR) is the projected percentage rate of return that a proposed project will earn, given its incremental cash flows and required initial cash outlay. To calculate IRR, find the discount rate that makes the project's NPV equal to zero. That rate of return is the IRR. A project is acceptable if its IRR is greater than or equal to the firm's required rate of return (the hurdle rate).

The IRR method assumes that intervening cash flows in a project are reinvested at a rate of return equal to the IRR. This can result in overly optimistic expectations when the IRR is high and the intervening cash flows can't be reinvested at that rate. To produce more realistic results, the Modified Internal Rate of Return (MIRR) method was developed. The MIRR method calls for assuming that the intervening cash flows from a project are reinvested at a rate of return equal to the cost of capital. To find the MIRR, first calculate how much you would end up with at the end of a project, assuming the intervening cash flows were invested at the cost of capital. The result is called the project's terminal value. Next, calculate the annual rate of return it would take to produce that end amount from the beginning investment. That rate is the MIRR.

3.   Describe capital rationing and how firms decide which projects to select.

The practice of placing dollar limits on the total size of the capital budget is called capital rationing. Under capital rationing, the firm will select the combination of projects that yields the highest NPV without exceeding the capital budget limit.

4.   Measure the risk of a capital budgeting project.

To measure the risk of a capital budgeting project, we determine how the project would affect the risk of the firm's existing asset portfolio. We compare the coefficient of variation of possible returns of the firm's asset portfolio with the proposed project and without it. The difference between the two coefficients of variation is the measure of the risk of the capital budgeting project.

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5. Explain risk-adjusted discount rates.

To compensate for the degree of risk in capital budgeting, firms may adjust the discount rate for each project according to risk. The more risk is increased, the higher the discount rate. The more risk is decreased, the lower the discount rate. Rates adjusted for the risk of projects are called risk-adjusted discount rates (RADRs).

Equations Introduced in This Chapter

Equation 10-1a. NPV Formula, Algebraic Method:

$$NPV = \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \ldots + \frac{CF_n}{(1+k)^n} - \text{Initial Investment}$$

where: CF = Cash flow at the indicated times
   k = Discount rate, or required rate of return for the project
   n = Life of the project measured in the number of time periods

Equation 10-1b. NPV Formula, Table Method:

$$NPV = CF_1(PVIF_{k,\,1}) + CF_2(PVIF_{k,\,2}) + \ldots + CF_n(PVIF_{k,\,n}) - \text{Initial Investment}$$

where: PVIF = Present Value Interest Factor
   k = Discount rate, or required rate of return for the project
   n = Life of the project measured in the number of time periods

Equation 10-2. Formula for IRR Using Equation 10-1a:

$$NPV = 0 = \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \ldots + \frac{CF_n}{(1+k)^n} - \text{Initial Investment}$$

where: k = the IRR value

Fill in cash flows (CFs) and periods (n). Then choose values for k by trial and error until the NPV equals zero. The value of k that causes the NPV to equal zero is the IRR.

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Self-Test

ST-1.   What is the NPV of the following project?

Initial investment	$50,000
Net cash flow at the end of year 1	$20,000
Net cash flow at the end of year 2	$40,000
Discount rate	11%

ST-2.   What is the IRR of the project in ST-1?

ST-3.   You've been assigned to evaluate a project for your firm that requires an initial investment of $200,000, is expected to last for 10 years, and is expected to produce after-tax net cash flows of $44,503 per year. If your firm's required rate of return is 14 percent, should the project be accepted?

ST-4.   What is the IRR of the project in ST-3?

ST-5.   What is the MIRR of the project in ST-3?

ST-6.   Assume you have decided to reinvest your portfolio in zero-coupon bonds. You like zero-coupon bonds because they pay off a known amount, $1,000 at maturity, and involve no other cash flows other than the purchase price. Assume your required rate of return is 12 percent. If you buy some 10-year zero-coupon bonds for $400 each today, will the bonds meet your return requirements? (Hint: Compute the IRR of the investment given the cash flows involved.)

ST-7.   Joe, the cut-rate bond dealer, has offered to sell you some zero-coupon bonds for $300. (Remember, zero-coupon bonds pay off $1,000 at maturity and involve no other cash flows other than the purchase price.) If the bonds mature in 10 years and your required rate of return for cut-rate bonds is 20 percent, what is the NPV of Joe's deal?

Review Questions

1. How do we calculate the payback period for a proposed capital budgeting project? What are the main criticisms of the payback method?

2. How does the net present value relate to the value of the firm?

3. What are the advantages and disadvantages of the internal rate of return method?

4. Provide three examples of mutually exclusive projects.

5. What is the decision rule for accepting or rejecting proposed projects when using net present value?

6. What is the decision rule for accepting or rejecting proposed projects when using internal rate of return?

7. What is capital rationing? Should a firm practice capital rationing? Why?

8. Explain how to resolve a ranking conflict between the net present value and the internal rate of return. Why should the conflict be resolved as you explained?

9. Explain how to measure the firm risk of a capital budgeting project.

10. Why is the coefficient of variation a better risk measure to use than the standard deviation when evaluating the risk of capital budgeting projects?

11. Explain why we measure a project's risk as the change in the CV.

12. Explain how using a risk-adjusted discount rate improves capital budgeting decision making compared with using a single discount rate for all projects.

Build Your Communication Skills

CS-1. Arrange to have a person who has no training in finance visit the class on the next day after the net present value (NPV) evaluation method has been covered. Before the visitor arrives, choose two teams who will brief the visitor on the results of their evaluation of a capital budgeting project. The visitor will play the role of the CEO of the firm. The CEO must choose one capital budgeting project for the firm to undertake. The CEO will choose the project based on the recommendations of Team A and Team B.

Now have each team describe the following projects:

| | Team A | Team B |
	Solar Energy Satellite (a plan to beam energy down from orbit)	Fresh Water Extraction Project (a plan to extract drinking water from the sea)
Initial Investment	$40 million	$40 million
NPV	$10 million	$ 8 million
IRR	12%	18%

After the projects have been described, have the CEO ask each team for its recommendation about which project to adopt. The CEO may also listen to input from the class at large.

When the debate is complete, have the CEO select one of the projects (based on the class's recommendations). Then critique the class's recommendations.

CS-2. Divide the class into groups of two members each. Have each pair prepare a written description (closed book) of how to measure the risk of a capital budgeting project. When the explanations are complete, select a volunteer from each group to present their findings to the class.

Problems

Payback ☞

10-1. Three separate projects each have an initial cash outlay of $10,000. The cash flow for Peter's Project is $4,000 per year for three years. The cash flow for Paul's Project is $2,000 in years 1 and 3, and $8,000 in year 2. Mary's Project has a cash flow of $10,000 in year 1, followed by $1,000 each year for years 2 and 3.

   a. Use the payback method to calculate how many years it will take for each project to recoup the initial investment.

   b. Which project would you consider most liquid?

Net Present Value ☞

10-2. You have just paid $20 million in the secondary market for the winning Powerball lottery ticket. The prize is $2 million at the end of each year for the next 25 years. If your required rate of return is 8 percent, what is the net present value (NPV) of the deal?

Internal Rate of Return ☞

10-3. What is the internal rate of return (IRR) of the Powerball deal in question 2?

Modified Internal Rate of Return ☞

10-4. What is the modified internal rate of return (MIRR) of the Powerball deal in question 2?

10-5.   RejuveNation needs to estimate how long the payback period would be for their new facility project. They have received two proposals and need to decide which one is best. Project Weights will have an initial investment of $200,000 and generate positive cash flows of $100,000 at the end of year 1, $75,000 at the end of year 2, $50,000 at the end of year 3, and $100,000 at the end of year 4. Project Waters will have an initial investment of $300,000 and will generate positive cash flows of $200,000 at the end of year 1 and $150,000 at the end of years 2, 3, and 4. What is the payback period for Project Waters? What is the payback period for Project Weights? Which project should RejuveNation choose?

🐟 **Payback Period**

10-6.   Calculate the NPV of each project in problem 10-5 using RejuveNation's cash flows and a 10 percent discount rate.

🐟 **NPV**

10-7.   The Bedford Falls Bridge Building Company is considering the purchase of a new crane. George Bailey, the new manager, has had some past management experience while he was the chief financial officer of the local savings and loan. The cost of the crane is $17,291.42, and the expected incremental cash flows are $5,000 at the end of year 1, $8,000 at the end of year 2, and $10,000 at the end of year 3.

🐟 **NPV and IRR**

   a. Calculate the net present value if the required rate of return is 12 percent.

   b. Calculate the internal rate of return.

   c. Should Mr. Bailey purchase this crane?

10-8.   Lin McAdam and Lola Manners, managers of the Winchester Company, do not practice capital rationing. They have three independent projects they are evaluating for inclusion in this year's capital budget. One is for a new machine to make rifle stocks. The second is for a new forklift to use in the warehouse. The third project involves the purchase of automated packaging equipment. The Winchester Company's required rate of return is 13 percent. The initial investment (a negative cash flow) and the expected positive net cash flows for years 1 through 4 for each project follow:

🐟 **NPV**

<div align="center">

Expected Net Cash Flow

Year	Rifle Stock Machine	Forklift	Packaging Equipment
0	$(9,000)	$(12,000)	$(18,200)
1	2,000	5,000	0
2	5,000	4,000	5,000
3	1,000	6,000	10,000
4	4,000	2,000	12,000

</div>

   a. Calculate the net present value for each project.

   b. Which project(s) should be undertaken? Why?

NPV and IRR ☞   **10-9.**   The Trask Family Lettuce Farm is located in the fertile Salinas Valley of California. Adam Trask, the head of the family, makes all the financial decisions that affect the farm. Because of an extended drought, the family needs more water per acre than what the existing irrigation system can supply. The quantity and quality of lettuce produced are expected to increase when more water is supplied. Cal and Aron, Adam's sons, have devised two different solutions to their problem. Cal suggests improvements to the existing system. Aron is in favor of a completely new system. The negative cash flow associated with the initial investment and expected positive net cash flows for years 1 through 7 for each project follow. Adam has no other alternatives and will choose one of the two projects. The Trask Family Lettuce Farm has a required rate of return of 12 percent for these projects.

	Expected Net Cash Flow	
Year	**Cal's Project**	**Aron's Project**
0	$(100,000)	$(300,000)
1	22,611	63,655
2	22,611	63,655
3	22,611	63,655
4	22,611	63,655
5	22,611	63,655
6	22,611	63,655
7	22,611	63,655

   a. Calculate the net present value for each project.

   b. Calculate the internal rate of return for each project.

   c. Which project should Adam choose? Why?

   d. Is there a conflict between the decisions indicated by the NPVs and the IRRs?

IRR and MIRR ☞   **10-10.**   Buzz Lightyear has been offered an investment in which he expects to receive payments of $4,000 at the end of each of the next 10 years in return for an initial investment of $10,000 now.

   a. What is the IRR of the proposed investment?

   b. What is the MIRR of the proposed investment? Assume a cost capital of 15%.

   c. Why is the MIRR thought of as a more realistic indication of a project's potential than the IRR?

Payback, NPV, ☞   **10-11.**   Dave Hirsh publishes his own manuscripts and is unsure which of two
and IRR   new printers he should purchase. He is a novelist living in Parkman, Illinois. Having slept through most of his Finance 300 course in college, he is unfamiliar with cash flow analysis. He enlists the help of the finance professor at the local university, Dr. Gwen French, to assist him. Together they estimate the following expected initial investment (a negative cash flow) and net positive cash flows for years 1 through 3 for each machine. Dave only needs one printer and estimates it will be worthless after three years of heavy use. Dave's required rate of return for this project is 10 percent.

	Expected Net Cash Flow	
Year	**Cal's Project**	**Aron's Project**
0	$(2,000)	$(2,500)
1	900	1,500
2	1,100	1,300
3	1,300	800

    a. Calculate the payback period for each printer.

    b. Calculate the net present value for each printer.

    c. Calculate the internal rate of return for each printer.

    d. Which printer do you think Dr. French will recommend? Why?

    e. Suppose Dave's required rate of return were 16 percent. Does the decision about which printer to purchase change?

10-12. Matrix.com has designed a virtual-reality program that is indistinguishable from real life to those experiencing it. The program will cost $20 million to develop (paid up front), but the payoff is substantial: $1 million at the end of year 1, $2 million at the end of year 2, $5 million at the end of year 3, and $6 million at the end of each year thereafter, through year 10. Matrix.com's weighted average cost of capital is 15 percent. Given these conditions, what are the NPV, IRR, and MIRR of the proposed program?

                                                          NPV, IRR, and MIRR

10-13. Project A has an initial investment of $11,000, and it generates positive cash flows of $4,000 each year for the next six years. Project B has an initial investment of $17,000, and it generates positive cash flows of $4,500 each year for the next six years. Assume the discount rate, or required rate of return, is 13 percent.

                                                           Independent and Mutually Exclusive Projects

    a. Calculate the net present value of each project. If Project A and Project B are mutually exclusive, which one would you select?

    b. Assume Projects A and B are independent. Based on NPV, which one(s) would you select?

    c. Calculate the internal rate of return for each project.

    d. Using IRR for your decision, which project would you select if Project A and Project B were mutually exclusive? Would your answer change if these two projects were independent instead of mutually exclusive?

    e. Project C was added to the potential capital budget list at the last minute as a mutually exclusive alternative to Project B. It has an initial cash outlay of $17,000, and it generates its only positive cash flow of $37,500 in the sixth year. Years 1 through 5 have $0 cash flow. Calculate Project C's NPV. Which alternative, Project B or C, would you choose?

    f. Now calculate the IRR of Project C and compare it with Project B's IRR. Based solely on the IRRs, which project would you select?

    g. Because Projects B and C are mutually exclusive, would you recommend that Project B or Project C be added to the capital budget for this year? Explain your choice.

NPV, and IRR ☛

10-14. Joanne Crale is an independent petroleum geologist. She is taking advantage of every opportunity to lease the mineral rights of land she thinks lies over oil reserves. She thinks there are many reservoirs with oil reserves left behind by major corporations when they plugged and abandoned fields in the 1960s. Ms. Crale knows that with today's technology, production can be sustained at much lower reservoir pressures than was possible in the 1960s. She would like to lease the mineral rights from George Hansen and Ed McNeil, the landowners. Her net cost for this current oil venture is $5 million. This includes costs for the initial leasing and three planned development wells. The expected positive net cash flows for the project are $1.85 million each year for four years. On depletion, she anticipates a negative cash flow of $250,000 in year 5 because of reclamation and disposal costs. Because of the risks involved in petroleum exploration, Ms. Crale's required rate of return is 16 percent.

  a. Calculate the net present value for Ms. Crale's project.

  b. Calculate the internal rate of return for this project.

  c. Would you recommend the project?

NPV Profile ☛
Analysis

10-15. Silkwood Power Company is considering two projects with the following predicted cash flows:

	Hydroelectric Upgrade	**Geothermal Upgrade**
Initial Investment CFs	($100,000)	($100,000)
End of year 1 CFs	20,000	80,000
End of year 2 CFs	30,000	40,000
End of year 3 CFs	40,000	30,000
End of year 4 CFs	90,000	10,000

  a. Construct a chart showing the NPV profiles of both projects.

  b. Which project(s) would be accepted if the company's cost of capital were 6 percent?

  c. Which project(s) would be accepted if the company's cost of capital were 15 percent?

  d. At what cost of capital would Silkwood value each project equally?

  e. At what cost of capital would the hydroelectric project become unacceptable?

  f. At what cost of capital would the geothermal project become unacceptable?

Challenge ☛
Problem

10-16. The product development managers at World Series Innovations are about to recommend their final capital budget for next year. They have a self-imposed budget limit of $100,000. Five independent projects are being considered. Vernon Simpson, the chief scientist and CEO, has minimal financial analysis experience and relies on his managers to recommend the projects that will increase the value of the firm by the greatest amount. Given the following summary of the five projects, which ones should the managers recommend?

Projects	Initial Cash Outlay	Net Present Value
Chalk Line Machine	($10,000)	$4,000
Gel Padded Glove	($25,000)	$3,600
Insect Repellent	($35,000)	$3,250
Titanium Bat	($40,000)	$2,500
Recycled Base Covers	($20,000)	$2,100

10-17. A project you are considering has an initial investment of $5,669.62. It has positive net cash flows of $2,200 each year for three years. Your required rate of return is 12 percent.

Various Capital Budgeting Issues

EXCEL SPREADSHEET

a. Calculate the net present value of this project.

b. Would you undertake this project if you had the cash available?

c. What would the discount rate have to be before this project's NPV would be positive? Construct an NPV profile with discount rates of 0 percent, 5 percent, and 10 percent to answer this question.

d. What other method might you use to determine at what discount rate the net present value would become greater than 0?

10-18. The internal rate of return (IRR) of a capital investment, Project B, is expected to have the following values with the associated probabilities for an economic life of five years. Calculate the expected value, standard deviation, and coefficient of variation of the IRR distribution.

Expected IRR

IRR	Probability of Occurrence
0%	0.05
1%	0.10
3%	0.20
6%	0.30
9%	0.20
11%	0.10
12%	0.05

10-19. Four capital investment alternatives have the following expected IRRs and standard deviations.

Capital Budgeting Risk

Project	Expected IRR	Standard Deviation
A	14%	2%
B	16%	6%
C	11%	5%
D	14%	4%

If the firm's existing portfolio of assets has an expected IRR of 13 percent with a standard deviation of 3 percent, identify the lowest- and the highest-risk projects in the preceding list. You may assume the correlation coefficient of returns from each project relative to the existing portfolio is .50 and the investment in each project would constitute 20 percent of the firm's total portfolio.

10-20. Assume that a company has an existing Portfolio A, with an expected
IRR of 10 percent and standard deviation of 2 percent. The company is
considering adding a Project B, with expected IRR of 11 percent and
standard deviation of 3 percent, to its portfolio. Also assume that the amount
invested in Portfolio A is $700,000 and the amount to be invested in Project
B is $200,000.

   a. If the degree of correlation between returns from Portfolio A and Project
B is .90 (r = +.9), calculate:

      1. the coefficient of variation of Portfolio A

      2. the expected IRR of the new combined portfolio

      3. the standard deviation of the new combined portfolio

      4. the coefficient of variation of the new combined portfolio

   b. What is the change in the coefficient of variation as a result of adopting
Project B?

   c. Assume the firm classifies projects as high risk if they raise the
coefficient of variation of the portfolio 2 percentage points or more,
as low risk if they lower the coefficient of variation of the portfolio
2 percentage points or more, and as average risk if they change the
coefficient of variation of the portfolio by less than 2 percent in either
direction. In what risk classification would Project B be?

   d. The required rate of return for average-risk projects for the company in
this problem is 13 percent. The company policy is to adjust the required
rate of return downward by 3 percent for low-risk projects and to raise it
3 percent for high-risk projects (average-risk projects are evaluated at the
average required rate of return). The expected cash flows from Project B
are as follows:

CF of Initial Investment	($200,000)
end of year 1	55,000
end of year 2	55,000
end of year 3	55,000
end of year 4	100,000

    Calculate the NPV of Project B when its cash flows are discounted at:

    1. the average required rate of return

    2. the high-risk discount rate

    3. the low-risk discount rate

10-21. Dorothy Gale is thinking of purchasing a Kansas amusement park, Glinda's
Gulch. She already owns at least one park in each of the surrounding states
and wants to expand her operations into Kansas. She estimates the IRR from
the project may be one of a number of values, each with a certain probability
of occurrence. Her estimated IRR values and the probability of their
occurrence follow.

Glinda's Gulch

Possible IRR Value	Probability
2%	.125
5%	.20
9%	.35
13%	.20
16%	.125

Now Dorothy wants to estimate the risk of the project. Assume she has asked you to do the following for her:

a. Calculate the mean, or expected, IRR of the project.

b. Calculate the standard deviation of the possible IRRs.

c. Assume the expected IRR of Dorothy's existing portfolio of parks is 8 percent, with a standard deviation of 2 percent. Calculate the coefficient of variation of Dorothy's existing portfolio.

d. Calculate the expected IRR of Dorothy's portfolio if the Glinda's Gulch project is adopted. For this calculation assume that Dorothy's new portfolio will consist of 80 percent existing parks and 20 percent new Glinda's Gulch.

e. Again assuming that Dorothy's new portfolio will consist of 80 percent existing parks and 20 percent new Glinda's Gulch, calculate the standard deviation of Dorothy's portfolio if the Glinda's Gulch project is added. Because Glinda's Gulch is identical to Dorothy's other parks, she estimates the returns from Glinda's Gulch and her existing portfolio are perfectly positively correlated (r = +1.0).

f. Calculate the coefficient of variation of Dorothy's new portfolio with Glinda's Gulch.

g. Compare the coefficient of variation of Dorothy's existing portfolio with and without the Glinda's Gulch project. How does the addition of Glinda's Gulch affect the riskiness of the portfolio?

10-22. Four capital investment alternatives have the following expected IRRs and standard deviations.

Risk in Capital Budgeting

Project	Expected IRR	Standard Deviation
A	18%	9%
B	15%	5%
C	11%	3%
D	8%	1%

The firm's existing portfolio of projects has an expected IRR of 12 percent and a standard deviation of 4 percent.

    a. Calculate the coefficient of variation of the existing portfolio.

    b. Calculate the coefficient of variation of the portfolio if the firm invests 10 percent of its assets in Project A. You may assume that there is no correlation between the returns from Project A and the returns from the existing portfolio (r = 0). (Note: In order to calculate the coefficient of variation, you must first calculate the expected IRR and standard deviation of the portfolio with Project A included.)

    c. Using the same procedure as in b and given the same assumptions, calculate the coefficient of variation of the portfolio if it includes, in turn, Projects B, C, and D.

    d. Which project has the highest risk? Which has the lowest risk?

Firm Risk ☞

EXCEL
SPREADSHEET

10-23. A firm has an existing portfolio of projects with a mean, or expected, IRR of 15 percent. The standard deviation of this estimate is 5 percent. The existing portfolio is worth $820,000. The addition of a new project, PROJ1, is being considered. PROJ1's expected IRR is 18 percent with a standard deviation of 9 percent. The initial investment for PROJ1 is expected to be $194,000. The returns from PROJ1 and the existing portfolio are perfectly positively correlated (r = +1.0).

    a. Calculate the coefficient of variation for the existing portfolio.

    b. If PROJ1 is added to the existing portfolio, calculate the weight (proportion) of the existing portfolio in the combined portfolio.

    c. Calculate the weight (proportion) of PROJ1 in the combined portfolio.

    d. Calculate the standard deviation of the combined portfolio. Is the standard deviation of the combined portfolio higher or lower than the standard deviation of the existing portfolio?

    e. Calculate the coefficient of variation of the combined portfolio.

    f. If PROJ1 is added to the existing portfolio, will the firm's risk increase or decrease?

Risk in Capital ☞
Budgeting

EXCEL
SPREADSHEET

10-24. VOTD Pharmaceuticals is considering the mass production of a new sleeping pill. Neely O'Hara, VOTD's financial analyst, has gathered all the available information from the finance, production, advertising, and marketing departments and has estimated that the yearly net incremental cash flows will be $298,500. She estimates the initial investment for this project will be $2 million. The pills are expected to be marketable for 10 years, and Neely does not expect that any of the investment costs will be recouped at the end of the 10-year period. VOTD's required rate of return for average-risk projects is 8 percent. Two percent is added to the required rate of return for high-risk projects.

    a. Calculate the net present value of the project if VOTD management considers it to be average risk.

    b. A lot of uncertainty is associated with the potential competition of new drugs that VOTD's competitors might introduce. Because of this uncertainty, VOTD management has changed the classification of this project to high risk. Calculate the net present value at the risk-adjusted discount rate.

c. Assuming the risk classification does not change again, should this project be adopted?

d. What will the yearly net incremental cash flow have to be to have a positive NPV when using the high-risk discount rate?

10-25. Aluminum Building Products Company (ABPC) is considering investing in either of the two mutually exclusive projects described as follows:

Project 1. Buying a new set of roll-forming tools for its existing roll-forming line to introduce a new cladding product. After its introduction, the product will need to be promoted. This means that cash inflows from additional production will start some time after and will gradually pick up in subsequent periods.

Project 2. Modifying its existing roll-forming line to increase productivity of its available range of cladding products. Cash inflows from additional production will start immediately and will reduce over time as the products move through their life cycle.

Sarah Brown, project manager of ABPC, has requested that you do the necessary financial analysis and give your opinion about which project ABPC should select. The projects have the following net cash flow estimates:

Year	Project 1	Project 2
0	($200,000)	($200,000)
1	0	90,000
2	0	70,000
3	20,000	50,000
4	30,000	30,000
5	40,000	10,000
6	60,000	10,000
7	90,000	10,000
8	100,000	10,000

Both these projects have the same economic life of eight years and average risk characteristics. ABPC's weighted average cost of capital, or hurdle rate, is 7.2 percent.

a. Which project would you recommend Ms. Brown accept to maximize value of the firm? (Hint: Calculate and compare NPVs of both projects.)

b. What are the IRRs of each project? Which project should be chosen using IRR as the selection criterion?

c. Draw the NPV profiles of both projects. What is the approximate discount rate at which both projects would have the same NPV? What is that NPV?

d. Does the selection remain unaffected for i) WACC > 5.4 percent; ii) WACC > 8.81 percent; and iii) WACC > 14.39 percent?

e. Further market survey research indicates that both projects have lower-than-average risk and, hence, the risk-adjusted discount rate should be 5 percent. What happens to the ranking of the projects using NPV and IRR as the selection criteria? Explain the conflict in ranking, if any.

f. Answer questions a, d, and e, assuming the projects are independent of each other.

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Answers to Self-Test

ST-1. The NPV of the project can be found using Equation 10-1a as follows:

$$NPV = \left[\frac{\$20,000}{(1 + .11)^1}\right] + \left[\frac{\$40,000}{(1 + .11)^2}\right] - \$50,000$$

$$= \left[\frac{\$20,000}{1.11}\right] + \left[\frac{\$40,000}{1.2321}\right] - \$50,000$$

$$= \$18,018 + \$32,465 - \$50,000$$

$$= \$483$$

ST-2. To find the IRR, we solve for the k value that results in an NPV of zero in the NPV formula (Equation 10-1a) by trial and error:

Try using k = 11%:

$$0 = \left[\frac{\$20,000}{(1 + .11)^1}\right] + \left[\frac{\$40,000}{(1 + .11)^2}\right] - \$50,000$$

$$= \left[\frac{\$20,000}{1.11}\right] + \left[\frac{\$40,000}{1.2321}\right] - \$50,000$$

$$= \$18,018 + \$32,465 - \$50,000$$

$$= \$483$$

The result does not equal zero. Try again using a slightly higher discount rate:
Second, try using k = 11.65%:

$$0 = \left[\frac{\$20,000}{(1 + .1165)^1}\right] + \left[\frac{\$40,000}{(1 + .1165)^2}\right] - \$50,000$$

$$= \left[\frac{\$20,000}{1.1165}\right] + \left[\frac{\$40,000}{1.24657225}\right] - \$50,000$$

$$= \$17,913 + \$32,088 - \$50,000$$

$$= \$1$$

Close enough. The IRR of the project is almost exactly 11.65 percent.

ST-3. The NPV of the project must be calculated to see if it is greater or less than zero. Because the project in this problem is an annuity, its NPV can be found most easily using a modified version of Equation 10-1b as follows:

$$NPV = PMT(PVIFA_{k, n}) - \text{Initial Investment}$$

We include this problem (and some others that follow) to show you that Equations 10-1a and 10-1b can be modified to take advantage of the present value of an annuity formulas covered in Chapter 8, Equations 8-4a and 8-4b.

The problem could also be solved using the basic forms of Equations 10-1a and 10-1b, but those would take longer.

In this problem, the initial investment is $200,000, the annuity payment is $44,503, the discount rate, k, is 14 percent, and the number of periods, n, is 10.

$$NPV = \$44,503(PVIFA_{14\%, 10}) - \$200,000$$

$$= \$44,503(5.2161) - \$200,000$$

$$= \$232,132 - \$200,000$$

$$= \$32,132$$

Because the NPV is positive, the project should be accepted.

ST-4.   The IRR is found by setting the NPV formula to zero and solving for the IRR rate, k. In this case, there are no multiple cash flows, so the equation can be solved algebraically. For convenience we use Equation 10-1b modified for an annuity:

$$0 = \$44,503\left(PVIFA_{k\%, 10}\right) - \$200,000$$

$$\$200,000 = \$44,503\left(PVIFA_{k\%, 10}\right)$$

$$\frac{\$200,000}{\$44,503} = PVIFA_{k\%, 10}$$

$$4.49408 = PVIFA_{k\%, 10}$$

Finding the PVIFA in Table IV in the Appendix, we see that 4.4941 (there are only four decimal places in the table) occurs in the year 10 row in the 18 percent column. Therefore, the IRR of the project is 18 percent.

ST-5.   To calculate the MIRR, first calculate the project's terminal value. Next, calculate the annual rate of return it would take to produce that end amount from the beginning investment. That rate is the MIRR.

For the project in ST-3:

Step 1: Calculate the project's terminal value. Note—The terminal value could be calculated by adding up the future values of each individual cash flow, as was done in the MIRR explanation on pages 276–278. In this case, however, that would be a lengthy procedure because there are 10 cash flows. Recognizing that the cash flows are an annuity, we can make use of the future value of an annuity formula, Equation 8-3a, to calculate the terminal value:

$$FV = \$44,503 \; \frac{(1+.14)^{10} \quad 1}{.14}$$

$$FV = \$44.503 \left[19.3372951\right]$$

$$FV = \text{Project's Terminal Value} = \$860,568$$

Step 2: Calculate the annual rate of return that will produce the terminal value from the initial investment.

Project's Terminal Value: $860,568
PV of Initial Investment: $200,000
per Equation 8-6:

$$\text{MIRR} = \left[\frac{860,568}{200,000}\right]^{\frac{1}{10}} - 1$$

$$\text{MIRR} = .1571, \text{ or } 15.71\%$$

ST-6. There are only two cash flows in this problem, a $400 investment at time 0 and the $1,000 payoff at the end of year 10. To find the IRR, we set the NPV formula to zero, fill in the two cash flows and periods, and solve for the IRR rate, k, that makes the right-hand side of the equation equal zero. Equation 10-1b is more convenient to use in this case:

$$0 = \$1,000\left(\text{PVIF}_{k\%, 10}\right) - \$400$$

$$\$400 = \$1,000\left(\text{PVIF}_{k\%, 10}\right)$$

$$\frac{\$400}{\$1,000} = \text{PVIF}_{k\%, 10}$$

$$.4000 = \text{PVIF}_{k\%, 10}$$

If we find the PVIF in Table II inside the book cover, we see that .4000 occurs in the year 10 row between the 9 percent and 10 percent columns. Therefore, the IRR of the project is between 9 percent and 10 percent. (The exact value of the IRR is 9.596 percent). Because your required rate of return in this problem was 12 percent, the bonds would not meet your requirements.

ST-7. Solve for the NPV using Equation 10-1a or b. Equation 10-1a would be the more convenient version for this problem. The cash flows are a $300 investment at time 0 and a $1,000 future value (FV) in 10 years. The discount rate is 20 percent.

$$\text{NPV} = \left[\frac{\$1,000}{(1 + .20)^{10}}\right] - \$300$$

$$= \left[\frac{\$1,000}{6.191736}\right] - \$300$$

$$= \$161.51 - \$300$$

$$= -\$138.49$$

Appendix 10A

Wrinkles in Capital Budgeting

In this appendix we discuss three situations that change the capital budgeting decision process. First, we examine nonsimple projects, projects that have a negative initial cash flow, in addition to one or more negative future cash flows. Next, we explore projects that have multiple IRRs. Finally, we discuss how to compare mutually exclusive projects with unequal project lives.

Nonsimple Projects

Most capital budgeting projects start with a negative cash flow—the initial investment—at t_0, followed by positive future cash flows. Such projects are called **simple projects. Nonsimple projects** are projects that have one or more negative future cash flows after the initial investment.

 To illustrate a nonsimple project, consider Project N, the expected cash flows for a nuclear power plant project. The initial investment of $500 million is a negative cash flow at t_0, followed by positive cash flows of $25 million per year for 30 years as electric power is generated and sold. At the end of the useful life of the project, the storage of nuclear fuel and the shutdown safety procedures require cash outlays of $100 million at the end of year 31. The timeline depicted in Figure 10A-1 shows the cash flow pattern:

 With a 20 percent discount rate, an initial investment of $500 million, a 30-year annuity of $25 million, and a shutdown cash outlay of $100 million in year 31, the NPV of Project N follows (in millions), per Equation 10-1a:

$$NPV_n = \$25\left[\frac{1 - \frac{1}{(1 + .20)^{30}}}{.20}\right] - \left[\frac{\$100}{(1 + .20)^{31}}\right] - \$500$$

$$= \left(\$25 \times 4.9789364\right) - \left[\frac{\$100}{284.8515766}\right] - \$500$$

$$= \$124.47341 - \$.35106 - \$500$$

$$= -\$375.8776$$

 We find that at a discount rate of 20 percent, Project N has a negative net present value of minus $375.88, so the firm considering Project N should reject it.
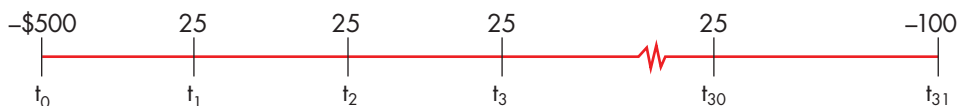

Figure 10A-1 Cash Flow Timeline for Project N (in millions)

Multiple IRRs

Some projects may have more than one internal rate of return. That is, a project may have several different discount rates that result in a net present value of zero.

Here is an example. Suppose Project Q requires an initial cash outlay of $160,000 and is expected to generate a positive cash flow of $1 million in year 1. In year 2, the project will require an additional cash outlay in the amount of $1 million. The cash flows for Project Q are shown on the timeline in Figure 10A-2.

We find the IRR of Project Q by using the trial-and-error procedure, Equation 10-2. When k = 25 percent, the NPV is zero.

$$0 = \left[\frac{\$1,000,000}{(1 + .25)^1}\right] - \left[\frac{\$1,000,000}{(1 + .25)^2}\right] - \$160,000$$

$$= \left[\frac{\$1,000,000}{1.25}\right] - \left[\frac{\$1,000,000}{1.5625}\right] - \$160,000$$

$$= \$800,000 - \$640,000 - \$160,000$$

$$= \$0$$

Because 25 percent causes the NPV of Project Q to be zero, the IRR of the project must be 25 percent. But wait! If we had tried k = 400%, the IRR calculation would look like this:

$$0 = \left[\frac{\$1,000,000}{(1 + 4.00)^1}\right] - \left[\frac{\$1,000,000}{(1 + 4.00)^2}\right] - \$160,000$$

$$= \left[\frac{\$1,000,000}{5.00}\right] - \left[\frac{\$1,000,000}{25.00}\right] - \$160,000$$

$$= \$200,000 - \$40,000 - \$160,000$$

$$= \$0$$

Because 400 percent results in an NPV of zero, 400 percent must also be the IRR of the Project Q. Figure 10A-3 shows the net present value profile for Project Q. By examining this graph, we see how 25 percent and 400 percent both make the net present value equal to zero.

As the graph shows, Project Q's net present value profile crosses the horizontal axis (has a zero value) in two different places, at discount rates of 25 percent and 400 percent.

Project Q had two IRRs because the project's cash flows changed from negative to positive (from t_0 to t_1) and then from positive to negative at (from t_1 to t_2). It turns out that a nonsimple project may have (but does not have to have) as many IRRs as there are sign changes. In this case, two sign changes resulted in two internal rates of return.

	t_0	t_1	t_2
Cash flows:	−$160,000	$1,000,000	−$1,000,000

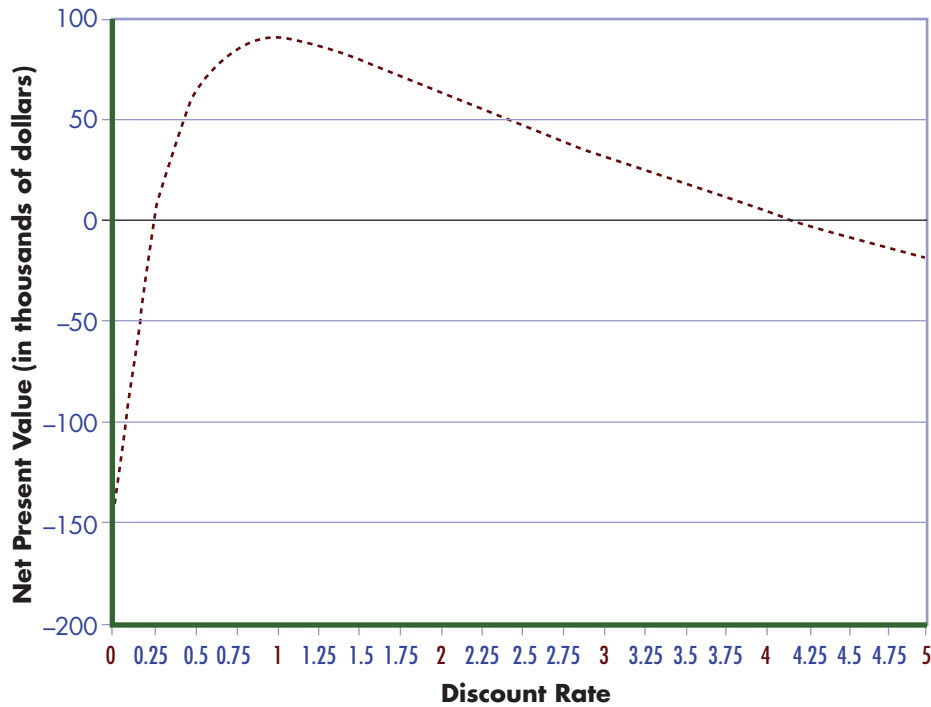**Figure 10A-2** Cash Flow Timeline for Project Q

Figure 10A-3
Net Present Value Profile
for Project Q

The net present value profile
for project Q crosses the zero
line twice, showing that two
IRRs exist.

Whenever we have two or more IRRs for a project, the IRR method is not a useful decision-making tool. Remember the IRR accept/reject decision rule: Firms should accept projects with IRRs higher than the discount rate and reject projects with IRRs lower than the discount rate. With more than one discount rate, decision makers will not know which IRR to use for the accept/reject decision. In projects that have multiple IRRs, then, switch to the NPV method.
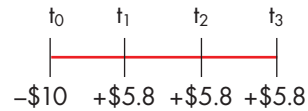
Mutually Exclusive Projects with Unequal Project Lives

When mutually exclusive projects have different expected useful lives, selecting among the projects requires more than comparing the projects' NPVs. To illustrate, suppose you are a business manager considering a new business telephone system. One is the Cheap Talk System, which requires an initial cash outlay of $10,000 and is expected to last three years. The other is the Rolles Voice System, which requires an initial cash outlay of $50,000 and is expected to last 12 years. The Cheap Talk System is expected to generate positive cash flows of $5,800 per year for each of its three years of life. The Rolles Voice System is expected to generate positive cash flows of $8,000 per year for each of its 12 years of life. The cash flows associated with each project are summarized in Figure10A-4.
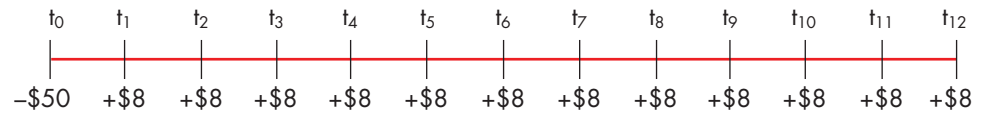
Cheap Talk

t_0 t_1 t_2 t_3

−$10   +$5.8 +$5.8 +$5.8

Rolles

t_0 t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9 t_{10} t_{11} t_{12}

−$50   +$8   +$8   +$8   +$8   +$8   +$8   +$8   +$8   +$8   +$8   +$8   +$8

To decide which project to choose, we first compute and compare their NPVs. Assume the firm's required rate of return is 10 percent. We solve for the NPVs as follows:

1. NPV of Cheap Talk.

$$NPV_{CT} = \$5,800 \left[\frac{1 - \frac{1}{(1 + .10)^3}}{.10} \right] - \$10,000$$

$$= (\$5,800 \times 2.48685) - \$10,000$$

$$= \$14.424 - \$10,000$$

$$= \$4,424$$

2. NPV of Rolles Voice.

$$NPV_{Rolles} = \$8,000 \left[\frac{1 - \frac{1}{(1 + .10)^{12}}}{.10} \right] - \$50,000$$

$$= (\$8,000 \times 6.81369) - \$50,000$$

$$= \$54,510 - \$50,000$$

$$= \$4,510$$

We find that Project Cheap Talk has an NPV of $4,424, compared with Project Rolles' NPV of $4,510. We might conclude based on this information that the Rolles Voice System should be selected over the Cheap Talk system because it has the higher NPV. However, before making that decision, we must assess how Project Cheap Talk's NPV would change if its useful life were 12 years, not three years.

Comparing Projects with Unequal Lives

Two possible methods that allow financial managers to compare projects with unequal lives are the replacement chain approach and the equivalent annual annuity (EAA) approach.

The Replacement Chain Approach

The replacement chain approach assumes each of the mutually exclusive projects can be replicated, until a common time period has passed in which the projects can be compared. The NPVs for the series of replicated projects are then compared to the project with the longer life. An example illustrates this process. Project Cheap Talk could be repeated four times in the same time span as the 12-year Rolles Voice project. If a business replicated project Cheap Talk four times, the cash flows would look as depicted in Figure 10A-5.

The NPV of this series of cash flows, assuming the discount rate is 10 percent, is (in thousands) $12,121. Each cash flow, be it positive or negative, is discounted back the appropriate number of years to get the NPV of the four consecutive investments in the Cheap Talk system.

The NPV of $12,121 for Cheap Talk system is the sum of the NPVs of the four repeated Cheap Talk projects, such that the project series would have a life of 12 years, the same life as the Rolles Voice System Project. We are now comparing apples to apples. Cheap Talk's replacement chain NPV is $12,121, whereas the NPV of Project Rolles Voice is $4,510 over the same 12-year period. If a firm invested in project Cheap Talk four successive times, it would create more value than investing in one project Rolles Voice.

The Equivalent Annual Annuity (EAA)

The equivalent annual annuity (EAA) approach converts the NPVs of the mutually exclusive projects into their equivalent annuity values. The equivalent annual annuity is the amount of the annuity payment that would give the same present value as the actual future cash flows for that project. The EAA approach assumes that you could repeat the mutually exclusive projects indefinitely as each project came to the end of its life.

The equivalent annuity value (EAA) is calculated by dividing the NPV of a project by the present value interest factor for an annuity (PVIFA) that applies to the project's life span.

Formula for an Equivalent Annual Annuity (EAA)

$$EAA = \frac{NPV}{PVIFA_{k,\,n}}$$

(10A-1)

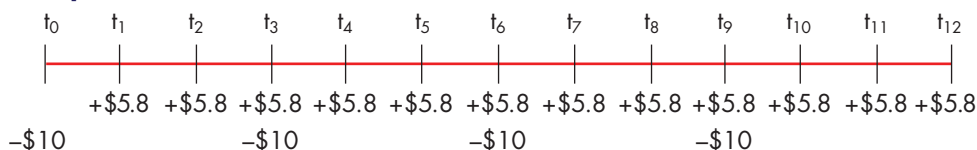**Cheap Talk**

Figure 10A-5   Cash Flows for the Cheap Talk System Repeated 4 times (in thousands)

where: k = Discount rate used to calculate the NPV
          n = Life span of the project

The NPVs of Cheap Talk ($4,424) and Rolles Voice ($4,510) were calculated earlier, assuming a required rate of return of 10 percent. With the project's NPV and the discount rate, we calculate each project's EAA, per Equation 10A-1, as follows:

1. EAA of Project Cheap Talk:

$$\text{EAA}_{\text{CT}} = \frac{\$4,424}{2.48685}$$

$$= \$1,778.96$$

2. EAA of Project Rolles

$$\text{EAA}_{\text{Rolles}} = \frac{\$4,510}{6.81369}$$

$$= \$661.90$$

Take Note

Note that the unequal lives problem is only an issue when the projects under consideration are mutually exclusive. If the projects were independent, we would adopt both.

The EAA approach decision rule calls for choosing whichever mutually exclusive project has the highest EAA. Our calculations show that Project Cheap Talk has an EAA of $1,778.96 and Project Rolles Voice System has an EAA of $661.90. Because Project Cheap Talk's EAA is higher than Project Rolles's, Project Cheap Talk should be chosen.

Both the replacement chain and the EAA approach assume that mutually exclusive projects can be replicated. If the projects can be replicated, then either the replacement chain or the equivalent annual annuity methods should be used because they lead to the same correct decision. Note in our case that the EAA method results in the same project selection (Project Cheap Talk) as the replacement chain method. If the projects cannot be replicated, then the normal NPVs should be used as the basis for the capital budgeting decision.

Equations Introduced in This Appendix

Equation 10A-1.   The Formula for an Equivalent Annual Annuity:

$$\text{EAA} = \frac{\text{NPV}}{\text{PVIFA}_{k,\,n}}$$

where: k = Discount rate used to calculate the NPV
          n = Life span of the project

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