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8 The Time Value of Money

"Time is money."

—Benjamin Franklin

Get a Free $1,000 Bond with Every Car Bought This Week!

There is a car dealer who appears on television regularly. He does his own commercials. He is quite loud and is also very aggressive. According to him you will pay way too much if you buy your car from anyone else in town. You might have a car dealer like this in your hometown.

One of the authors of this book used to watch and listen to the television commercials for a particular car dealer. One promotion struck him as being particularly interesting. The automobile manufacturers had been offering cash rebates to buyers of particular cars but this promotion had recently ended. This local car dealer seemed to be picking up where the manufacturers had left off. He was offering "a free $1,000 bond with every new car purchased" during a particular week. This sounded pretty good.

The fine print of this deal was not revealed until you were signing the final sales papers. Even then you had to look close since the print was small. It turns out the "$1,000 bond" offered to each car buyer was what is known as a "zero coupon bond." This means that there are no periodic interest payments. The buyer of the bond pays a price less than the face value of $1,000 and then at maturity the issuer pays $1,000 to the holder of the bond. The investor's return is entirely equal to the difference between the lower price paid for the bond and the $1,000 received at maturity.

How much less than $1,000 did the dealer have to pay to get this bond he was giving away? The amount paid by the dealer is what the bond would be worth (less after paying commissions) if the car buyer wanted to sell this bond now. It turns out that this bond had a maturity of 30 years. This is how

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Save up to 85%
1 Year - $12 Save 80%
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long the car buyer would have to wait to receive the $1,000. The value of the bond at the time the car was purchased was about $57. This is what the car dealer paid for each of these bonds and is the amount the car buyer would get from selling the bond. It's a pretty shrewd marketing gimmick when a car dealer can buy something for $57 and get the customers to believe they are receiving something worth $1,000.

In this chapter you will become armed against such deceptions. It's all in the time value of money.

Source: This is inspired by an actual marketing promotion. Some of the details have been changed, and all identities have been hidden, so the authors do not get sued.

Learning Objectives

After reading this chapter, you should be able to:

1. Explain the time value of money and its importance in the business world.
2. Calculate the future value and present value of a single amount.
3. Find the future and present values of an annuity.
4. Solve time value of money problems with uneven cash flows.
5. Solve for the interest rate, number or amount of payments, or the number of periods in a future or present value problem.

Chapter Overview

A dollar in hand today is worth more than a promise of a dollar tomorrow. This is one of the basic principles of financial decision making. Time value analysis is a crucial part of financial decisions. It helps answer questions about how much money an investment will make over time and how much a firm must invest now to earn an expected payoff later.

In this chapter we will investigate why money has time value, as well as learn how to calculate the future value of cash invested today and the present value of cash to be received in the future. We will also discuss the present and future values of an annuity—a series of equal cash payments at regular time intervals. Finally, we will examine special time value of money problems, such as how to find the rate of return on an investment and how to deal with a series of uneven cash payments.

Why Money Has Time Value

The time value of money means that money you hold in your hand today is worth more than the same amount of money you expect to receive in the future. Similarly, a given amount of money you must pay out today is a greater burden than the same amount paid in the future.

In Chapter 2 we learned that interest rates are positive in part because people prefer to consume now rather than later. Positive interest rates indicate, then, that money has time value. When one person lets another borrow money, the first person requires compensation in exchange for reducing current consumption. The person who borrows the money is willing to pay to increase current consumption. The cost paid by the borrower to the lender for reducing consumption, known as an *opportunity cost*, is the real rate of interest.

The real rate of interest reflects compensation for the **pure time value of money.** The real rate of interest does not include interest charged for expected inflation or the risk factors discussed in Chapter 2. Recall from the interest rate discussion in Chapter 2 that many factors—including the pure time value of money, inflation, default risk, illiquidity risk, and maturity risk—determine market interest rates.

The required rate of return on an investment reflects the pure time value of money, an adjustment for expected inflation, and any risk premiums present.

Measuring the Time Value of Money

Financial managers adjust for the time value of money by calculating the **future value** and the present value. Future value and present value are mirror images of each other. Future value is the value of a starting amount at a future point in time, given the rate of growth per period and the number of periods until that future time. How much will $1,000 invested today at a 10 percent interest rate grow to in 15 years? **Present value** is the value of a future amount today, assuming a specific required interest rate for a number of periods until that future amount is realized. How much should we pay today to obtain a promised payment of $1,000 in 15 years if investing money today would yield a 10 percent rate of return per year?

The Future Value of a Single Amount

To calculate the future value of a single amount, we must first understand how money grows over time. Once money is invested, it earns an interest rate that compensates for the time value of money and, as we learned in Chapter 2, for default risk, inflation, and other factors. Often, the interest earned on investments is compound interest—interest earned on interest and on the original principal. In contrast, *simple interest* is interest earned only on the original principal.

To illustrate compound interest, assume the financial manager of SaveCom decided to invest $100 of the firm's excess cash in an account that earns an annual interest rate of 5 percent. In one year, SaveCom will earn $5 in interest, calculated as follows:

$$
\begin{aligned}
\text{Balance at the end of year 1} &= \text{Principal} + \text{Interest}\\
&= \$100 + (100 \text{ ¥ } .05)\\
&= \$100 \text{ ¥ } (1 + .05)\\
&= \$100 \text{ ¥ } 1.05\\
&= \$105
\end{aligned}
$$

The total amount in the account at the end of year 1, then, is $105.

But look what happens in years 2 and 3. In year 2, SaveCom will earn 5 percent of 105. The $105 is the original principal of $100 plus the first year's interest—so the interest earned in year 2 is $5.25, rather than $5.00. The end of year 2 balance is $110.25—$100 in original principal and $10.25 in interest. In year 3, SaveCom will earn 5 percent of $110.25, or $5.51, for an ending balance of $115.76, shown as follows:

	Beginning Balance	×	(1 + Interest Rate)	=	Ending Balance	Interest
Year 1	$100.00	×	1.05	=	$105.00	$5.00
Year 2	$105.00	×	1.05	=	$110.25	$5.25
Year 3	$110.25	×	1.05	=	$115.76	$5.51

In our example, SaveCom earned $5 in interest in year 1, $5.25 in interest in year 2 ($110.25 – $105.00), and $5.51 in year 3 ($115.76 – $110.25) because of the compounding effect. If the SaveCom deposit earned interest only on the original principal, rather than on the principal and interest, the balance in the account at the end of year 3 would be $115 ($100 + ($5 × 3) = $115). In our case the compounding effect accounts for an extra $.76 ($115.76 – $115.00 = .76).

The simplest way to find the balance at the end of year 3 is to multiply the original principal by 1 plus the interest rate per period (expressed as a decimal), 1 + k, raised to the power of the number of compounding periods, n.[1] Here's the formula for finding the future value—or ending balance—given the original principal, interest rate per period, and number of compounding periods:

Future Value for a Single Amount
Algebraic Method

$$FV = PV \times (1 + k)^n \tag{8-1a}$$

where: FV = Future Value, the ending amount

PV = Present Value, the starting amount, or original principal

k = Rate of interest per period (expressed as a decimal)

n = Number of time periods

In our SaveCom example, PV is the original deposit of $100, k is 5 percent, and n is 3. To solve for the ending balance, or FV, we apply Equation 8-1a as follows:

[1]The compounding periods are usually years but not always. As you will see later in the chapter, compounding periods can be months, weeks, days, or any specified period of time.

$$\begin{aligned} FV &= PV \times (1 + k)^n \\ &= \$100 \times (1.05)^3 \\ &= \$100 \times 1.1576 \\ &= \$115.76 \end{aligned}$$

We may also solve for future value using a financial table. Financial tables are a compilation of values, known as interest factors, that represent a term, $(1 + k)^n$ in this case, in time value of money formulas. Table I in the Appendix at the end of the book is developed by calculating $(1 + k)^n$ for many combinations of k and n.

Table I in the Appendix at the end of the book is the future value interest factor (FVIF) table. The formula for future value using the FVIF table follows:

Future Value for a Single Amount
Table Method

$$FV = PV \times \left(FVIF_{k,\,n}\right) \tag{8-1b}$$

where: FV = Future Value, the ending amount

PV = Present Value, the starting amount

$FVIF_{k,n}$ = Future Value Interest Factor given interest rate, k, and number of periods, n, from Table I

In our SaveCom example, in which $100 is deposited in an account at 5 percent interest for three years, the ending balance, or FV, according to Equation 8-1b, is as follows:

$$\begin{aligned} FV &= PV \times \left(FVIF_{k,\,n}\right) \\ &= \$100 \times \left(FVIF_{5\%,\,3}\right) \\ &= \$100 \times 1.1576 \text{ (from the FVIF table)} \\ &= \$115.76 \end{aligned}$$

Take Note
Because future value interest factors are rounded to four decimal places in Table I, you may get a slightly different solution compared to a problem solved by the algebraic method.

To solve for FV using a financial calculator, we enter the numbers for PV, n, and k (k is depicted as I/Y on the TI BAII PLUS calculator; on other calculators it may be symbolized by i or I), and ask the calculator to compute FV. The keystrokes follow.

TI BAII PLUS Financial Calculator Solution

Step 1: First press [2nd] [CLR TVM]. This clears all the time value of money keys of all previously calculated or entered values.

Step 2: Press [2nd] [P/Y] 1 [ENTER], [2nd] [QUIT]. This sets the calculator to the mode where one payment per year is expected, which is the assumption for the problems in this chapter.

Step 3: Input values for principal (PV), interest rate (k or I/Y on the calculator), and number of periods (n).

100 [+/–] [PV] 5 [I/Y] 3 [N] [CPT] [FV] **Answer: 115.76**

In the SaveCom example, we input –100 for the present value (PV), 3 for number of periods (N), and 5 for the interest rate per year (I/Y). Then we ask the calculator to compute the future value, FV. The result is $115.76. Our TI BAII PLUS is set to display two decimal places. You may choose a greater or lesser number if you wish.

We have learned three ways to calculate the future value of a single amount: the algebraic method, the financial table method, and the financial calculator method. In the next section, we see how future values are related to changes in the interest rate, k, and the number of time periods, n.

The Sensitivity of Future Values to Changes in Interest Rates or the Number of Compounding Periods

Future value has a positive relationship with the interest rate, k, and with the number of periods, n. That is, as the interest rate increases, future value increases. Similarly, as the number of periods increases, so does future value. In contrast, future value decreases with decreases in k and n values.

It is important to understand the sensitivity of future value to k and n because increases are exponential, as shown by the $(1 + k)^n$ term in the future value formula. Consider this: A business that invests $10,000 in a savings account at 5 percent for 20 years will have a future value of $26,532.98. If the interest rate is 8 percent for the same 20 years, the future value of the investment is $46,609.57. We see that the future value of the investment increases as k increases. Figure 8-1a shows this graphically.

Now let's say that the business deposits $10,000 for 10 years at a 5 percent annual interest rate. The future value of that sum is $16,288.95. Another business deposits $10,000 for 20 years at the same 5 percent annual interest rate. The future value of that $10,000 investment is $26,532.98. Just as with the interest rate, the higher the number of periods, the higher the future value. Figure 8-1b shows this graphically.

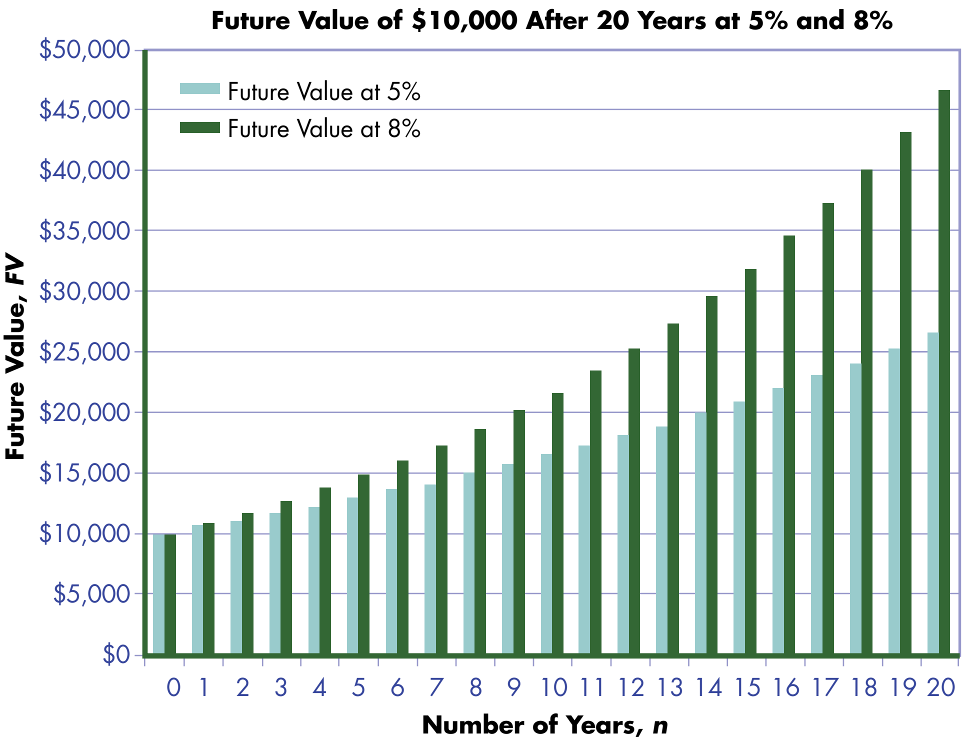

Figure 8-1a Future Value at Different Interest Rates

Figure 8-1a shows the future value of $10,000 after 20 years at interest rates of 5 percent and 8 percent.

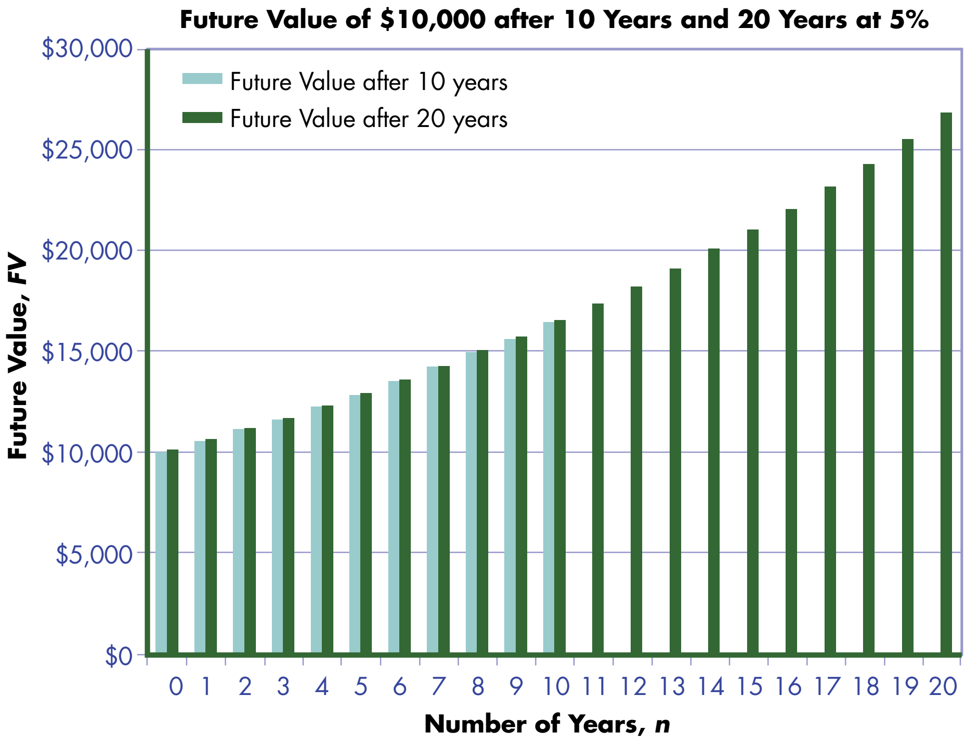

Figure 8-1b Future Value at Different Times

Figure 8-1b shows the future value of $10,000 after 10 years and 20 years at an interest rate of 5 percent

The Present Value of a Single Amount

Present value is today's dollar value of a specific future amount. With a bond, for instance, the issuer promises the investor future cash payments at specified points in time. With an investment in new plant or equipment, certain cash receipts are expected. When we calculate the present value of a future promised or expected cash payment, we discount it (mark it down in value) because it is worth less if it is to be received later rather than now. Similarly, future cash outflows are less burdensome than present cash outflows of the same amount. Future cash outflows are similarly discounted (made less negative). In present value analysis, then, the interest rate used in this discounting process is known as the discount rate. The discount rate is the required rate of return on an investment. It reflects the lost opportunity to spend or invest now (the opportunity cost) and the various risks assumed because we must wait for the funds.

Discounting is the inverse of compounding. Compound interest causes the value of a beginning amount to increase at an increasing rate. Discounting causes the present value of a future amount to decrease at an increasing rate.

To demonstrate, imagine the SaveCom financial manager needed to know how much to invest now to generate $115.76 in three years, given an interest rate of 5 percent. Given what we know so far, the calculation would look like this:

$$\begin{aligned} FV &= PV \text{ ¥ } (1 + k)^n \\ \$115.76 &= PV \text{ ¥ } 1.05^3 \\ \$115.76 &= PV \text{ ¥ } 1.157625 \\ PV &= \$100.00 \end{aligned}$$

To simplify solving present value problems, we modify the future value for a single amount equation by multiplying both sides by $1/(1 + k)^n$ to isolate PV on one side of the equal sign. The present value formula for a single amount follows:

The Present Value of a Single Amount Formula
Algebraic Method

$$PV = FV \text{ ¥ } \frac{1}{(1 + k)^n} \tag{8-2a}$$

where: PV = Present Value, the starting amount

FV = Future Value, the ending amount

k = Discount rate of interest per period (expressed as a decimal)

n = Number of time periods

Interactive Module

Go to the Interactive Spreadsheets you downloaded for chapter 8. Follow the instructions there. Look at the 3-D graph. See what happens to FV and to PV as k and n values change.

Applying this formula to the SaveCom example, in which its financial manager wanted to know how much the firm should pay today to receive \$115.76 at the end of three years, assuming a 5 percent discount rate starting today, the following is the present value of the investment:

$$PV = FV \text{ ¥ } \frac{1}{(1 + k)^n}$$

$$= \$115.76 \text{ ¥ } \frac{1}{(1 + .05)^3}$$

$$= \$115.76 \text{ ¥ } .86384$$

$$= \$100.00$$

SaveCom should be willing to pay \$100 today to receive \$115.76 three years from now at a 5 percent discount rate.

To solve for PV, we may also use the Present Value Interest Factor Table in Table II in the Appendix at the end of the book. A present value interest factor, or PVIF, is calculated and shown in Table II. It equals $1/(1 + k)^n$ for given combinations of k and n. The table method formula, Equation 8-2b, follows:

The Present Value of a Single Amount Formula
Table Method

$$PV = FV \times (PVIF_{k,\,n}) \tag{8-2b}$$

where: PV = Present Value

FV = Future Value

$PVIF_{k,n}$ = Present Value Interest Factor given discount rate, k, and number of periods, n, from Table II

In our example, SaveCom's financial manager wanted to solve for the amount that must be invested today at a 5 percent interest rate to accumulate \$115.76 within three years. Applying the present value table formula, we find the following solution:

$$\begin{aligned} PV &= FV \text{ ¥ } (PVIF_{k,\,n}) \\ &= \$115.76 \text{ ¥ } (PVIF_{5\%,\,3}) \\ &= \$115.76 \times .8638 \text{ (from the PVIF table)} \\ &= \$99.99 \text{ (slightly lower than \$100 due to the rounding to four places in the table)} \end{aligned}$$

The present value of \$115.76, discounted back three years at a 5 percent discount rate, is \$100.

To solve for present value using a financial calculator, enter the numbers for future value, FV, the number of periods, n, and the interest rate, k—symbolized as I/Y on the calculator—then hit the CPT (compute) and PV (present value) keys. The sequence follows:

TI BAII PLUS Financial Calculator Solution

Step 1: Press [2nd] [CLR TVM] to clear previous values.

Step 2: Press [2nd] [P/Y] 1 [ENTER], [2nd] [QUIT] to ensure the calculator is in the mode for annual interest payments.

Step 3: Input the values for future value, the interest rate, and number of periods, and compute PV.

115.76 [PV] 5 [I/Y] 3 [N] [CPT] [PV] **Answer: –100.00**

The financial calculator result is displayed as a negative number to show that the present value sum is a cash outflow—that is, that sum will have to be invested to earn \$115.76 in three years at a 5 percent annual interest rate.

We have examined how to find present value using the algebraic, table, and financial calculator methods. Next, we see how present value analysis is affected by the discount rate, k, and the number of compounding periods, n.

The Sensitivity of Present Values to Changes in the Interest Rate or the Number of Compounding Periods

In contrast with future value, present value is inversely related to k and n values. In other words, present value moves in the opposite direction of k and n. If k increases, present value decreases; if k decreases, present value increases. If n increases, present value decreases; if n decreases, present value increases.

Consider this: A business that expects a 5 percent annual return on its investment (k = 5%) should be willing to pay \$3,768.89 today (the present value) for \$10,000 to be received 20 years from now. If the expected annual return is 8 percent for the same 20 years, the present value of the investment is only \$2,145.48. We see that the present value of the investment decreases as k increases. The way the present value of the \$10,000 varies with changes in the required rate of return is shown graphically in Figure 8-2a.

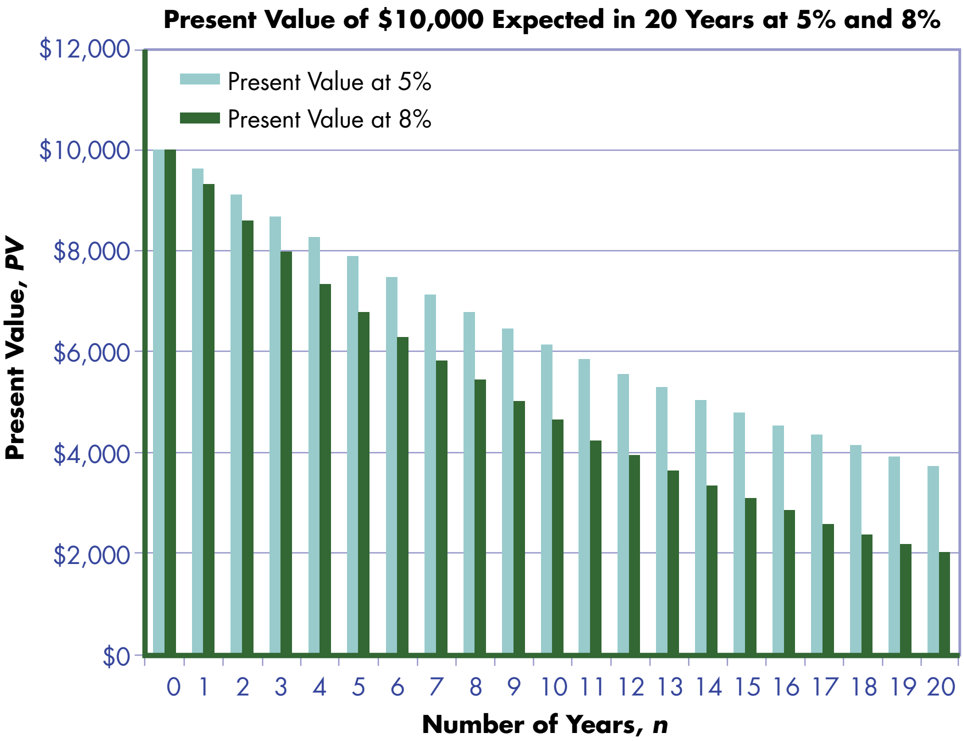

Figure 8-2a Present Value at Different Interest Rates

Figure 8-2a shows the present value of $10,000 to be received in 20 years at interest rates of 5 percent and 8 percent.

Now let's say that a business expects to receive $10,000 ten years from now. If its required rate of return for the investment is 5 percent annually, then it should be willing to pay $6,139 for the investment today (the present value is $6,139). If another business expects to receive $10,000 twenty years from now and it has the same 5 percent annual required rate of return, then it should be willing to pay $3,769 for the investment (the present value is $3,769). Just as with the interest rate, the greater the number of periods, the lower the present value. Figure 8-2b shows how it works.

In this section we have learned how to find the future value and the present value of a single amount. Next, we will examine how to find the future value and present value of several amounts.

Working with Annuities

Financial managers often need to assess a series of cash flows rather than just one. One common type of cash flow series is the **annuity**—a series of equal cash flows, spaced evenly over time.

Professional athletes often sign contracts that provide annuities for them after they retire, in addition to the signing bonus and regular salary they may receive during their playing years. Consumers can purchase annuities from insurance companies as a means of providing retirement income. The investor pays the insurance company a lump sum now in order to receive future payments of equal size at regularly spaced time intervals (usually monthly). Another example of an annuity is the interest on a bond. The interest payments are usually equal dollar amounts paid either annually or semiannually during the life of the bond.

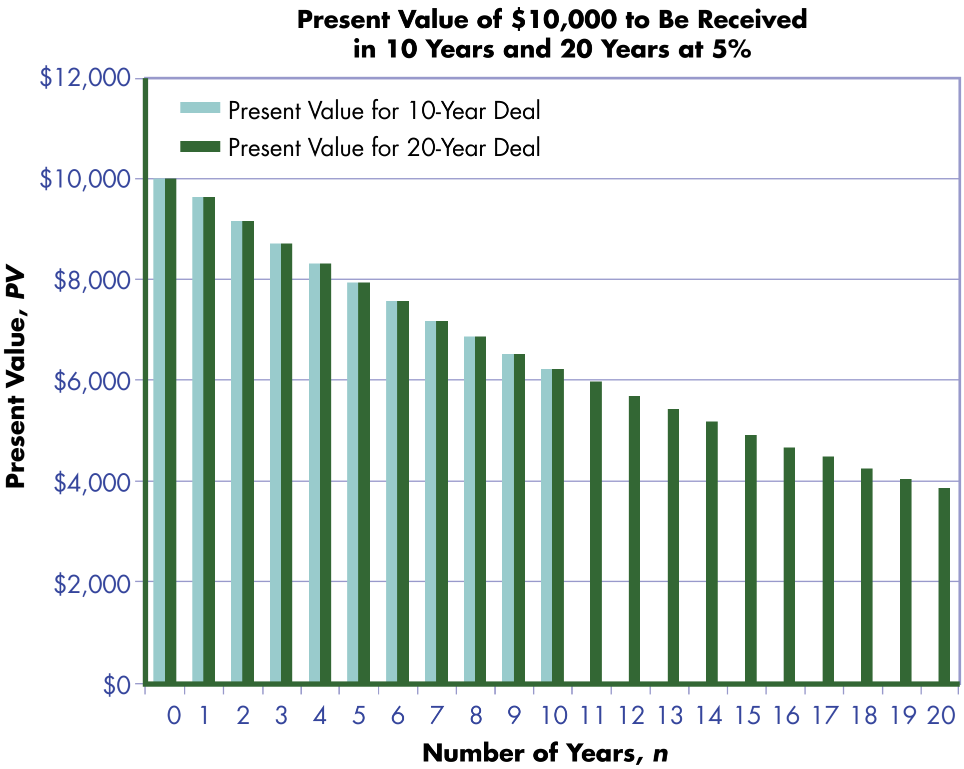

Figure 8-2b Present Value at Different Times

Figure 8-2b shows the present value of $10,000 to be received in 10 years and 20 years at interest rates of 5 percent and 8 percent.

Annuities are a significant part of many financial problems. You should learn to recognize annuities and determine their value, future or present. In this section we will explain how to calculate the future value and present value of annuities in which cash flows occur at the end of the specified time periods. Annuities in which the cash flows occur at the end of each of the specified time periods are known as *ordinary annuities*. Annuities in which the cash flows occur at the beginning of each of the specified time periods are known as *annuities due*.

Future Value of an Ordinary Annuity

Financial managers often plan for the future. When they do, they often need to know how much money to save on a regular basis to accumulate a given amount of cash at a specified future time. The future value of an annuity is the amount that a given number of annuity payments, n, will grow to at a future date, for a given periodic interest rate, k.

For instance, suppose the SaveCom Company plans to invest $500 in a money market account at the end of each year for the next four years, beginning one year from today. The business expects to earn a 5 percent annual rate of return on its investment. How much money will SaveCom have in the account at the end of four years? The problem is illustrated in the timeline in Figure 8-3. The t values in the timeline represent the end of each time period. Thus, t_1 is the end of the first year, t_2 is the end of the second year, and so on. The symbol t_0 is now, the present point in time.

Because the $500 payments are each single amounts, we can solve this problem one step at a time. Looking at Figure 8-4, we see that the first step is to calculate the future value of the cash flows that occur at t_1, t_2, t_3, and t_4 using the future value formula

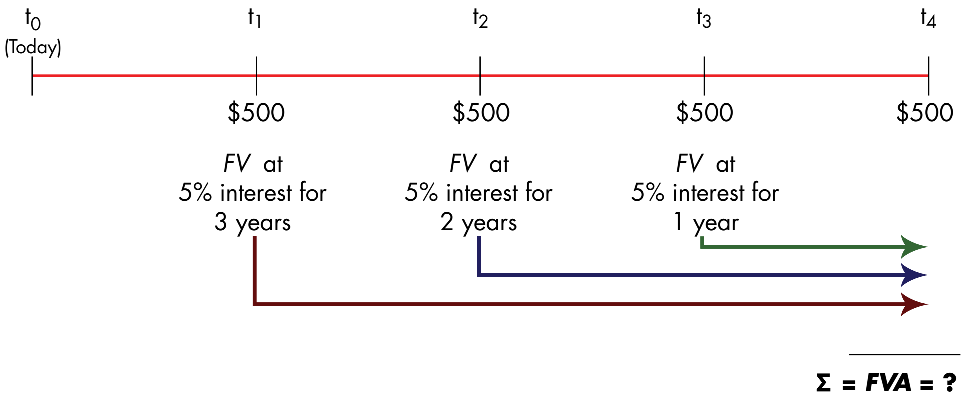

Figure 8-3 SaveCom Annuity Timeline

for a single amount. The next step is to add the four values together. The sum of those values is the annuity's future value.

As shown in Figure 8-4, the sum of the future values of the four single amounts is the annuity's future value, $2,155.05. However, the step-by-step process illustrated in Figure 8-4 is time-consuming even in this simple example. Calculating the future value of a 20- or 30-year annuity, such as would be the case with many bonds, would take an enormous amount of time. Instead, we can calculate the future value of an annuity easily by using the following formula:

Future Value of an Annuity
Algebraic Method

$$FVA = PMT \times \left[\frac{(1+k)^n - 1}{k}\right] \tag{8-3a}$$

where: FVA = Future Value of an Annuity

PMT = Amount of each annuity payment

k = Interest rate per time period expressed as a decimal

n = Number of annuity payments

Using Equation 8-3a in our SaveCom example, we solve for the future value of the annuity at 5 percent interest (k = 5%) with four $500 end-of-year payments (n = 4 and PMT = $500), as follows:

$$\begin{aligned} FVA &= 500 \times \left[\frac{(1+.05)^4 - 1}{.05}\right] \\ &= 500 \times 4.3101 \\ &= \$2{,}155.05 \end{aligned}$$

For a $500 annuity with a 5 percent interest rate and four annuity payments, we see that the future value of the SaveCom annuity is $2,155.05.

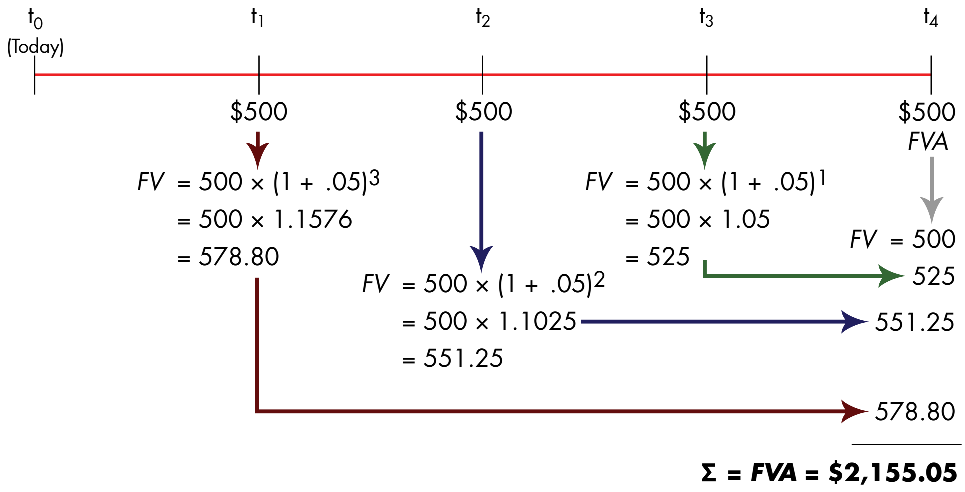

Figure 8-4 Future Value of the SaveCom Annuity

To find the future value of an annuity with the table method, we must find the **future value interest factor for an annuity** (FVIFA), found in Table III in the Appendix at the end of the book. The $FVIFA_{k,\,n}$ is the value of $[(1 + k)^n - 1] \div k$ for different combinations of k and n.

Future Value of an Annuity Formula
Table Method

$$FVA = PMT \times FVIFA_{k,\,n} \tag{8-3b}$$

where: FVA = Future Value of an Annuity

PMT = Amount of each annuity payment

$FVIFA_{k,\,n}$ = Future Value Interest Factor for an Annuity from Table III

k = Interest rate per period

n = Number of annuity payments

In our SaveCom example, then, we need to find the FVIFA for a discount rate of 5 percent with four annuity payments. Table III in the Appendix shows that the $FVIFA_{k\,=\,5\%,\;n\,=\,4}$ is 4.3101. Using the table method, we find the following future value of the SaveCom annuity:

$$\begin{aligned} FVA &= 500 \times FVIFA_{5\%,\,4} \\ &= 500 \times 4.3101 \text{ (from the FVIFA table)} \\ &= \$2{,}155.05 \end{aligned}$$

To find the future value of an annuity using a financial calculator, key in the values for the annuity payment (PMT), n, and k (remember that the notation for the interest rate on the TI BAII PLUS calculator is I/Y, not k). Then compute the future value of the annuity (FV on the calculator). For a series of four $500 end-of-year (ordinary annuity) payments where n = 4 and k = 5 percent, the computation is as follows:

TI BAII PLUS Financial Calculator Solution

Step 1: Press [2nd] [CLR TVM] to clear previous values.

Step 2: Press [2nd] [P/Y] 1 [ENTER], [2nd] [BGN] [2nd] [SET] [2nd] [SET]. Repeat [2nd] [SET] until END shows in the display [2nd] [QUIT] to set the annual interest rate mode and to set the annuity payment to end of period mode.

Step 3: Input the values and compute.

0 [PV] 5 [I/Y] 4 [N] 500 [+/–] [PMT] [CPT] [FV] **Answer: 2,155.06**

In the financial calculator inputs, note that the payment is keyed in as a negative number to indicate that the payments are cash outflows—the payments flow out from the company into an investment.

> **Take Note**
> Note that slight differences occur between the table method, algebraic method, and calculator solution. This is because our financial tables round interest factors to four decimal places, whereas the other methods generally use many more significant figures for greater accuracy.

The Present Value of an Ordinary Annuity

Because annuity payments are often promised (as with interest on a bond investment) or expected (as with cash inflows from an investment in new plant or equipment), it is important to know how much these investments are worth to us today. For example, assume that the financial manager of Buy4Later, Inc. learns of an annuity that promises to make four annual payments of $500, beginning one year from today. How much should the company be willing to pay to obtain that annuity? The answer is the present value of the annuity.

Because an annuity is nothing more than a series of equal single amounts, we could calculate the present value of an annuity with the present value formula for a single amount and sum the totals, but that would be a cumbersome process. Imagine calculating the present value of a 50-year annuity! We would have to find the present value for each of the 50 annuity payments and total them.

Fortunately, we can calculate the present value of an annuity in one step with the following formula:

The Present Value of an Annuity Formula
Algebraic Method

$$PVA = PMT \times \left[\frac{1 - \dfrac{1}{(1+k)^n}}{k} \right] \tag{8-4a}$$

where: PVA = Present Value of an Annuity

PMT = Amount of each annuity payment

k = Discount rate per period expressed as a decimal

n = Number of annuity payments

Using our example of a four-year ordinary annuity with payments of $500 per year and a 5 percent discount rate, we solve for the present value of the annuity as follows:

$$PVA = 500 \times \left[\frac{1 - \frac{1}{(1+.05)^4}}{.05} \right]$$

$$= 500 \times 3.54595$$

$$= \$1,772.97$$

The present value of the four-year ordinary annuity with equal yearly payments of $500 at a 5 percent discount rate is $1,772.97.

We can also use the financial table for the present value interest factor for an annuity (PVIFA) to solve present value of annuity problems. The PVIFA table is found in Table IV in the Appendix at the end of the book. The formula for the table method follows:

The Present Value of an Annuity Formula
Table Method

$$PVA = PMT \times PVIFA_{k,\,n} \tag{8-4b}$$

where: PVA = Present Value of an Annuity

PMT = Amount of each annuity payment

$PVIFA_{k,\,n}$ = Present Value Interest Factor for an Annuity from Table IV

k = Discount rate per period

n = Number of annuity payments

Applying Equation 8-4b, we find that the present value of the four-year annuity with $500 equal payments and a 5 percent discount rate is as follows:[2]

$$PVA = 500 \times PVIFA_{5\%,\,4}$$

$$= 500 \times 3.5460$$

$$= \$1,773.00$$

We may also solve for the present value of an annuity with a financial calculator. Simply key in the values for the payment, PMT, number of payment periods, n, and the interest rate, k—symbolized by I/Y on the TI BAII PLUS calculator—and ask the calculator to compute PVA (PV on the calculator). For the series of four $500 payments where n = 4 and k = 5 percent, the computation follows:

[2]The $.03 difference between the algebraic result and the table formula solution is due to differences in rounding.

TI BAII PLUS Financial Calculator Solution

Step 1: Press 2nd CLR TVM to clear previous values.

Step 2: Press 2nd P/Y 1 ENTER, 2nd BGN 2nd SET 2nd SET. Repeat 2nd SET until END shows in the display 2nd QUIT to set the annual interest rate mode and to set the annuity payment to end of period mode.

Step 3: Input the values and compute.

5 I/Y 4 N 500 PMT CPT PV **Answer: –1,772.97**

The financial calculator present value result is displayed as a negative number to signal that the present value sum is a cash outflow—that is, $1,772.97 will have to be invested to earn a 5 percent annual rate of return on the four future annual annuity payments of $500 each to be received.

Future and Present Values of Annuities Due

Sometimes we must deal with annuities in which the annuity payments occur at the beginning of each period. These are known as annuities due, in contrast to ordinary annuities in which the payments occurred at the end of each period, as described in the preceding section.

Annuities due are more likely to occur when doing future value of annuity (FVA) problems than when doing present value of annuity (PVA) problems. Today, for instance, you may start a retirement program, investing regular equal amounts each month or year. Calculating the amount you would accumulate when you reach retirement age would be a future value of an annuity due problem. Evaluating the present value of a promised or expected series of annuity payments that began today would be a present value of an annuity due problem. This is less common because car and mortgage payments almost always start at the end of the first period making them ordinary annuities.

Whenever you run into an FVA or a PVA of an annuity due problem, the adjustment needed is the same in both cases. Use the FVA or PVA of an ordinary annuity formula shown earlier, then multiply your answer by $(1 + k)$. We multiply the FVA or PVA formula by $(1 + k)$ because annuities due have annuity payments earning interest one period sooner. So, higher FVA and PVA values result with an annuity due. The first payment occurs sooner in the case of a future value of an annuity due. In present value of annuity due problems, each annuity payment occurs one period sooner, so the payments are discounted less severely.

In our SaveCom example, the future value of a $500 ordinary annuity, with $k = 5\%$ and $n = 4$, was $2,155.06. If the $500 payments occurred at the beginning of each period instead of at the end, we would multiply $2,155.06 by 1.05 $(1 + k = 1 + .05)$. The product, $2,262.81, is the future value of the annuity due. In our earlier Buy4Later, Inc., example, we found that the present value of a $500 ordinary annuity, with $k = 5\%$ and $n = 4$, was $1,772.97. If the $500 payments occurred at the beginning of each period instead of at the end, we would multiply $1,772.97 by 1.05 $(1 + k = 1 + .05)$ and find that the present value of Buy4Later's annuity due is $1,861.62.

The financial calculator solutions for these annuity due problems are shown next.

Future Value of a Four-Year, $500 Annuity Due, k = 5%

TI BAII PLUS Financial Calculator Solution

Step 1: Press [2nd] [CLR TVM] to clear previous values.

Step 2: Press [2nd] [P/Y] 1 [ENTER], [2nd] [BGN] [2nd] [SET] [2nd] [SET]. Repeat [2nd] [SET] command until the display shows BGN, [2nd] [QUIT] to set to the annual interest rate mode and to set the annuity payment to beginning of period mode.

Step 3: Input the values for the annuity due and compute.

5 [I/Y] 4 [N] 500 [PMT] [CPT] [FV] **Answer: –2,262.82**

Present Value of a Four-Year, $500 Annuity Due, k = 5%

TI BAII PLUS Financial Calculator Solution

Step 1: Press [2nd] [CLR TVM] to clear previous values.

Step 2: Press [2nd] [P/Y] 1 [ENTER], [2nd] [BGN] [2nd] [SET] [2nd] [SET]. Repeat [2nd] [SET] command until the display shows BGN, [2nd] [QUIT] to set to the annual interest rate mode and to set the annuity payment to beginning of period mode.

Step 3: Input the values for the annuity due and compute.

5 [I/Y] 4 [N] 500 [PMT] [CPT] [PV] **Answer: –1,861.62**

In this section we discussed ordinary annuities and annuities due and learned how to compute the present and future values of the annuities. Next, we will learn what a perpetuity is and how to solve for its present value.

Perpetuities

An annuity that goes on forever is called a perpetual annuity or a perpetuity. Perpetuities contain an infinite number of annuity payments. An example of a perpetuity is the dividends typically paid on a preferred stock issue.

The future value of perpetuities cannot be calculated, but the present value can be. We start with the present value of an annuity formula, Equation 8-3a.

$$PVA = PMT \times \left[\frac{1 - \frac{1}{(1+k)^n}}{k} \right]$$

Now imagine what happens in the equation as the number of payments (n) gets larger and larger. The $(1 + k)^n$ term will get larger and larger, and as it does, it will cause the $1/(1 + k)^n$ fraction to become smaller and smaller. As n approaches infinity, the $(1 + k)^n$ term becomes infinitely large, and the $1/(1 + k)^n$ term approaches zero. The entire formula reduces to the following equation:

Present Value of Perpetuity

$$PVP = PMT \times \left[\frac{1 - 0}{k}\right]$$

or

$$PVP = PMT \times \left(\frac{1}{k}\right) \tag{8-5}$$

where: PVP = Present Value of a Perpetuity

k = Discount rate expressed as a decimal

Neither the table method nor the financial calculator can solve for the present value of a perpetuity. This is because the PVIFA table does not contain values for infinity and the financial calculator does not have an infinity key.

Suppose you had the opportunity to buy a share of preferred stock that pays $70 per year forever. If your required rate of return is 8 percent, what is the present value of the promised dividends to you? In other words, given your required rate of return, how much should you be willing to pay for the preferred stock? The answer, found by applying Equation 8-5, follows:

$$\begin{aligned} PVP &= PMT \times \left(\frac{1}{k}\right) \\ &= \$70 \times \left(\frac{1}{.08}\right) \\ &= \$875 \end{aligned}$$

The present value of the preferred stock dividends, with a k of 8 percent and a payment of $70 per year forever, is $875.

Present Value of an Investment with Uneven Cash Flows

Unlike annuities that have equal payments over time, many investments have payments that are unequal over time. That is, some investments have payments that vary over time. When the periodic payments vary, we say that the cash flow streams are uneven. For instance, a professional athlete may sign a contract that provides for an immediate $7 million signing bonus, followed by a salary of $2 million in year 1, $4 million in year 2, then $6 million in years 3 and 4. What is the present value of the promised payments that total $25 million? Assume a discount rate of 8 percent. The present value calculations are shown in Table 8-1.

As we see from Table 8-1, we calculate the present value of an uneven series of cash flows by finding the present value of a single amount for each series and summing the totals.

Table 8-1 The Present Value of an Uneven Stream of Cash Flows

Time	Cash Flow	PV of Cash Flow
t_0	$7,000,000	$7,000,000 ¥ $\frac{1}{1.08^0}$ = $7,000,000
t_1	$2,000,000	$2,000,000 ¥ $\frac{1}{1.08^1}$ = $1,851,852
t_2	$4,000,000	$4,000,000 ¥ $\frac{1}{1.08^2}$ = $3,429,355
t_3	$6,000,000	$6,000,000 ¥ $\frac{1}{1.08^3}$ = $4,762,993
t_4	$6,000,000	$6,000,000 ¥ $\frac{1}{1.08^4}$ = $4,410,179
		Sum of the PVs = $21,454,380

We can also use a financial calculator to find the present value of this uneven series of cash flows. The worksheet mode of the TI BAII PLUS calculator is especially helpful in solving problems with uneven cash flows. The C display shows each cash payment following CF_0, the initial cash flow. The F display key indicates the frequency of that payment. The keystrokes follow.

TI BAII PLUS Financial Calculator PV Solution
Uneven Series of Cash Flows

Keystrokes	Display
CF	CF_0 = old contents
2nd CLR Work	CF_0 = 0.00
7000000 ENTER	7,000,000.00
↓ 2000000 ENTER	C01 = 2,000,000.00
↓	F01 = 1.00
↓ 4000000 ENTER	C02 = 4,000,000.00
↓	F02 = 1.00
↓ 6000000 ENTER	C03 = 6,000,000.00
↓ 2 ENTER	F03 = 2.00
NPV	I = 0.00
8 ENTER	I = 8.00
↓ CPT	NPV = 21,454,379.70

Take Note

We used the NPV (net present value) key on our calculator to solve this problem. NPV will be discussed in Chapter 10.

We see from the calculator keystrokes that we are solving for the present value of a single amount for each payment in the series except for the last two payments, which are the same. The value of F03, the frequency of the third cash flow after the initial cash flow, was 2 instead of 1 because the $6 million payment occurred twice in the series (in years 3 and 4).

We have seen how to calculate the future value and present value of annuities, the present value of a perpetuity, and the present value of an investment with uneven cash flows. Now we turn to time value of money problems in which we solve for k, n, or the annuity payment.

Special Time Value of Money Problems

Financial managers often face time value of money problems even when they know both the present value and the future value of an investment. In those cases, financial managers may be asked to find out what return an investment made—that is, what the interest rate is on the investment. Still other times financial managers must find either the number of payment periods or the amount of an annuity payment. In the next section, we will learn how to solve for k and n. We will also learn how to find the annuity payment (PMT).

Finding the Interest Rate

Financial managers frequently have to solve for the interest rate, k, when firms make a long-term investment. The method of solving for k depends on whether the investment is a single amount or an annuity.

Finding k of a Single-Amount Investment Financial managers may need to determine how much periodic return an investment generated over time. For example, imagine that you are head of the finance department of GrabLand, Inc. Say that GrabLand purchased a house on prime land 20 years ago for $40,000. Recently, GrabLand sold the property for $106,131. What average annual rate of return did the firm earn on its 20-year investment?

First, the future value—or ending amount—of the property is $106,131. The present value—the starting amount—is $40,000. The number of periods, n, is 20. Armed with those facts, you could solve this problem using the table version of the future value of a single amount formula, Equation 8-1b, as follows:

$$\text{FV} = \text{PV} \text{ ¥ } (\text{FVIF}_{k,\,n})$$

$$\$106{,}131 = \$40{,}000 \text{ ¥ } (\text{FVIF}_{k\,=\,?,\;n\,=\,20})$$

$$\$106{,}131 \div \$40{,}000 = \text{FVIF}_{k\,=\,?,\;n\,=\,20}$$

$$2.6533 = \text{FVIF}_{k\,=\,?,\;n\,=\,20}$$

Now find the FVIF value in Table I, shown in part on page 208. The whole table is in the Appendix at the end of the book. You know n = 20, so find the n = 20 row on the left-hand side of the table. You also know that the FVIF value is 2.6533, so move across the n = 20 row until you find (or come close to) the value 2.6533. You find the 2.6533 value in the k = 5% column. You discover, then, that GrabLand's property investment had an interest rate of 5 percent.

Future Value Interest Factors, Compounded at k Percent for n Periods, Part of Table I

Number of Periods, n	Interest Rate, k 0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
18	1.0000	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599
19	1.0000	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159
20	1.0000	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275
25	1.0000	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.8347

Solving for k using the FVIF table works well when the interest rate is a whole number, but it does not work well when the interest rate is not a whole number. To solve for the interest rate, k, we rewrite the algebraic version of the future value of a single-amount formula, Equation 8-1a, to solve for k:

The Rate of Return, k

$$k = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1 \tag{8-6}$$

where: k = Rate of return expressed as a decimal

FV = Future Value

PV = Present Value

n = Number of compounding periods

Let's use Equation 8-6 to find the average annual rate of return on GrabLand's house investment. Recall that the company bought it 20 years ago for $40,000 and sold it recently for $106,131. We solve for k applying Equation 8-6 as follows:

$$\begin{aligned} k &= \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1 \\ &= \left(\frac{\$106{,}131}{\$40{,}000}\right)^{\frac{1}{20}} - 1 \\ &= 2.653275^{.05} - 1 \\ &= 1.05 - 1 \\ &= .05, \text{ or } 5\% \end{aligned}$$

Equation 8-6 will find any rate of return or interest rate given a starting value, PV, an ending value, FV, and a number of compounding periods, n.

To solve for k with a financial calculator, key in all the other variables and ask the calculator to compute k (depicted as I/Y on your calculator). For GrabLand's house-buying example, the calculator solution follows:

TI BAII PLUS Financial Calculator Solution

Step 1: Press **2nd** **CLR TVM** to clear previous values.

Step 2: Press **2nd** **P/Y** 1 **ENTER**, **2nd** **QUIT**.

Step 3: Input the values and compute.

40000 **+/–** **PV** 106131 **FV** 20 **N** **CPT** **I/Y** **Answer: 5.00**

Remember when using the financial calculator to solve for the rate of return, you must enter cash outflows as a negative number. In our example, the $40,000 PV is entered as a negative number because GrabLand spent that amount to invest in the house.

Finding k for an Annuity Investment Financial managers may need to find the interest rate for an annuity investment when they know the starting amount (PVA), n, and the annuity payment (PMT), but they do not know the interest rate, k. For example, suppose GrabLand wanted a 15-year, $100,000 amortized loan from a bank. An amortized loan is a loan that is paid off in equal amounts that include principal as well as interest.[3] According to the bank, GrabLand's payments will be $12,405.89 per year for 15 years. What interest rate is the bank charging on this loan?

To solve for k when the known values are PVA (the $100,000 loan proceeds), n (15), and PMT (the loan payments $12,405.89), we start with the present value of an annuity formula, Equation 8-3b, as follows:

$$\text{PVA} = \text{PMT} \times \left(\text{PVIFA}_{k,\,n}\right)$$

$$\$100{,}000 = \$12{,}405.89 \times \left(\text{PVIFA}_{k\,=\,?,\ n\,=\,20}\right)$$

$$8.0607 = \text{PVIFA}_{k\,=\,?,\ n\,=\,20}$$

Now refer to the PVIFA values in Table IV, shown in part below. The whole table is in the Appendix at the end of the book. You know n = 15, so find the n = 15 row on the left-hand side of the table. You have also determined that the PVIFA value is 8.0607 ($100,000/$12,405 = 8.0607), so move across the n = 15 row until you find (or come close to) the value of 8.0607. In our example, the location on the table where n = 15 and the PVIFA is 8.0607 is in the k = 9% column, so the interest rate on GrabLand's loan is 9 percent.

Present Value Interest Factors for an Annuity, Discounted at k Percent for n Periods, Part of Table IV

Number of Periods, n	Interest Rate, k										
	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
13	13.0000	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034
14	14.0000	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667
15	15.0000	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061
16	16.0000	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237

[3]Amortize comes from the Latin word *mortalis*, which means "death." You will kill off the entire loan after making the scheduled payments.

To solve this problem with a financial calculator, key in all the variables but k, and ask the calculator to compute k (depicted as I/Y on the TI calculator) as follows:

TI BAII PLUS Financial Calculator Solution

Step 1: Press [2nd] [CLR TVM] to clear previous values.

Step 2: Press [2nd] [P/Y] 1 [ENTER], [2nd] [BGN] [2nd] [SET] [2nd] [SET]. Repeat [2nd] [SET] Until END shows in the display

Step 3: Input the values and compute.

100000 [PV] 15 [N] 12405.89 [+/–] [PMT] [CPT] [I/Y] **Answer: 9.00**

In this example the PMT was entered as a negative number to indicate that the loan payments are cash outflows, flowing away from the firm. The missing interest rate value was 9 percent, the interest rate on the loan.

Finding the Number of Periods

Suppose you found an investment that offered you a return of 6 percent per year. How long would it take you to double your money? In this problem you are looking for n, the number of compounding periods it will take for a starting amount, PV, to double in size (FV = 2 × PV).

To find n in our example, start with the formula for the future value of a single amount and solve for n as follows:

$$FV = PV \times \left(FVIF_{k,\,n}\right)$$

$$2 \times PV = PV \times \left(FVIF_{k\,=\,6\%,\,n\,=\,?}\right)$$

$$2.0 = FVIF_{k\,=\,6\%,\,n\,=\,?}$$

Now refer to the FVIF values, shown below in part of Table I. You know k = 6%, so scan across the top row to find the k = 6% column. Knowing that the FVIF value is 2.0, move down the k = 6% column until you find (or come close to) the value 2.0. Note that it occurs in the row in which n = 12. Therefore, n in this problem, and the number of periods it would take for the value of an investment to double at 6 percent interest per period, is 12.

Future Value Interest Factors, Compounded at k Percent for n Periods, Part of Table I

Number of Periods, n	Interest Rate, k										
	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
11	1.0000	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531
12	1.0000	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384
13	1.0000	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523
14	1.0000	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975

This problem can also be solved on a financial calculator quite quickly. Just key in all the known variables (PV, FV, and I/Y) and ask the calculator to compute n.

TI BAII PLUS Financial Calculator Solution

Step 1: Press [2nd] [CLR TVM] to clear previous values.

Step 2: Press [2nd] [P/Y] 1 [ENTER], [2nd] [QUIT].

Step 3: Input the values and compute.

1 [+/−] [PV] 2 [FV] 6 [I/Y] [CPT] [N] **Answer: 11.90**

In our example n = 12 when $1 is paid out and $2 received with a rate of interest of 6 percent. That is, it takes approximately 12 years to double your money at a 6 percent annual rate of interest.

Solving for the Payment

Lenders and financial managers frequently have to determine how much each payment—or installment—will need to be to repay an amortized loan. For example, suppose you are a business owner and you want to buy an office building for your company that costs $200,000. You have $50,000 for a down payment and the bank will lend you the $150,000 balance at a 6 percent annual interest rate. How much will the annual payments be if you obtain a 10-year amortized loan?

As we saw earlier, an amortized loan is repaid in equal payments over time. The period of time may vary. Let's assume in our example that your payments will occur annually, so that at the end of the 10-year period you will have paid off all interest and principal on the loan (FV = 0).

Because the payments are regular and equal in amount, this is an annuity problem. The present value of the annuity (PVA) is the $150,000 loan amount, the annual interest rate (k) is 6 percent, and n is 10 years. The payment amount (PMT) is the only unknown value.

Because all the variables but PMT are known, the problem can be solved by solving for PMT in the present value of an annuity formula, equation 8-4a, as follows:

$$\text{PVA} = \text{PMT} \times \left[\frac{1 - \dfrac{1}{(1+k)^n}}{k} \right]$$

$$\$150{,}000 = \text{PMT} \times \left[\frac{1 - \dfrac{1}{(1.06)^{10}}}{.06} \right]$$

$$\$150{,}000 = \text{PMT} \times 7.36009$$

$$\frac{\$150{,}000}{7.36009} = \text{PMT}$$

$$\$20{,}380.19 = \text{PMT}$$

We see, then, that the payment for an annuity with a 6 percent interest rate, an n of 10, and a present value of $150,000 is $20,380.19.

We can also solve for PMT using the table formula, Equation 8-4b, as follows:

$$\text{PVA} = \text{PMT} \times \left(\text{PVIFA}_{k,\, n}\right)$$

$$\$150{,}000 = \text{PMT} \times \left(\text{PVIFA}_{6\%,\ 10\text{ years}}\right)$$

$$\$150{,}000 = \text{PMT} \times 7.3601 \text{ (look up PVIFA, Table IV)}$$

$$\frac{\$150{,}000}{7.3601} = \text{PMT}$$

$$\$20{,}380.16 = \text{PMT (note the \$0.03 rounding error)}$$

The table formula shows that the payment for a loan with the present value of $150,000 at an annual interest rate of 6 percent and an n of 10 is $20,380.16.

With the financial calculator, simply key in all the variables but PMT and have the calculator compute PMT as follows:

TI BAII PLUS Financial Calculator Solution

Step 1: Press [2nd] [CLR TVM] to clear previous values.

Step 2: Press [2nd] [P/Y] 1 [ENTER], [2nd] [BGN] [2nd] [SET]. Repeat [2nd] [SET] command until the display shows END, [2nd] [QUIT] to set to the annual interest rate mode and to set the annuity payment to end of period mode.

Step 3: Input the values for the annuity due and compute.

150,000 [PV] 6 [I/Y] 10 [N] [CPT] [PMT] Answer: –20,380.19

The financial calculator will display the payment, $20,380.19, as a negative number because it is a cash outflow.

Loan Amortization

As each payment is made on an amortized loan, the interest due for that period is paid, along with a repayment of some of the principal that must also be "killed off." After the last payment is made, all the interest and principal on the loan have been paid. This step-by-step payment of the interest and principal owed is often shown in an *amortization table*. The amortization table for the ten-year 6 percent annual interest rate loan of $150,000 that was discussed in the previous section is shown in Table 8-2. The annual payment, calculated in the previous section, is $20,380.19.

We see from Table 8-2 how the balance on the $150,000 loan is killed off a little each year until the balance at the end of year 10 is zero. The payments reflect an increasing amount going toward principal and a decreasing amount going toward interest over time.

Table 8-2 Amortization Table for a $150,000 Loan, 6 Percent Annual Interest Rate, 10-Year Term

Loan Amortization Schedule

Amount Borrowed: **$150,000**
Interest Rate: **6.0%**
Term: **10 years**
Required Payments: **$20,380.19** *(found using Equation 8-4a)*

	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
			Col. 1 x .06	*Col. 2 - Col. 3*	*Col. 1 - Col. 4*
Year	**Beginning Balance**	**Total Payment**	**Payment of Interest**	**Payment of Principal**	**Ending Balance**
1	$150,000.00	$ 20,380.19	$ 9,000.00	$ 11,380.19	$138,619.81
2	$138,619.81	$ 20,380.19	$ 8,317.19	$ 12,063.01	$126,556.80
3	$126,556.80	$ 20,380.19	$ 7,593.41	$ 12,786.79	$113,770.02
4	$113,770.02	$ 20,380.19	$ 6,826.20	$ 13,553.99	$100,216.02
5	$100,216.02	$ 20,380.19	$ 6,012.96	$ 14,367.23	$ 85,848.79
6	$ 85,848.79	$ 20,380.19	$ 5,150.93	$ 15,229.27	$ 70,619.52
7	$ 70,619.52	$ 20,380.19	$ 4,237.17	$ 16,143.02	$ 54,476.50
8	$ 54,476.50	$ 20,380.19	$ 3,268.59	$ 17,111.60	$ 37,364.90
9	$ 37,364.90	$ 20,380.19	$ 2,241.89	$ 18,138.30	$ 19,226.60
10	$ 19,226.60	$ 20,380.19	$ 1,153.60	$ 19,226.60	$ 0.00

Compounding More Than Once per Year

So far in this chapter, we have assumed that interest is compounded *annually*. However, there is nothing magical about annual compounding. Many investments pay interest that is compounded semiannually, quarterly, or even daily. Most banks, savings and loan associations, and credit unions, for example, compound interest on their deposits more frequently than annually.

Suppose you deposited $100 in a savings account that paid 12 percent annual interest, compounded annually. After one year you would have $112 in your account ($\$112 = \$100 \times 1.12^1$).

Now, however, let's assume the bank used *semiannual compounding*. With semiannual compounding you would receive half a year's interest (6 percent) after six months. In the second half of the year, you would earn interest both on the interest earned in the first six months *and* on the original principal. The total interest earned during the year on a $100 investment at 12 percent annual interest would be:

	$ 6.00	(interest for the first six months)
+	$ .36	(interest on the $6 interest during the second 6 months)[4]
+	$ 6.00	(interest on the principal during the second six months)
=	$ 12.36	total interest in year 1

Interactive Module
Go to the Interactive Spreadsheets you downloaded for chapter 8. Follow the instructions there. See step by step how an amortized loan is paid off over time. Move about the cells and see how the variables relate to each other.

[4]The $.36 was calculated by multiplying $6 by half of 12%: $6.00 × .06 = $.36.

At the end of the year, you will have a balance of $112.36 if the interest is compounded semiannually, compared with $112.00 with annual compounding—a difference of $.36.

Here's how to find answers to problems in which the compounding period is less than a year: Apply the relevant present value or future value equation, but adjust k and n so they reflect the actual compounding periods.

To demonstrate, let's look at our example of a $100 deposit in a savings account at 12 percent for one year with semiannual compounded interest. Because we want to find out what the future value of a single amount will be, we use that formula to solve for the future value of the account after one year. Next, we divide the annual interest rate, 12 percent, by two because interest will be compounded twice each year. Then, we multiply the number of years n (one in our case) by two because with semiannual interest there are two compounding periods in a year. The calculation follows:

$$
\begin{aligned}
FV &= PV \text{ ¥ } (1 + k/2)^{n \text{ ¥ } 2} \\
&= \$100 \text{ ¥ } (1 + .12/2)^{1 \text{ ¥ } 2} \\
&= \$100 \text{ ¥ } (1 + .06)^{2} \\
&= \$100 \text{ ¥ } 1.1236 \\
&= \$112.36
\end{aligned}
$$

The future value of $100 after one year, earning 12 percent annual interest compounded semiannually, is $112.36.

To use the table method for finding the future value of a single amount, find the $FVIF_{k,\,n}$ in Table I in the Appendix at the end of the book. Then, divide the k by two and multiply the n by two as follows:

$$
\begin{aligned}
FV &= PV \text{ ¥ } \left(FVIF_{k/2,\, n \text{ ¥ } 2}\right) \\
&= \$100 \text{ ¥ } \left(FVIF_{12\%/2,\, 1 \text{ ¥ } 2 \text{ periods}}\right) \\
&= \$100 \text{ ¥ } \left(FVIF_{6\%,\ 2 \text{ periods}}\right) \\
&= \$100 \text{ ¥ } 1.1236 \\
&= \$112.36
\end{aligned}
$$

To solve the problem using a financial calculator, divide the k (represented as I/Y on the TI BAII PLUS calculator) by two and multiply the n by two. Next, key in the variables as follows:

TI BAII PLUS Financial Calculator Solution

Step 1: Press [2nd] [CLR TVM] to clear previous values.

Step 2: Press [2nd] [P/Y] 1 [ENTER], [2nd] [QUIT].

Step 3: Input the values and compute.

100 [+/–] [PV] 6 [I/Y] 2 [N] [CPT] [FV] **Answer: 112.36**

The future value of $100 invested for two periods at 6 percent per period is $112.36.[5]

Other compounding rates, such as quarterly or monthly rates, can be found by modifying the applicable formula to adjust for the compounding periods. With a quarterly compounding period, then, annual k should be divided by four and annual n multiplied by four. For monthly compounding, annual k should be divided by twelve and annual n multiplied by twelve. Similar adjustments could be made for other compounding periods.

Annuity Compounding Periods

Many annuity problems also involve compounding or discounting periods of less than a year. For instance, suppose you want to buy a car that costs $20,000. You have $5,000 for a down payment and plan to finance the remaining $15,000 at 6 percent annual interest for four years. What would your monthly loan payments be?

First, change the stated annual rate of interest, 6 percent, to a monthly rate by dividing by 12 (6%/12 = 1/2% or .005). Second, multiply the four-year period by 12 to obtain the number of months involved (4 × 12 = 48 months). Now solve for the annuity payment size using the annuity formula.

In our case, we apply the present value of an annuity formula, equation 8-4a, as follows:

$$PVA = PMT \times \left[\frac{1 - \frac{1}{(1+k)^n}}{k} \right]$$

$$\$15{,}000 = PMT \times \left[\frac{1 - \frac{1}{(1.005)^{48}}}{.005} \right]$$

$$\$15{,}000 = PMT \times 42.5803$$

$$\frac{\$150{,}000}{42.5803} = PMT$$

$$\$352.28 = PMT$$

The monthly payment on a $15,000 car loan with a 6 percent annual interest rate (.5 percent per month) for four years (48 months) is $352.28.

Solving this problem with the PVIFA table in Table IV in the Appendix at the end of the book would be difficult because the .5 percent interest rate is not listed in the PVIFA table. If the PVIFA were listed, we would apply the table formula, make the adjustments to reflect the monthly interest rate and the number of periods, and solve for the present value of the annuity.

On a financial calculator, we would first adjust the k and n to reflect the same time period—monthly, in our case—and then input the adjusted variables to solve the problem as follows:

[5]Note that we “lied” to our TI BAII PLUS calculator. It asks us for the interest rate per year (I/Y). We gave it the semiannual interest rate of 6 percent, not the annual interest rate of 12 percent. Similarly, n was expressed as the number of semiannual periods, two in one year. As long as we are consistent in expressing the k and n values according to the number of compounding or discounting periods per year, the calculator will give us the correct answer.

TI BAII PLUS Financial Calculator Solution

Step 1: Press 2nd CLR TVM to clear previous values.

Step 2: Press 2nd P/Y 1 ENTER, 2nd BGN 2nd SET 2nd SET. Repeat 2nd SET command until the display shows END, 2nd QUIT to set to the annual interest rate mode and to set the annuity payment to end of period mode.

Step 3: Input the values for the annuity due and compute.

15,000 PV .5 I/Y 48 N CPT PMT **Answer: –352.28**

Note that once again we have lied to our TI BAII PLUS financial calculator. The interest rate we entered was not the 6 percent rate per year but rather the .5 percent rate per month. We entered not the number of years, four, but rather the number of months, 48. Because we were consistent in entering the k and n values in monthly terms, the calculator gave us the correct monthly payment of 2352.28 (an outflow of $352.28 per month).

Continuous Compounding

The effect of increasing the number of compounding periods per year is to increase the future value of the investment. The more frequently interest is compounded, the greater the future value. The smallest compounding period is used when we do continuous compounding—compounding that occurs every tiny unit of time (the smallest unit of time imaginable).

Recall our $100 deposit in an account at 12 percent for one year with annual compounding. At the end of year 1, our balance was $112. With semiannual compounding, the amount increased to $112.36.

When continuous compounding is involved, we cannot divide k by infinity and multiply n by infinity. Instead, we use the term e, which you may remember from your math class. We define e as follows:

$$e = \lim_{h \to \infty} \left(1 + \frac{1}{h}\right)^h = 2.71828$$

The value of e is the natural antilog of 1 and is approximately equal to 2.71828. This number is one of those like pi (approximately equal to 3.14159), which we can never express exactly but can approximate. Using e, the formula for finding the future value of a given amount of money, PV, invested at annual rate, k, for n years, with continuous compounding, is as follows:

Future Value with Continuous Compounding

$$FV = PV \times e^{(k \times n)} \tag{8-7}$$

where k (expressed as a decimal) and n are expressed in annual terms

Applying Equation 8-7 to our example of a $100 deposit at 12 percent annual interest with continuous compounding, at the end of one year we would have the following balance:

$$FV = \$100 \times 2.71828^{(.12 \times 1)}$$
$$= \$112.75$$

The future value of \$100, earning 12 percent annual interest compounded continuously, is \$112.75.

As this section demonstrates, the compounding frequency can impact the value of an investment. Investors, then, should look carefully at the frequency of compounding. Is it annual, semiannual, quarterly, daily, or continuous? Other things being equal, the more frequently interest is compounded, the more interest the investment will earn.

What's Next

In this chapter we investigated the importance of the time value of money in financial decisions and learned how to calculate present and future values for a single amount, for ordinary annuities, and for annuities due. We also learned how to solve special time value of money problems, such as finding the interest rate or the number of periods.

The skills acquired in this chapter will be applied in later chapters, as we evaluate proposed projects, bonds, and preferred and common stock. They will also be used when we estimate the rate of return expected by suppliers of capital. In the next chapter, we will turn to the cost of capital.

Summary

1. Explain the time value of money and its importance in the business world.

Money grows over time when it earns interest. Money expected or promised in the future is worth less than the same amount of money in hand today. This is because we lose the opportunity to earn interest when we have to wait to receive money. Similarly, money we owe is less burdensome if it is to be paid in the future rather than now. These concepts are at the heart of investment and valuation decisions of a firm.

2. Calculate the future value and present value of a single amount.

To calculate the future value and the present value of a single dollar amount, we may use the algebraic, table, or calculator methods. Future value and present value are mirror images of each other. They are compounding and discounting, respectively. With future value, increases in k and n result in an exponential increase in future value. Increases in k and n result in an exponential decrease in present value.

3. Find the future and present values of an annuity.

Annuities are a series of equal cash flows. An annuity that has payments that occur at the end of each period is an ordinary annuity. An annuity that has payments that occur at the beginning of each period is an annuity due. A perpetuity is a perpetual annuity.

To find the future and present values of an ordinary annuity, we may use the algebraic, table, or financial calculator method. To find the future and present values of an annuity due, multiply the applicable formula by $(1 + k)$ to reflect the earlier payment.

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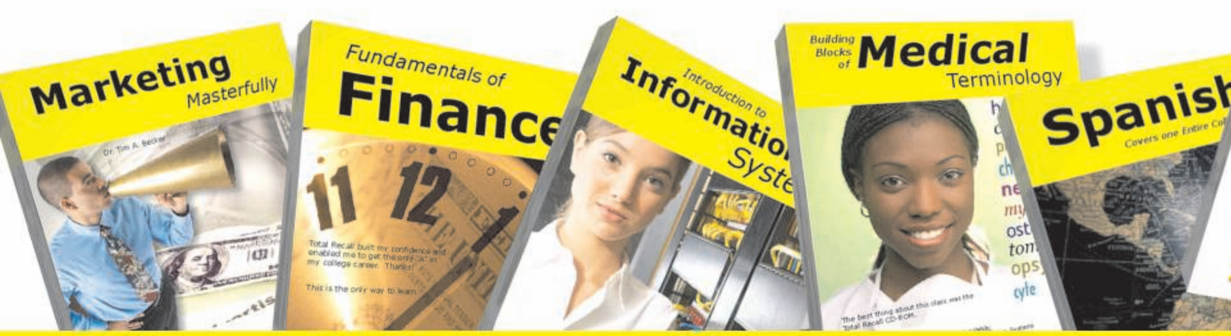

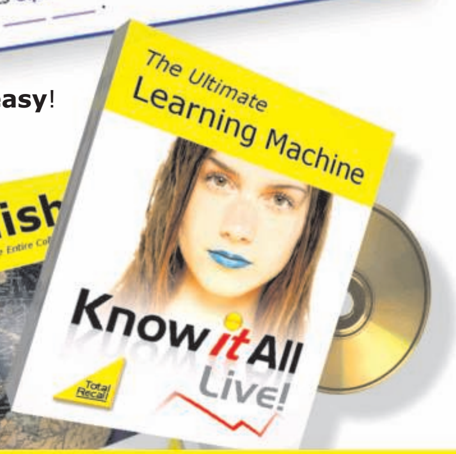

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4. Solve time value of money problems with uneven cash flows.

To solve time value of money problems with uneven cash flows, we find the value of each payment (each single amount) in the cash flow series and total each single amount. Sometimes the series has several cash flows of the same amount. If so, calculate the present value of those cash flows as an annuity and add the total to the sum of the present values of the single amounts to find the total present value of the uneven cash flow series.

5. Solve special time value of money problems, such as finding the interest rate, number or amount of payments, or number of periods in a future or present value problem.

To solve special time value of money problems, we use the present value and future value equations and solve for the missing variable, such as the loan payment, k, or n. We may also solve for the present and future values of single amounts or annuities in which the interest rate, payments, and number of time periods are expressed in terms other than a year. The more often interest is compounded, the larger the future value.

Equations Introduced in This Chapter

Equation 8-1a. Future Value of a Single Amount—Algebraic Method:

$$FV = PV \times (1 + k)^n$$

where: FV = Future Value, the ending amount

PV = Present Value, the starting amount, or original principal

k = Rate of interest per period (expressed as a decimal)

n = Number of time periods

Equation 8-1b. Future Value of a Single Amount—Table Method:

$$FV = PV \times \left(FVIF_{k,\,n}\right)$$

where: FV = Future Value, the ending amount

PV = Present Value, the starting amount

$FVIF_{k,n}$ = Future Value Interest Factor given interest rate, k, and number of periods, n, from Table I

Equation 8-2a. Present Value of a Single Amount—Algebraic Method:

$$PV = FV \text{ ¥ } \frac{1}{(1 + k)^n}$$

where: PV = Present Value, the starting amount

FV = Future Value, the ending amount

k = Discount rate of interest per period (expressed as a decimal)

n = Number of time periods

Equation 8-2b. Present Value of a Single Amount—Table Method:

$$PV = FV \times \left(PVIF_{k,\,n}\right)$$

where: PV = Present Value

FV = Future Value

$PVIF_{k,\,n}$ = Present Value Interest Factor given discount rate, k, and number of periods, n, from Table II

Equation 8-3a. Future Value of an Annuity—Algebraic Method:

$$FVA = PMT \times \left[\frac{(1+k)^n - 1}{k}\right]$$

where: FVA = Future Value of an Annuity

PMT = Amount of each annuity payment

k = Interest rate per time period expressed as a decimal

n = Number of annuity payments

Equation 8-3b. Future Value of an Annuity—Table Method:

$$FVA = PMT \times FVIFA_{k,\,n}$$

where: FVA = Future Value of an Annuity

PMT = Amount of each annuity payment

$FVIFA_{k,\,n}$ = Future Value Interest Factor for an Annuity from Table III

k = Interest rate per period

n = Number of annuity payments

Equation 8-4a. Present Value of an Annuity—Algebraic Method:

$$PVA = PMT \times \left[\frac{1 - \frac{1}{(1+k)^n}}{k}\right]$$

where: PVA = Present Value of an Annuity

PMT = Amount of each annuity payment

k = Discount rate per period expressed as a decimal

n = Number of annuity payments

Equation 8-4b. Present Value of an Annuity—Table Method:

$$PVA = PMT \times PVIFA_{k,\,n}$$

where: PVA = Present Value of an Annuity

PMT = Amount of each annuity payment

$PVIFA_{k,\,n}$ = Present Value Interest Factor for an Annuity from Table IV

k = Discount rate per period

n = Number of annuity payments

Equation 8-5. Present Value of a Perpetuity:

$$PVP = PMT \times \left(\frac{1}{k}\right)$$

where: PVP = Present Value of a Perpetuity

k = Discount rate expressed as a decimal

Equation 8-6. Rate of Return:

$$k = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

where: k = Rate of return expressed as a decimal

FV = Future Value

PV = Present Value

n = Number of compounding periods

Equation 8-7. Future Value with Continuous Compounding:

$$FV = PV \times e^{(k \times n)}$$

where: FV = Future Value

PV = Present Value

e = Natural antilog of 1

k = Stated annual interest rate expressed as a decimal

n = Number of years

Self-Test

ST-1. Jed is investing $5,000 into an eight-year certificate of deposit (CD) that pays 6 percent annual interest with annual compounding. How much will he have when the CD matures?

ST-2. Tim has found a 2005 Toyota 4-Runner on sale for $19,999. The dealership says that it will finance the entire amount with a one-year loan, and the monthly payments will be $1,776.98. What is the annualized interest rate on the loan (the monthly rate times 12)?

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ST-3. Heidi's grandmother died and provided in her will that Heidi will receive $100,000 from a trust when Heidi turns 21 years of age, 10 years from now. If the appropriate discount rate is 8 percent, what is the present value of this $100,000 to Heidi?

ST-4. Zack wants to buy a new Ford Mustang automobile. He will need to borrow $20,000 to go with his down payment in order to afford this car. If car loans are available at a 6 percent annual interest rate, what will Zack's monthly payment be on a four-year loan?

ST-5. Bridget invested $5,000 in a growth mutual fund, and in 10 years her investment had grown to $15,529.24. What annual rate of return did Bridget earn over this 10-year period?

ST-6. If Tom invests $1,000 a year beginning today into a portfolio that earns a 10 percent return per year, how much will he have at the end of 10 years? (Hint: Recognize that this is an annuity due problem.)

Review Questions

1. What is the time value of money?

2. Why does money have time value?

3. What is compound interest? Compare compound interest to discounting.

4. How is present value affected by a change in the discount rate?

5. What is an annuity?

6. Suppose you are planning to make regular contributions in equal payments to an investment fund for your retirement. Which formula would you use to figure out how much your investments will be worth at retirement time, given an assumed rate of return on your investments?

7. How does continuous compounding benefit an investor?

8. If you are doing PVA and FVA problems, what difference does it make if the annuities are ordinary annuities or annuities due?

9. Which formula would you use to solve for the payment required for a car loan if you know the interest rate, length of the loan, and the borrowed amount? Explain.

Build Your Communication Skills

CS-1. Obtain information from four different financial institutions about the terms of their basic savings accounts. Compare the interest rates paid and the frequency of the compounding. For each account, how much money would you have in 10 years if you deposited $100 today? Write a one- to two-page report of your findings.

CS-2. Interview a mortgage lender in your community. Write a brief report about the mortgage terms. Include in your discussion comments about what interest rates are charged on different types of loans, why rates differ, and what fees are charged in addition to the interest and principal that mortgagees must pay to this lender. Describe the advantages and disadvantages of some of the loans offered.

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Problems

Future Value ☛ **8-1.** What is the future value of $1,000 invested today if you earn 7 percent annual interest for five years?

Future Value ☛ **8-2.** Calculate the future value of $50,000 ten years from now if the annual interest rate is

a. 0 percent
b. 5 percent
c. 10 percent
d. 20 percent

Future Value ☛ **8-3.** How much will you have in 10 years if you deposit $5,000 today and earn 8 percent annual interest?

Future Value ☛ **8-4.** Calculate the future value of $100,000 fifteen years from now based on the following interest rates:

a. 3 percent
b. 6 percent
c. 9 percent
d. 12 percent

Future Value ☛ **8-5.** Calculate the future values of the following amounts at 10 percent for twenty-five years:

a. 50,000
b. 75,000
c. 100,000
d. 125,000

Future Value ☛ **8-6.** Calculate the future value of $60,000 at 12 percent for the following years:

a. 5 years
b. 10 years
c. 15 years
d. 20 years

Present Value ☛ **8-7.** What is the present value of $20,000 to be received ten years from now using a 12 percent annual discount rate?

Present Value ☛ **8-8.** Calculate the present value of $60,000 to be received twenty years from now at an annual discount rate of:

a. 0 percent
b. 5 percent
c. 10 percent
d. 20 percent

8-9. Norton is going to receive a graduation present of $9,000 from his grandparents in four years. If the discount rate is 8 percent, what is this gift worth today?

Present Value

8-10. Calculate the present values of $25,000 to be received in ten years using the following annual discount rates:

Present Value

a. 3 percent
b. 6 percent
c. 9 percent
d. 12 percent

8-11. Calculate the present values of the following using a 6 percent discount rate at the end of 15 years:

Present Value

a. $50,000
b. $75,000
c. $100,000
d. $125,000

8-12. Calculate the present value of $80,000 at a 9 percent discount rate to be received in:

Present Value

a. 5 years
b. 10 years
c. 15 years
d. 20 years

8-13. What is the present value of a $500 ten-year annual ordinary annuity at a 6 percent annual discount rate?

Present Value of an Ordinary Annuity

8-14. Calculate the present value of a $10,000 thirty-year annual ordinary annuity at an annual discount rate of:

Present Value of an Ordinary Annuity

a. 0 percent
b. 10 percent
c. 20 percent
d. 50 percent

8-15. What is the present value of a ten-year ordinary annuity of $20,000, using a 7 percent discount rate?

Present Value of an Ordinary Annuity

8-16. Find the present value of a five-year ordinary annuity of $10,000, using the following discount rates:

Present Value of an Ordinary Annuity

a. 9 percent
b. 13 percent
c. 15 percent
d. 21 percent

Future Value of an Ordinary Annuity

8-17. What is the future value of a five-year annual ordinary annuity of $500, using a 9 percent interest rate?

Future Value of an Ordinary Annuity

8-18. Calculate the future value of a twelve-year, $6,000 annual ordinary annuity, using an interest rate of:

a. 0 percent

b. 2 percent

c. 10 percent

d. 20 percent

Future Value of an Ordinary Annuity

8-19. What is the future value of a ten-year ordinary annuity of $5,000, using an interest rate of 6 percent?

Future Value of an Ordinary Annuity

8-20. What is the future value of an eight-year ordinary annuity of $5,000, using an 11 percent interest rate?

Future Value of an Ordinary Annuity

8-21. Find the future value of the following five-year ordinary annuities, using a 10 percent interest rate.

a. $1,000

b. $10,000

c. $75,000

d. $125,000

Future Value of an Annuity Due

8-22. Starting today, you invest $1,200 a year into your individual retirement account (IRA). If your IRA earns 12 percent a year, how much will be available at the end of 40 years?

Future Value of an Annuity Due

8-23. John will deposit $500 at the beginning of each year for five years into an account that has an interest rate of 8 percent. How much will John have to withdraw in five years?

Future Value of an Annuity Due

8-24. An account manager has found that the future value of $10,000, deposited at the end of each year, for five years at an interest rate of 6 percent will amount to $56,370.93. What is the future value of this scenario if the account manager deposits the money at the beginning of each year?

Present Value of an Annuity Due

8-25. If your required rate of return is 12 percent, how much will an investment that pays $80 a year at the beginning of each of the next 20 years be worth to you today?

Present Value of an Annuity Due

8-26. Sue has won the lottery and is going to receive $30,000 for 25 years; she received her first check today. The current discount rate is 9 percent. Find the present value of her winnings.

Present Value of an Annuity Due

8-27. Kelly pays a debt service of $1,300 a month and will continue to make this payment for 15 years. What is the present value of these payments discounted at 7 percent if she mails her first payment in today?

Solving for k

8-28. You invested $50,000, and 10 years later the value of your investment has grown to $185,361. What is your compounded annual rate of return over this period?

8-29. You invested $1,000 five years ago, and the value of your investment has fallen to $773.78. What is your compounded annual rate of return over this period? **Solving for k**

8-30. What is the rate of return on an investment that grows from $50,000 to $246,795 in 10 years? **Solving for k (Interest)**

8-31. What is the present value of a $50 annual perpetual annuity using a discount rate of 8 percent? **Present Value of a Perpetuity**

8-32. A payment of $80 per year forever is made with a discount rate of 9 percent. What is the present value of these payments? **Present Value of a Perpetuity**

8-33. You are valuing a preferred stock that makes a dividend payment of $65 per year, and the current discount rate is 8.5 percent. What is the value of this share of preferred stock? **Present Value of a Perpetuity (Preferred Stock)**

8-34. What is the future value of $10, earning 8 percent annual interest, 200 years later? **Future Value**

8-35. Joe's Dockyard is financing a new boat with an amortizing loan of $24,000, which is to be repaid in 10 annual installments of $4,247.62 each. What interest rate is Joe paying on the loan? **Solving for a Loan Interest Rate**

8-36. On June 1, 2006, Sue purchased a home for $220,000. She put $20,000 down on the house and obtained a 30-year fixed-rate mortgage for the remaining $200,000. Under the terms of the mortgage, Sue must make payments of $1,330.61 a month for the next 30 years starting June 30. What is the effective annual interest rate on Sue's mortgage? **Solving for the Monthly Interest Rate on a Mortgage**

8-37. What is the amount you have to invest today at 7 percent annual interest to be able to receive $10,000 after **Present Value**

a. 5 years?

b. 10 years?

c. 20 years?

8-38. How much money would Ruby Carter need to deposit in her savings account at Great Western Bank today in order to have $16,850.58 in her account after five years? Assume she makes no other deposits or withdrawals and the bank guarantees an 11 percent annual interest rate, compounded annually. **Future Value**

8-39. If you invest $20,000 today, how much will you receive after **Future Value**

a. 7 years at a 5 percent annual interest rate?

b. 10 years at a 7 percent annual interest rate?

8-40. The Microsoft stock you purchased twelve years ago for $55 a share is now worth $67.73. What is the compounded annual rate of return you have earned on this investment? **Solving for k**

8-41. Amy Jolly deposited $1,000 in a savings account. The annual interest rate is 10 percent, compounded semiannually. How many years will it take for her money to grow to $2,653.30? **Solving for n**

Present Value of an Annuity Due

8-42. Beginning a year from now, Bernardo O'Reilly will receive $20,000 a year from his pension fund. There will be fifteen of these annual payments. What is the present value of these payments if a 6 percent annual interest rate is applied as the discount rate?

Future Value of an Annuity Due

8-43. If you invest $4,000 per year into your pension fund, earning 9 percent annually, how much will you have at the end of twenty years? You make your first payment of $4,000 today.

Future Value of an Annuity Due

8-44. What would you accumulate if you were to invest $100 every quarter for five years into an account that returned 8 percent annually? Your first deposit would be made one quarter from today. Interest is compounded quarterly.

Future Value of an Annuity Due

8-45. If you invest $2,000 per year for the next ten years at an 8 percent annual interest rate, beginning one year from today compounded annually, how much are you going to have at the end of the tenth year?

Future Value of an Annuity Due (Challenge Problem)

8-46. It is the beginning of the quarter and you intend to invest $300 into your retirement fund at the end of every quarter for the next thirty years. You are promised an annual interest rate of 8 percent, compounded quarterly.

a. How much will you have after thirty years upon your retirement?

b. How long will your money last if you start withdrawing $6,000 at the end of every quarter after you retire?

Solving for a Loan Payment

8-47. A $30,000 loan obtained today is to be repaid in equal annual installments over the next seven years starting at the end of this year. If the annual interest rate is 10 percent, compounded annually, how much is to be paid each year?

Present Value of a Perpetuity

8-48. Allie Fox is moving to Central America. Before he packs up his wife and son he purchases an annuity that guarantees payments of $10,000 a year in perpetuity. How much did he pay if the annual interest rate is 12 percent?

Future Value

8-49. Matt and Christina Drayton deposited $500 into a savings account the day their daughter, Joey, was born. Their intention was to use this money to help pay for Joey's wedding expenses when and if she decided to get married. The account pays 5 percent annual interest with continuous compounding. Upon her return from a graduation trip to Hawaii, Joey surprises her parents with the sudden announcement of her planned marriage to John Prentice. The couple set the wedding date to coincide with Joey's twenty-third birthday. How much money will be in Joey's account on her wedding day?

Time to Double Your Money

8-50. You deposit $1,000 in an account that pays 8 percent interest, compounded annually. How long will it take to double your money?

Time to Pay Off a Credit Card Balance

8-51. Upon reading your most recent credit card statement, you are shocked to learn that the balance owed on your purchases is $4,000. Resolving to get out of debt once and for all, you decide not to charge any more purchases and to make regular monthly payments until the balance is zero. Assuming that the bank's credit card interest rate is 19.5 percent and the most you can afford to pay each month is $200, how long will it take you to pay off your debt?

8-52. Joanne and Walter borrow $14,568.50 for a new car before they move to Stepford, Connecticut. They are required to repay the amortized loan with four annual payments of $5,000 each. What is the interest rate on their loan?

Solving for k

8-53. Norman Bates is planning for his eventual retirement from the motel business. He plans to make quarterly deposits of $1,000 into an IRA starting three months from today. The guaranteed annual interest rate is 8 percent, compounded quarterly. He plans to retire in 15 years.

Future Value of an Annuity (Challenge Problem)

a. How much money will be in his retirement account when he retires?

b. Norman also supports his mother. At Norman's retirement party, Mother tells him they will need $2,000 each month in order to pay for their living expenses. Using the preceding interest rate and the total account balance from part a, for how many years will Norman keep Mother happy by withdrawing $6,000 at the end of each quarter? It is very important that Norman keep Mother happy.

8-54. Jack Torrance comes to you for financial advice. He hired the Redrum Weed-N-Whack Lawn Service to trim the hedges in his garden. Because of the large size of the project (the shrubs were really out of control), Redrum has given Jack a choice of four different payment options. Which of the following four options would you recommend that Jack choose? Why?

Challenge Problem

- Option 1. Pay $5,650 cash immediately.
- Option 2. Pay $6,750 cash in one lump sum two years from now.
- Option 3. Pay $800 at the end of each quarter for two years.
- Option 4. Pay $1,000 immediately plus $5,250 in one lump sum two years from now.

Jack tells you he can earn 8 percent interest, compounded quarterly, on his money. You have no reason to question his assumption.

8-55. Sarah has $30,000 for a down payment on a house and wants to borrow $120,000 from a mortgage banker to purchase a $150,000 house. The mortgage loan is to be repaid in monthly installments over a thirty-year period. The annual interest rate is 9 percent. How much will Sarah's monthly mortgage payments be?

Solving for a Mortgage Loan Payment

8-56. The Robinsons have found the house of their dreams. They have $50,000 to use as a down payment and they want to borrow $250,000 from the bank. The current mortgage interest rate is 6 percent. If they make equal monthly payments for fifteen years, how much will their monthly mortgage payment be?

Solving for a Mortgage Loan Payment

8-57. Slick has his heart set on a new Miata sportscar. He will need to borrow $18,000 to get the car he wants. The bank will loan Slick the $18,000 at an annual interest rate of 6 percent.

Solving for Auto Loan Payments

a. How much would Slick's monthly car payments be for a four-year loan?

b. How much would Slick's monthly car payments be if he obtains a six-year loan at the same interest rate?

Challenge Problem (Value of Missing Cash Flow)

EXCEL SPREADSHEET

8-58. Assume the following set of cash flows:

Time 1	Time 2	Time 3	Time 4
\$100	\$150	?	\$100

At a discount rate of 10 percent, the total present value of all the cash flows above, including the missing cash flow, is \$320.74. Given these conditions, what is the value of the missing cash flow?

Loan Amortization Table

EXCEL SPREADSHEET

8-59. You are considering financing the purchase of an automobile that costs \$22,000. You intend to finance the entire purchase price with a five-year amortized loan with an 8 percent annual interest rate.

a. Calculate the amount of the monthly payments for this loan.

b. Construct an amortization table for this loan using the format shown in Table 8-2. Use monthly payments.

c. If you elected to pay off the balance of the loan at the end of the thirty-sixth month how much would you have to pay?

Answers to Self-Test

ST-1. $FV = \$5000 \times 1.06^8 = \$7{,}969.24$ = Jed's balance when the eight-year CD matures.

ST-2.

$$\$19{,}999 = \$1{,}1776.89 \text{ ¥ } \left(PVIFA_{k=?,\, n=12}\right)$$

$$11.2551 = PVIFA_{k=?,\, n=12}$$

k = 1% monthly (from the PVIFA Table at the end of the n = 12 row,

k = 1%)

Annual Rate = 1% × 12 = 12%

ST-3.

$$PV = \$100{,}000 \times (1/1.08^{10}) = \$46{,}319.35$$

= the present value of Heidi's \$100,000

ST-4.

$$\$20{,}000 = PMT \times [1 - (1/1.005^{48})] / .005$$

$$\$20{,}000 = PMT \times 42.5803$$

PMT = \$469.70 = Zack's car loan payment

ST-5.

$$\$5{,}000 \times FVIF_{k=?,\, n=10} = \$15{,}529.24$$

$FVIF_{k=?,\, n=10} = 3.1058$, therefore k = 12% = Bridget's annual rate of return on the mutual fund.

ST-6.

$$FV = PMT \text{ ¥ } \left(FVIFA_{k\%,\, n}\right) \text{ ¥ } (1+k)$$

$$= \$1{,}000 \text{ ¥ } \left(FVIFA_{10\%,\, 10}\right) \text{ ¥ } (1+.10)$$

$$= \$1{,}000 \times 15.9374 \times 1.10$$

$$= \$17{,}531.14$$

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