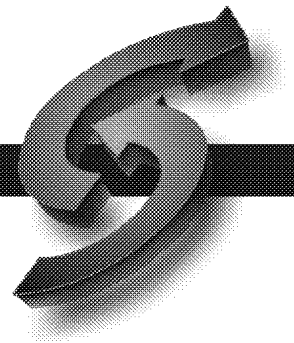




**Latin America Training
and Development Center**

**Basics of
Corporate Finance**
May 1994

Self-Instruction Series



Basics of Corporate Finance

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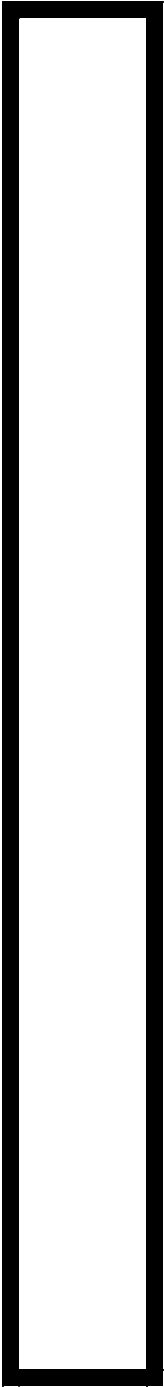


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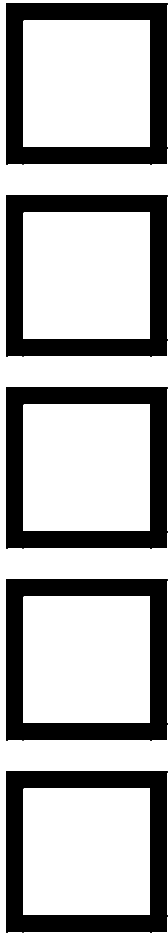
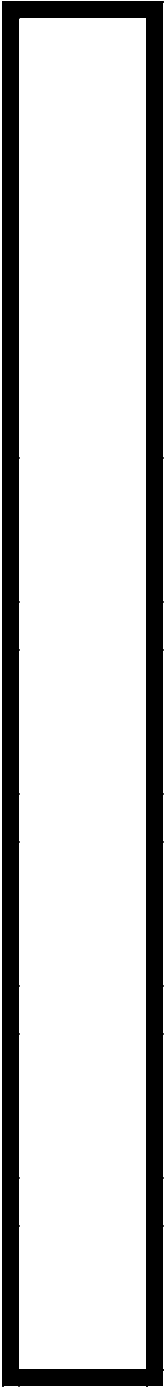
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Introduction

INTRODUCTION: BASICS OF CORPORATE FINANCE

COURSE OVERVIEW

Basics of Corporate Finance serves as an introductory course for students beginning their study of finance and financial markets. The ideas and calculations presented in this workbook serve as the foundation for continued study in the areas related to corporate finance and the capital and derivative markets. The purpose of this course is to help the student build a working vocabulary of the financial world and to understand the basic computations used by analysts working in the corporate finance field. The terms and ideas covered in this course will provide the background necessary for the student to understand concepts presented in future courses.

This workbook serves as a foundation for other courses in the series. It is comprised of ten units.

UNIT 1: Financial Statement Analysis

The first unit in the workbook is an introduction to the financial statements of a company. This unit briefly explains the purpose of each financial statement and provides definitions for the common items found in each of the statements. The unit also introduces ratio analysis and its use in the analysis of the finances and operations of a firm. The unit focuses on definitions of key terms, although some simple mathematical calculations and relationships are introduced and explained.

UNIT 2: Financial Markets and Interest Rates

The next unit is a brief overview of the financial markets and the components of interest rates. This unit will help you understand how financial markets operate and to identify the participants in the markets. A brief discussion of interest rates and the role they play in the financial markets is also included. The unit's focus is mainly to identify and define key terms; it does not require any mathematical calculations.

UNIT 3: Time Value of Money

The time value of money is introduced in this unit. You will learn how and why the value of money changes over time. You will also be introduced to the ideas of present value and future value and how those computations are used to evaluate a potential investment. The topics in this unit are quantitative, with several formulas introduced and explained. All of the computations can be completed using a financial calculator. You will learn how to identify the key variables necessary for input into the calculator to find the proper solution. The concepts presented in this unit are important because much of the remainder of the workbook builds upon them.

UNIT 4: Valuing Financial Assets

An introduction to the process of valuing financial assets is provided in this unit. These simple methods for valuation are based on the ideas presented in Unit Three. The unit provides an explanation of some of the basic terms associated with financial assets. Basic formulas used to place a value on simple financial assets (bonds, preferred stock, and common stock) are also demonstrated. The unit requires some mathematical calculations, but all are simple and straight-forward. The ideas for valuing these securities serve as building blocks when more complex securities are being considered.

UNIT 5: Introduction to Capital Budgeting

The basic ideas and methodologies surrounding capital budgeting are introduced in Unit Five. You will see how the idea of present value can be used to evaluate alternatives for capital investment when resources are scarce. The most important points in this unit are the calculation of net present value and internal rate of return. These two computations are important for the evaluation of many types of projects and securities. Most financial calculators will perform the computations, and we will demonstrate how to identify the key variables needed for input into your calculator.

UNIT 6: Corporate Valuation – Risk

In this unit, some real-world complications, including risk and uncertainty, are applied to some of the simple ideas presented earlier. More specifically, you will see how an analyst may make assumptions concerning the future operations of a firm in order to estimate future cash flows. From these estimates, a value can be placed upon the firm or project. Some key statistical terms are presented. These ideas and terms are used to build the foundation for the capital asset pricing model (CAPM). This model is the most widely-used method for calculating the value of a firm. The unit has many calculations and you should feel comfortable identifying the key variables needed to use the CAPM.

UNIT 7: Corporate Valuation – Cost of Capital

The main focus of this unit is a discussion of each component of capital and the costs associated with the use of each component. You will see how these costs are combined in the proper proportions to find the weighted-average cost of capital (WACC). This is the appropriate rate to be used to discount a set of future cash flows. This unit contains both key terms and important computations used later in the course.

UNIT 8: Corporate Valuation – Estimating Corporate Value

The ideas presented in Units Six and Seven are brought together in this unit. An estimate of corporate value can be found by forecasting a set of cash flows and discounting them at the weighted average cost of capital. You will be introduced to a relatively simple method for forecasting cash flows based on a set of assumptions concerning the future operations and finances of a company. Other methods for estimating corporate value are presented and the relative strengths and weaknesses of each are discussed. The unit requires some simple calculations, including applications of the WACC and present value.

UNIT 9: Fixed Income Securities

In this unit you will revisit the debt markets discussed briefly in Units Two and Four through the introduction of the mathematics which surround fixed income securities (bonds). The calculations of yield and rate of return concerning bonds making fixed interest payments are introduced and the relationship between the yield and the price of a bond are discussed. The unit also includes a discussion of duration and its calculation. All of the computations are relatively straightforward; many financial calculators can perform most of the calculations.

UNIT 10: Derivative Securities

The final unit in the corporate finance workbook focuses on some derivative securities that are used in managing exposure to risk. The unit includes brief introductions to options and swaps and a simple explanation of how they are used. This unit is included so that you can begin to understand some complex securities that are encountered in future courses. For a more thorough discussion of risk management and the use of swaps and options, refer to the workbooks designed to cover these topics. This unit is included only to provide a brief introduction to the topics.

COURSE OBJECTIVES

When you complete this workbook, you will be able to:

- Understand the basic concepts of the three main financial statements
- Recognize the significance of the time value of money in the financial planning process
- Identify techniques used by financial planners to evaluate, compare, and select investment alternatives
- Recognize the basic valuation concepts and calculations that apply to corporate valuation
- Identify fixed income securities that may be included in an investment portfolio and derivative securities that are used to manage risk

THE WORKBOOK

This self-instruction workbook has been designed to give you complete control over your own learning. The material is broken into workable sections, each containing everything you need to master the content. You can move through the workbook at your own pace, and go back to review ideas that you didn't completely understand the first time. Each unit contains:



Unit Objectives – which point out important elements in the unit that you are expected to learn.



Text – which is the "heart" of the workbook. This section explains the content in detail.

Key Terms – which also appear in the Glossary. They appear in **bold face** the first time they appear in the text.

Instructional Mapping – terms or phrases in the left margin which highlight significant points in the lesson.



Practice Exercises and Progress Checks – help you practice what you have learned and check your progress. Appropriate questions or problems are presented at strategic places within the units and at the end of each unit. You will not be graded on these by anyone else; they are to help you evaluate your progress. Each set of questions is followed by an Answer Key. If you have an incorrect answer, we encourage you to review the corresponding text and then try the question again.

In addition to these unit elements, the workbook includes the:

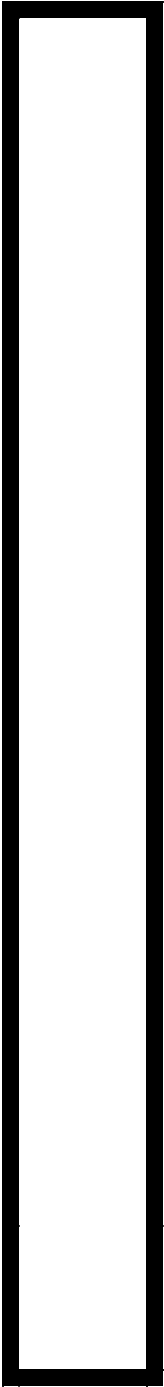
Glossary – which contains all of the key terms used in the workbook.

Index – which helps you locate the glossary item in the workbook.

This workbook is designed to provide you with a background in the key points of corporate finance. Upon completing the workbook, you should feel comfortable with the ideas and calculations presented in the course. The practice problems in each unit will help you assess your understanding of the material in that unit. Try to work each problem before looking at the solutions. That will help you identify the sections that may require more review on your part. Since this corporate finance workbook really serves as the foundation for the other workbooks and courses in the series, it is important that you understand the main ideas that you will study.

As we have mentioned, many sections in this workbook contain mathematical formulas and calculations. It is important that you understand the formulas and feel comfortable making these computations. It is not critical that you memorize every formula. The goal of these sections is to help you recognize the relevant information contained in a problem and be able to input that data into your calculator. Whenever possible, we will also discuss the calculations that can be made on a business calculator. In those cases, you will need to review your owner's manual for the specific instructions for your calculator.

This is a self-instructional course; your progress will not be supervised. We expect you to complete the course to the best of your ability and at your own speed. Now that you know what to expect, you are ready to begin. Please turn to Unit One. Good Luck!



Unit 1

UNIT I: FINANCIAL STATEMENT ANALYSIS

INTRODUCTION

Companies use financial statements to record their asset investments, report their profitability, and describe their cash flow. Analysts evaluate financial statements (often called a company's "books") for clues about a company's performance. There are three types of financial statements:

- Balance Sheet
- Income Statement
- Cash Flow Statement

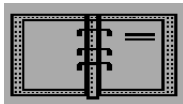
We include an overview of financial statement analysis in the Corporate Finance Workbook to help you understand the terminology that is used by persons working in finance and to introduce some calculations that will apply later in the course, particularly in the units that discuss corporate valuation. For a more in-depth study, you can take the Citibank self-instruction course on Financial Statement Analysis.



UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize the three main types of financial statements
- Identify the relationships between accounts on the Balance Sheet, line items on the Income Statement, and the calculation of the Cash Flow Statement
- Calculate common ratios used in financial statement analysis and interpret the results



BALANCE SHEET

Conditions at a point in time

The Balance Sheet is a record of assets held by a business and capital used to pay for those assets. It is a snapshot of conditions at a specific point in time, generally at the end of a quarter or year. A typical Balance Sheet looks like the example for XYZ Corporation (Figure 1.1).

XYZ Corporation			
December 31, 1993		(In Millions \$)	
ASSETS		LIABILITIES AND EQUITY	
Cash	5.7	Accounts Payable	22.7
Marketable Securities	6.3	Notes Payable	31.5
Accounts Receivable	50.9	Accrued Wages and Taxes	2.3
Inventories	88.7	Other Current Liabilities	<u>4.9</u>
Prepaid Expenses	1.1	Total Current Liabilities	61.4
Other Current Assets	<u>0.3</u>		
Total Current Assets	153.0	Long-term Debt	107.4
		Preferred Stock	<u>12.3</u>
Gross Fixed Assets	158.8	Total Long-term Liabilities	181.1
Less Depreciation	<u>24.9</u>		
Net Fixed Assets	133.9	Common Stock (8 Mil. Outst.)	10.4
		Retained Earnings	<u>95.4</u>
		Total Common Equity	105.8
Total Assets	<u>286.9</u>	Total Liabilities and Equity	<u>286.9</u>

Figure 1.1: Balance Sheet

Let's look at a definition of each account in XYZ Corporation's Balance Sheet.

Assets

Assets are used to produce the products or generate the services that are sold by the company. Although all assets are stated in monetary terms, only cash represents actual money.

Short-term Assets

Current

Some asset accounts represent the current (short-term) assets of XYZ Corporation. They include:

- **CASH** – amount of cash held by the company in liquid bank accounts
- **MARKETABLE SECURITIES** – value of short-term investments held by the company
- **ACCOUNTS RECEIVABLE** – amount owed to the company by its customers who have purchased their products or services on credit terms
- **INVENTORIES** – amount of money the company has invested in raw materials, work-in-progress, and finished goods available for sale
- **PREPAID EXPENSES** – amount of expenses paid by the company before the expense is incurred. One example is rent on an office building paid at the beginning of the year for the entire year.
- **OTHER CURRENT ASSETS** – any other item related to production that does not fit into the above classifications

Long-term Assets

Investments for operations

XYZ Corporation also has long-term **investments** for the operation of its business. These asset accounts include:

- **GROSS FIXED ASSETS** – amount that the company paid for its property, manufacturing plants, and equipment when it acquired those assets at some time in the past
- **DEPRECIATION** – total amount of money that the company has charged as an expense for using the plants and equipment. Since this account is a deduction from an asset account, it is referred to as a *contra-asset* account.

- **NET FIXED ASSETS** – difference between GROSS FIXED ASSETS and DEPRECIATION. This is often called the *book value* of the fixed assets.

*Assets listed
in order of
liquidity*

The assets on the Balance Sheet are listed in the order of their **liquidity**, which is the amount of time it would typically take to convert them to cash. For example, it is much easier to convert finished products in inventory to cash than to convert a manufacturing plant to cash.

A company may use more detail in reporting its assets on the Balance Sheet. For example, a company may list its property, plants, and equipment separately or break out its accounts receivable or inventories into more specific accounts. The basic concepts of reporting the company's financial position at a point in time remain the same regardless of the amount of detail. Remember, only the cash account represents actual money. A company expects the other assets to produce cash eventually, but the amount of cash produced may be higher or lower than the values presented on the Balance Sheet.

Liabilities (Debt) and Equity

*Capital used to
acquire assets*

To acquire assets, it is necessary for the company to raise capital to pay for them. Capital comes in two basic forms: debt and equity.

Debt vs. Equity

*Debt: money
owed by the
company*

In Balance Sheet analysis, debt is synonymous with liability. It represents money owed by the company to its creditors. Creditors expect the company to repay debt, with interest, at some specified future date. There are many kinds of debt: long-term and short-term, **zero-coupon** and interest-bearing, fixed rate and floating rate, secured and unsecured, and so on. We will discuss these in greater detail later in this workbook.

***Equity: claim
on ownership***

The other form of capital is equity. Equity represents money paid to the company in exchange for a claim on the ownership of the company. These claims are usually called shares or stock, and owners of claims are called shareholders or stockholders. Shares are issued in two classifications: preferred and common.

Liability / Equity Accounts

Now that you know the difference between debt and equity, let's look at a brief description of the accounts appearing on the LIABILITIES AND EQUITY side of XYZ Corporation's Balance Sheet (Figure 1.1).

Current liabilities

The first group of accounts includes all current liabilities which the company is obligated to pay within the next 90 days.

- **ACCOUNTS PAYABLE** – amount owed by the company for materials they have purchased on credit
- **NOTES PAYABLE** – amount the company has borrowed from banks in the form of short-term loans
- **ACCRUED WAGES AND TAXES** – for wages, the amount of wages earned by the employees, but not yet paid by the company; for taxes, the amount of taxes incurred by the business, but not yet paid to the respective governments by the company
- **OTHER CURRENT LIABILITIES** – any other obligations that the company is expected to pay in the short-term (usually around 90 days)

***Long-term
liabilities /
equity***

The company also has long-term sources of capital. These accounts include:

- **LONG-TERM DEBT** – amount the company has borrowed in the form of bonds sold to investors or banks
- **PREFERRED STOCK** – amount paid by investors to the company wanting a priority claim on the assets of the company

- **COMMON STOCK** – amount paid to the company by investors in exchange for a claim on the ownership of the company. Often, the number of outstanding shares is included on this line.
- **RETAINED EARNINGS** – value of the assets of the company in excess of the claims upon those assets (liabilities and stockholders' ownership). This does not represent cash held in the company.

The liability accounts are listed in the order in which they must be paid. Accounts payable are generally due within 30 days; stockholders' equity accounts represent ownership and never need to be paid off.

Assets Equal Liabilities Plus Equity

*Source of term
"balance" sheet*

Notice that the total of all assets is equal to the total of the liabilities and equity (286.9 at the bottom of each side of the Balance Sheet in our example). This is always true, hence the term "balance" sheet. The value of the assets equals the amount of money borrowed by the company plus the value of the owner's investment in the company.

The stockholders' (common) equity, also known as net worth, is the residual between the value of the assets and the value of the liabilities. In our example Balance Sheet:

$$\begin{array}{rcl} \text{Assets} & - & \text{Liabilities} & = & \text{Common Equity (Net Worth)} \\ 286.9 & - & 181.1 & = & 105.8 \end{array}$$

Suppose that assets decline in value; for example, some of the accounts receivable are written off as bad debts. Liabilities remain constant; the value of common equity is adjusted so that both sides of the Balance Sheet remain equal. If the value of assets increases, those benefits accrue solely to the stockholders (common equity).

If, for some reason, the company is no longer able to function profitably, its assets will likely be sold. The proceeds of these sales will first be used to pay the creditors of the company and then to repay the preferred stockholders' investment. Any remaining proceeds are divided among the common shareholders. You can see why stockholders' investments are often thought of as "risky."

Preferred stock – debt or equity?

Preferred stock is often difficult to classify as debt or equity. Many researchers and analysts consider preferred stock to be a hybrid – similar to a liability in some respects and similar to common equity in others.

Like bonds, preferred stock has a **par value**. The par value is printed on the stock certificate and represents the value of the stock at the time it was issued. Preferred dividends are similar to interest payments when they are fixed and paid at regular intervals.

A company usually must pay preferred dividends before paying common dividends. However, a company may defer payment of preferred dividends. This will not lead to bankruptcy, but forgoing interest payments probably will lead to trouble.

Typically, accountants classify preferred stock as equity and list it in the equity section of the Balance Sheet. A bondholder will also consider preferred stock as having similar characteristics to common equity. However, a common shareholder will classify preferred shares as liabilities because preferred shareholders have a priority claim over the common shareholders.

For the purposes of this course, we will consider preferred stock to behave like a liability, and equity calculations will not include the value of the preferred stock. Typically, the unit of the bank in which you work will have specific instructions on how to handle preferred stock in an analysis.

Summary

The Balance Sheet is a financial statement that includes a list of assets owned by the company in order to produce its products or supply its services and a list of liabilities and equities (sources of capital) used by the company to acquire those assets. During the normal course of business, many of these accounts are constantly increasing or decreasing. The Balance Sheet represents a snapshot of asset and liability accounts at a specified time.

You have completed the "Balance Sheet" section of *Financial Statement Analysis*. Please complete the Progress Check and then continue with the "Income Statement" section. If you answer any questions incorrectly, please review the appropriate text.

PROGRESS CHECK 1.1

Directions: Use the Balance Sheet for Fruit Packing, Inc. to answer the following questions. Calculate the answer to each question, then mark the correct answer. Check your solution with the Answer Key on the next page.

Fruit Packing, Inc. — Balance Sheet			
December 31, 1993		(In Millions \$)	
ASSETS		LIABILITIES AND EQUITY	
Cash	7.4	Accounts Payable - Fruit	10.2
Marketable Securities	0.6	Accounts Payable - Material	9.6
Accounts Receivable	23.7	Notes Payable	23.0
Raw Goods Inventory	19.5	Accrued Wages and Taxes	1.7
Finished Goods Inventory	22.4	Other Current Liabilities	<u>0.6</u>
Prepaid Expenses	1.5	Total Current Liabilities	45.1
Other Current Assets	<u>0.8</u>		
Total Current Assets	75.9	Long-term Debt	45.1
		Preferred Stock	<u>7.8</u>
Gross Fixed Assets	101.2	Total Long-term Liabilities	98.0
Less Depreciation	<u>19.9</u>		
Net Fixed Assets	_____	Common Stock	15.1
		Retained Earnings	<u>44.1</u>
		Total Common Equity	59.2
Total Assets	<u>157.2</u>	Total Liabilities and Equity	<u>157.2</u>

- The value for the NET FIXED ASSET account is missing. What should the correct amount be for the Balance Sheet to be correct?
 - _____ a) 121.1
 - _____ b) 177.1
 - _____ c) 137.3
 - _____ d) 81.3
 - _____ e) 56.0
- Suppose that \$1.2 million worth of apples spoiled before the company was able to package them. What asset account would be affected?
 - _____ a) Cash
 - _____ b) Accounts Receivable
 - _____ c) Raw Goods Inventory
 - _____ d) Prepaid Expenses
 - _____ e) Gross Fixed Assets

ANSWER KEY

1. The value for the NET FIXED ASSET account is missing. What should the correct amount be for the Balance Sheet to be correct?

d) 81.3

$$\text{GROSS FIXED ASSETS} - \text{DEPRECIATION} = \text{NET FIXED ASSETS}$$

$$101.2 - 19.9 = 81.3$$

2. Suppose that \$1.2 million worth of apples spoiled before the company was able to package them. What asset account would be affected?

c) Raw Goods Inventory

The company uses fresh fruit and packaging materials as raw materials to produce packaged fruit as its product for sale.

PROGRESS CHECK 1.1
(Continued)

3. A customer of Fruit Packing, Inc. is no longer able to pay its bills. The amount to be written off as a loss is \$1.9 million. What will the new ACCOUNTS RECEIVABLE amount be?
- _____ a) 25.6
 - _____ b) 21.8
 - _____ c) 17.6
 - _____ d) 20.5
 - _____ e) 5.5
4. This event will also affect an account on the other side of the Balance Sheet. What is this account?
- _____ a) Accounts Payable - Fruit
 - _____ b) Notes Payable
 - _____ c) Long-term Debt
 - _____ d) Preferred Stock
 - _____ e) Retained Earnings
5. What will the new amount of this account be?
- _____ a) 1.3
 - _____ b) 21.1
 - _____ c) 43.2
 - _____ d) 5.9
 - _____ e) 42.2

ANSWER KEY

3. A customer of Fruit Packing, Inc. is no longer able to pay its bills. The amount to be written off as a loss is \$1.9 million. What will the new ACCOUNTS RECEIVABLE amount be?

b) 21.8

$$23.7 - 1.9 = 21.8$$

4. This event will also affect an account on the other side of the Balance Sheet. What is this account?

e) Retained Earnings

The value of the stockholder's equity is affected by a change in the value of an asset. This value is found in the RETAINED EARNINGS account.

5. What will the new amount of this account be?

e) 42.2

$$44.1 - 1.9 = 42.2$$

INCOME STATEMENT

Summary of operations and profitability

The Income Statement provides a summary of a company's operations and profitability over a given period of time (at the end of a month, quarter, or year). It shows the value of the products and services sold by the company for the reporting period, the costs incurred in achieving those sales, and the distribution of the residual income. The Income Statement provides an analyst with clues about the profitability of a company's operations. A typical Income Statement is shown in Figure 1.2.

XYZ Corporation	
December 31, 1993	(In Millions \$)
Net Sales	287.6
Cost and Expenses:	
Labor and Materials	249.3
Depreciation	8.9
Selling Expenses	1.6
General and Administrative	3.2
Lease Payments	<u>2.1</u>
Total Operating Costs	<u>265.1</u>
Net Operating Income (EBIT)	22.5
Interest Expenses	<u>6.0</u>
Earnings Before Taxes	16.5
Taxes (at 40%)	<u>6.6</u>
Net Income Before Preferred Dividend	9.9
Preferred Dividend	<u>3.7</u>
Net Income to Common	6.2
Common Dividend	<u>4.0</u>
Earnings Retained	<u>2.2</u>

Figure 1.2: Income Statement

Description of accounts

Let's look at a description of each account included in the example Income Statement for XYZ Corporation.

- **NET SALES** – total value of all goods or services sold by the company as part of its normal business operations during the period

- **COSTS AND EXPENSES**

LABOR AND MATERIALS – total amount spent on labor wages and material costs to produce the items sold for the period. It is common for these costs to be separated into two accounts: material costs listed as **COST OF GOODS SOLD** and labor listed as **DIRECT LABOR COST**.

DEPRECIATION – amount charged by the company for the use of fixed assets to produce the items sold. This is not a cash expenditure. It is an estimate of the decline in value of the plant and equipment resulting from its use during the period.

SELLING EXPENSES – amount spent to sell the products or services produced (includes advertising, sales commissions, promotions, etc.)

GENERAL AND ADMINISTRATIVE – amount spent to manage the production and sales of the company's products or services (includes management salaries, office equipment, support staff, etc.)

LEASE PAYMENTS – amount spent on leasing plants, equipment, office space, etc.

- **NET OPERATING INCOME** – amount earned by the company's operations during the reporting period. The calculation is **NET SALES** minus **COSTS AND EXPENSES**. This value is also known as **EARNINGS BEFORE INTEREST AND TAXES (EBIT)**.

The Income Statement format varies between companies. The **COST AND EXPENSES** section may have more or less detail than our example. Common variations may include combining **SELLING EXPENSES** with **GENERAL AND ADMINISTRATIVE EXPENSES** or combining **DEPRECIATION EXPENSES** with **COST OF LABOR AND MATERIALS**.

The remaining Income Statement accounts are as follows.

- **INTEREST EXPENSES** – amount of interest paid on the company's loans, notes, and bonds during the period
- **EARNINGS BEFORE TAXES** – NET OPERATING INCOME minus INTEREST EXPENSES
- **TAXES** – amount of taxes paid by the company during the period. Many companies provide the total tax assessment percentage on the Income Statement.
- **NET INCOME BEFORE PREFERRED DIVIDEND** – amount of income available for distribution to shareholders; calculated as EARNINGS BEFORE TAXES minus TAXES
- **PREFERRED DIVIDEND** – amount distributed to preferred shareholders during the period
- **NET INCOME TO COMMON** – amount available for distribution to common shareholders or for internal use
- **COMMON DIVIDEND** – amount distributed to common shareholders during the period
- **EARNINGS RETAINED** – amount held by the business to continue operations

Shareholder Return on Investment

Appreciation in value / dividends

There are two ways that a shareholder can receive a return on investment. One is the appreciation in value of the shares of the company (based on the future prospects of the company). The other is a cash payment (dividend) paid by the company to the shareholders. Many companies do not pay dividends to common shareholders, especially growing companies with lots of investment opportunities. We will discuss the methodology for determining the value of stocks later in the course.

Depreciation

DEPRECIATION in the Income Statement is a charge against income based on an estimate of the percentage of the original cost for fixed assets that has been used up in the production process during the period covered by the Income Statement.

For example, suppose that a piece of equipment is purchased for \$500,000. For tax purposes, most governments do not allow companies to charge the entire \$500,000 as a business expense at the time of purchase. Instead, they require the purchase to be "capitalized," which means it is carried on the books as an asset.

However, equipment has a finite useful life to the company, so governments allow a portion of the equipment purchase price to be deducted from operating expenses each year of its useful life. Typically, governments will provide depreciation schedules that classify each type of equipment and dictate its estimated useful life.

Straight-line depreciation

In our example, the piece of equipment may have an estimated useful life of five years and, therefore, will have no value at the end of the five years. Using a straight-line depreciation method, the annual amount that the company may deduct is $(\$500,000 - \$0) / 5 \text{ yrs.} = \$100,000$. There are several different depreciation methods and any accounting text can provide a more detailed discussion of their calculations and uses.

Non-cash bookkeeping entry

Remember, DEPRECIATION is not a cash expense like LABOR AND MATERIAL costs; it is simply a bookkeeping entry on the Balance Sheet and on the Income Statement. The cash expense for fixed assets is incurred at the time of purchase.

- **On the Balance Sheet**, GROSS FIXED ASSETS (capitalized purchases of property, plants, and equipment) are listed at their purchase price. Accumulated DEPRECIATION (sum of all depreciation charges over the life of the assets currently on the company's Balance Sheet) is deducted to arrive at NET FIXED ASSETS. NET FIXED ASSETS can be considered as an estimate of the value of those assets for the remainder of their useful lives. These principles will also be important in the next section as we analyze the Cash Flow Statement.
- **On the Income Statement**, the total DEPRECIATION for the period for all CAPITALIZED ASSETS is deducted from earnings as an OPERATING COST.

Summary

The Income Statement summarizes a company's operational expenses and profitability over a given period of time. The total value of goods and services sold by a company after deducting all operating costs is the NET OPERATING INCOME. The deduction of interest, taxes, and shareholder dividends results in the amount of earnings retained by the company to support operations.

DEPRECIATION is a non-cash bookkeeping entry on the Balance Sheet and on the Income Statement. On the Balance Sheet, it is the sum of all depreciation charges over the life of the fixed assets. On the Income Statement, it represents the portion of the value of a fixed asset that has been used up for operations during the period.

You have completed the "Income Statement" section of *Financial Statement Analysis*. Please complete the Progress Check and then continue with the section on "Cash Flow Statement." If you answer any questions incorrectly, please review the appropriate text.

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 **PROGRESS CHECK 1.2**

Directions: Select the correct answer for each question. Check your solution with the Answer Key on the next page.

6. Suppose that a company has NET SALES of \$332 million, INTEREST EXPENSES of \$16 million, DEPRECIATION of \$6 million, \$225 million expenses for LABOR AND MATERIALS, and ADMINISTRATIVE EXPENSES of \$42 million. What is the EBIT for that company?
- _____ a) \$43 million
_____ b) \$49 million
_____ c) \$59 million
_____ d) \$65 million
_____ e) \$91 million
7. Use the information in question 6, and apply a tax rate of 40%. What is the NET INCOME available for distribution to the shareholders of this company?
- _____ a) \$23.6 million
_____ b) \$25.8 million
_____ c) \$35.4 million
_____ d) \$43.0 million
_____ e) \$59.0 million
8. XYZ Corporation purchases a widget maker for \$750,000. The piece of equipment has an estimated useful life of ten years. Using a straight-line depreciation method, the annual amount the company can deduct as an OPERATING COST is:
- _____ a) \$7,500 per year
_____ b) \$150,000 per year
_____ c) \$1,500 per year
_____ d) \$75,000 per year

ANSWER KEY

6. Suppose that a company has NET SALES of \$332 million, INTEREST EXPENSES of \$16 million, DEPRECIATION of \$6 million, \$225 million expenses for LABOR AND MATERIALS, and ADMINISTRATIVE EXPENSES of \$42 million. What is the EBIT for that company?

c) \$59 million

EBIT = NET SALES - OPERATING COSTS

	<i>(in millions)</i>	
Net Sales	\$332	
Less:		
LABOR AND MATERIALS	\$225	
DEPRECIATION	6	
ADMINISTRATIVE	42	
EBIT	<u>\$ 59</u>	

INTEREST EXPENSE is not an OPERATING COST

7. Use the information in question 6, and apply a tax rate of 40%. What is the NET INCOME available for distribution to the shareholders of this company?

b) \$25.8 million

Start with EBIT and deduct INTEREST and TAXES

	<i>(in millions)</i>	
EBIT	\$59.0	
Less:		
INTEREST	\$16.0	
TAXES (@40%)	17.2	<i>(59.0 - 16.0) x 0.40</i>
NET INCOME	<u>\$25.8</u>	

8. XYZ Corporation purchases a widget maker for \$750,000. The piece of equipment has an estimated useful life of ten years. Using a straight-line depreciation method, the annual amount the company can deduct as an OPERATING COST is:

d) \$75,000 per year

(\$750,000 , 10 years)

CASH FLOW STATEMENT

Sources and uses of funds

The financial statement that is used to show the sources and uses of a company's funds is called a Cash Flow Statement. It also may be called the "Statement of Changes in Financial Position" or the "Sources and Uses Statement," depending on its structure. The analyst uses the Cash Flow Statement to examine the flow of funds into a company and the use of those funds. We will use the terms "funds" and "cash" interchangeably throughout this section.

An analyst may have difficulty gaining access to the applicable information needed to determine a company's sources and uses of cash. This is especially true when studying a competitor. One solution is to construct a Cash Flow Statement based on publicly available information (such as the Income Statement and Balance Sheet from a company's annual report). The objective is to develop a framework that provides an accurate estimate of the company's cash flows. It is important for you to learn to construct a Cash Flow Statement as preparation for Unit Three – *Time Value of Money*.

We will continue to use the XYZ Corporation as our example. The company's Balance Sheet for the past two years is shown again in Figure 1.3 and the Income Statement is shown again in Figure 1.4. These two financial statements provide the information for the Cash Flow Statement, and you will want to refer to them as we move through the process.

XYZ Corporation					
December 31, 1993 (In Millions \$)					
ASSETS	1992	1993	LIABILITIES AND EQUITY	1992	1993
Cash	6.2	5.7	Accounts Payable	20.2	22.7
Marketable Securities	4.9	6.3	Notes Payable	29.6	31.5
Accounts Receivable	51.8	50.9	Accrued Wages and Taxes	2.1	2.3
Inventories	92.4	88.7	Other Current Liabilities	<u>5.6</u>	<u>4.9</u>
Prepaid Expenses	0.3	1.1	Total Current Liabilities	57.5	61.4
Other Current Assets	<u>0.2</u>	<u>0.3</u>			
Total Current Assets	155.8	153.0	Long-term Debt	102.7	107.4
			Preferred Stock	<u>12.2</u>	<u>12.3</u>
Gross Fixed Assets	136.2	158.8	Total Long-term Liabilities	172.4	181.1
Less Depreciation	<u>16.0</u>	<u>24.9</u>			
Net Fixed Assets	120.2	133.9	Common Stock (8 Mil. Outst.)	10.4	10.4
			Retained Earnings	<u>93.2</u>	<u>95.4</u>
			Total Common Equity	103.6	105.8
Total Assets	<u>276.0</u>	<u>286.9</u>	Total Liabilities and Equity	<u>276.0</u>	<u>286.9</u>

Figure 1.3: XYZ Corporation – Balance Sheet for Two Years

XYZ Corporation	
December 31, 1993 (In Millions \$)	
Net Sales	287.6
Cost and Expenses:	
Labor and Materials	249.3
Depreciation	8.9
Selling Expenses	1.6
General and Administrative	3.2
Lease Payments	<u>2.1</u>
Total Operating Costs	<u>265.1</u>
Net Operating Income (EBIT)	22.5
Interest Expenses	<u>6.0</u>
Earnings Before Taxes	16.5
Taxes (at 40%)	<u>6.6</u>
Net Income Before Preferred Dividend	9.9
Preferred Dividend	<u>3.7</u>
Net Income to Common	6.2
Common Dividend	<u>4.0</u>
Earnings Retained	<u>2.2</u>

Figure 1.4: XYZ Corporation – Income Statement

Now, let's see how we build the Cash Flow Statement. It is grouped into two main sections:

- Funds generated and used by operating activities
- Funds generated by financing activities

Operating Activities

We begin by identifying the operating activities that contribute to the company's cash flow. Then we classify each activity as a source or a use of cash.

Sources

*Balance Sheet:
increased
liabilities;
decreased assets*

First, let's see which accounts in the Balance Sheet represent sources of cash from operating activities. There are two general rules to determine if a change in a Balance Sheet account is a source of cash:

1. An **increase in a liability or equity account** is a source of cash.

For example, an increase in ACCOUNTS PAYABLE (from \$20.2 million in 1992 to \$22.7 million in 1993) indicates that in 1993, XYZ Company borrowed \$2.5 million more from creditors than it paid off. The company had access to that \$2.5 million for use in the business.

2. A **decrease in an asset account** is a source of cash.

For example, the decrease in the INVENTORY account (from \$92.4 million in 1992 to \$88.7 million in 1993) indicates that XYZ Corporation reduced its investment in inventories by \$3.7 million over the period. XYZ then had \$3.7 million to use in some other part of the company's operations.

Income Statement

There are two accounts in the Income Statement that represent sources of cash. The first, NET INCOME BEFORE PREFERRED DIVIDEND, is the cash generated by the sale of the product or services that the company has available for discretionary expenditures. We are assuming that the costs and expenses related to production, interest, and taxes are all non-discretionary expenses. For XYZ Corporation, we will use the line:

Net Income Before Preferred Dividend	\$9.9 million
---	----------------------

The other account we include in the Cash Flow Statement as a source of cash is DEPRECIATION. Remember, depreciation is deducted as an operating expense in the Income Statement. Since depreciation is not actually a cash outflow, we must add XYZ's 1993 DEPRECIATION figure of \$8.9 million into the Cash Flow Statement so that the cash flow sources are accurate.

Uses

Balance Sheet:
decreased
liabilities;
increased assets

As with the sources, there are two simple rules for calculating the uses of cash in a company.

1. A **decrease in a liability or equity account** is a use of funds.

Paying off a loan is one example. In our XYZ Corporation example, the OTHER CURRENT LIABILITIES account decreased by \$700,000 (from \$5.6 million in 1992 to \$4.9 million in 1993). This means that XYZ paid off \$700,000 of short-term liabilities.

2. An **increase in an asset account** is a use of cash.

The increase indicates that funds were used to purchase additional assets. XYZ purchased \$22.6 million worth of FIXED ASSETS during 1992 (from \$136.2 million in 1992 to \$158.8 million in 1993).

Financing Activities

Buying and selling of capital

Financing activities are related to the buying and selling of capital. Remember, a company has two alternatives for raising capital: debt and equity. This section of the Cash Flow Statement is a summary of the equity raised and the debt borrowed and paid off. Usually, only the net changes of each account are listed.

Cash flow – net increase / (decrease)

For example, XYZ Corporation's LONG-TERM BONDS account was \$102.7 million in 1992 and \$107.4 million in 1993. The company may have paid off \$25 million in bonds and issued \$29.7 million in new bonds, but the Cash Flow Statement will only show the net increase of \$4.7 million.

With that brief explanation about the sources and uses of funds, let's take a look at XYZ Corporation's Cash Flow Statement.

XYZ Corporation	
December 31, 1993	(In Millions \$)
OPERATING ACTIVITIES:	
Sources:	
Net Income Before Preferred Dividends	9.9
Depreciation	8.9
Incr. in Accounts Payable	2.5
Incr. in Accrued Wages and Taxes	0.2
Incr. in Other Current Liabilities	<u>(0.7)</u>
Total Sources from Operations	20.8
Uses:	
Incr. in Accounts Receivable	(0.7)
Incr. in Inventories	(3.7)
Incr. in Prepaid Expenses	0.8
Incr. in Other Current Assets	0.1
Incr. in Gross Fixed Assets	<u>22.6</u>
Total Uses from Operations	18.9
FINANCING ACTIVITIES:	
Incr. in Notes Payable	1.9
Incr. in Long-term Bonds	4.7
Incr. in Preferred Stock	0.1
Incr. of Common Stock	<u>0.0</u>
Net Funds from Financing	6.7
Total Funds from Operations and Financing	8.6
Less Common and Preferred Dividends	<u>7.7</u>
Incr. (Decr.) in Cash and Mkt Sec.	<u>0.9</u>

Figure 1.5: XYZ Corporation Cash Flow Statement

As you can see, the Cash Flow Statement is divided into OPERATING ACTIVITIES, Sources and Uses, and FINANCING ACTIVITIES. The final calculation is the NET INCREASE or DECREASE IN CASH AND MARKETABLE SECURITIES. Let's see how we get there.

***Operating activities:
Sources***

NET INCOME BEFORE PREFERRED DIVIDENDS (\$9.9 million) and the \$8.9 million DEPRECIATION charge are taken directly from the 1993 Income Statement.

The current liabilities from the Balance Sheet are listed individually (INCREASE IN ACCOUNTS PAYABLE, INCREASE IN ACCRUED WAGES AND TAXES, INCREASE IN OTHER CURRENT LIABILITIES) with their net change from 1992 to 1993. TOTAL SOURCES FROM OPERATIONS (\$20.8 million) is the sum of these sources.

***Operating activities:
Uses***

Next, the net change in each asset account except CASH and MARKETABLE SECURITIES is listed: INCREASE IN ACCOUNTS RECEIVABLE, INCREASE IN INVENTORIES, INCREASE IN PREPAID EXPENSES, INCREASE IN OTHER CURRENT ASSETS, INCREASE IN GROSS FIXED ASSETS. GROSS FIXED ASSETS is used instead of NET FIXED ASSETS because we have already adjusted for DEPRECIATION and we do not want to double count. TOTAL USES FROM OPERATIONS is the sum of these changes.

Financing activities

Finally, we summarize the financing activities by listing each net change from 1992 to 1993 for the potential sources of capital: INCREASE IN NOTES PAYABLE, INCREASE IN LONG-TERM BONDS, INCREASE IN PREFERRED STOCK, INCREASE OF COMMON STOCK. NET FUNDS FROM FINANCING is the sum of the financing activities.

Total funds from operations and financing

TOTAL FUNDS FROM OPERATIONS AND FINANCING is calculated by subtracting the uses from the sources and adding to that number the NET FUNDS FROM FINANCING. In our XYZ example, that calculation is:

	Total operating sources	\$20.8 million
-	Total operating uses	18.9 million
+	Net financing sources	6.7 million
	Total funds from operations and financing	\$ 8.6 million

***Increase
(Decrease) in
cash and
marketable
securities***

Finally, we subtract the total value of the COMMON and PREFERRED DIVIDENDS (from the Income Statement) to arrive at the net INCREASE or DECREASE in CASH and MARKETABLE SECURITIES.

$$\mathbf{\$8.6 \text{ million} - \$7.7 \text{ million} = \$0.9 \text{ million}}$$

Our check to make sure that the calculation is correct is to calculate the net change in the Balance Sheet accounts for CASH and MARKETABLE SECURITIES.

$$\mathbf{(\$5.7 \text{ million} + \$6.3 \text{ million}) - (\$6.2 \text{ million} + \$4.9 \text{ million}) = \$0.9 \text{ million}}$$

For our XYZ Corporation, we have calculated the sources and uses of cash correctly.

Summary

The Cash Flow Statement is a summary of the sources and uses of cash for a specified period of time. Different formats for presenting the information have other names, such as "Statement of Changes in Financial Position." Regardless of the format, the basic rules for calculating cash flow remain the same. Any basic finance or accounting text will give additional information on these other formats. The unit of the bank in which you work will have specific instructions on the format you should use for your analysis.

You have completed the "Cash Flow Statement" section of *Financial Statement Analysis*. Please complete Progress Check 1.3, then continue to the next section that covers "Financial Ratios." If you answer any questions incorrectly, please review the appropriate text.

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 **PROGRESS CHECK 1.3**

Directions: Use the Balance Sheet and Income Statement information presented below to answer the questions. Calculate the answer to each question, then check your solution with the Answer Key on the next page.

Fruit Packing, Inc. — Balance Sheet					
December 31, 1993		(In Millions \$)			
ASSETS	1992	1993	LIABILITIES AND EQUITY	1992	1993
Cash	6.7	7.4	Accounts Payable - Fruit	9.6	10.2
Marketable Securities	0.0	0.6	Accounts Payable - Material	9.2	9.6
Accounts Receivable	22.4	23.7	Notes Payable	21.8	23.0
Raw Goods Inventory	19.2	19.5	Accrued Wages and Taxes	1.5	1.7
Finished Goods Inventory	22.3	22.4	Other Current Liabilities	<u>0.7</u>	<u>0.6</u>
Prepaid Expenses	1.2	1.5	Total Current Liabilities	42.8	45.1
Other Current Assets	<u>0.8</u>	<u>0.8</u>			
Total Current Assets	72.6	75.9			
Gross Fixed Assets	97.5	101.2	Long-term Debt	46.1	45.1
Less Depreciation	<u>16.8</u>	<u>19.9</u>	Preferred Stock	<u>7.8</u>	<u>7.8</u>
Net Fixed Assets	80.7	81.3	Total Long-term Liabilities	96.7	98.0
			Common Stock	15.1	15.1
			Retained Earnings	<u>41.5</u>	<u>44.1</u>
			Total Common Equity	56.6	59.2
Total Assets	<u>153.3</u>	<u>157.2</u>	Total Liabilities and Equity	<u>153.3</u>	<u>157.2</u>

Fruit Packing, Inc. — Income Statement	
December 31, 1993	
(In Millions \$)	
Net Sales	150.6
Operating Expenses	101.2
Depreciation	<u>3.1</u>
EBIT	46.3
Interest Expense	<u>11.5</u>
Earnings Before Taxes	34.8
Taxes (@ 40%)	<u>13.9</u>
Net Income to Shareholders	20.9
Preferred Dividends	<u>6.3</u>
Net Income to Common	14.6
Common Dividends	<u>12.0</u>
Earnings Retained	<u>2.6</u>

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PROGRESS CHECK 1.3
(Continued)

Assume you are building a Cash Flow Statement for Fruit Packing, Inc.

9. How much is the total cash **SOURCE** from operations?

- _____ a) \$19.9 million
- _____ b) \$23.6 million
- _____ c) \$24.6 million
- _____ d) \$25.1 million
- _____ e) \$44.7 million

10. How much is the total cash **USE** from operations?

- _____ a) \$0.3 million
- _____ b) \$0.4 million
- _____ c) \$1.3 million
- _____ d) \$2.0 million
- _____ e) \$5.7 million

11. What is the total of net funds from **FINANCING**?

- _____ a) (\$1.0) million
- _____ b) \$0.2 million
- _____ c) \$1.0 million
- _____ d) \$1.2 million
- _____ e) \$1.3 million

ANSWER KEY

9. How much is the total cash **SOURCE** from operations?

d) \$25.1 million

(in millions \$)

NET SALES	20.9
DEPRECIATION	3.1
INCREASE ACCOUNTS PAYABLE	1.0
INCREASE ACCRUED WAGES AND TAXES	0.2
INCREASE OTHER CURRENT LIABILITIES	<u>(0.1)</u>
TOTAL SOURCES FROM OPERATIONS	25.1

10. How much is the total cash **USE** from operations?

e) \$5.7 million

(in millions \$)

INCREASE IN ACCOUNTS RECEIVABLE	1.3
INCREASE IN INVENTORIES	0.4
INCREASE IN PREPAID EXPENSES	0.3
INCREASE IN OTHER CURRENT ASSETS	0.0
INCREASE IN GROSS FIXED ASSETS	<u>3.7</u>
TOTAL USES FROM OPERATIONS	5.7

11. What is the total of net funds from **FINANCING**?

b) \$0.2 million

(in millions \$)

INCREASE IN NOTES PAYABLE	1.2
INCREASE IN LONG-TERM BONDS	<u>(1.0)</u>
INCREASE IN COMMON STOCK	0.0
INCREASE IN PREFERRED STOCK	<u>0.0</u>
TOTAL FUNDS FROM FINANCING	0.2

FINANCIAL RATIOS

Financial statements used to predict future earnings

So far, we have seen how the Balance Sheet reports a company's position at a point in time, how the Income Statement reports a company's operations over a period of time, and how the Cash Flow Statement reports a company's sources and uses of funds over that period. The real value of financial statement analysis is to use these statements to forecast a firm's future earnings.

From an investor's point of view, forecasting the future is the main purpose of financial statement analysis. From a manager's viewpoint, financial statement analysis is useful as a way to anticipate future conditions and, most important, as a starting point for developing strategies that influence a company's future course of business.

Ratios highlight relationships between accounts

An important step toward achieving these goals is to analyze the firm's financial ratios. Ratios are designed to highlight relationships between the financial statement accounts. These relationships begin to reveal how well a company is doing in its primary goal of creating value for its shareholders.

Useful with application of analytical techniques

The ratios, alone, usually give the analyst very little information. There are two **ratio analysis** techniques that provide additional insight into a company. The first technique is to compare the ratios of one company with other similar companies within the same industry. The second technique is to observe trends of the ratios over a period of time. These trends give clues about a company's performance.

Five ratio categories

The most common financial ratios can be grouped into five broad categories:

- Liquidity Ratios
- Asset Management Ratios
- Debt Management Ratios
- Profitability Ratios
- Market Value Ratios

We will discuss each of these groups of ratios and the formulas used to calculate them. Refer to XYZ Corporation's Balance Sheet and Income Statement as we calculate each ratio.

XYZ Corporation			
December 31, 1993 (In Millions \$)			
ASSETS		LIABILITIES AND EQUITY	
Cash	5.7	Accounts Payable	22.7
Marketable Securities	6.3	Notes Payable	31.5
Accounts Receivable	50.9	Accrued Wages and Taxes	2.3
Inventories	88.7	Other Current Liabilities	<u>4.9</u>
Prepaid Expenses	1.1	Total Current Liabilities	61.4
Other Current Assets	<u>0.3</u>		
Total Current Assets	153.0	Long-term Debt	107.4
		Preferred Stock	<u>12.3</u>
Gross Fixed Assets	158.8	Total Long-term Liabilities	181.1
Less Depreciation	<u>24.9</u>		
Net Fixed Assets	133.9	Common Stock (8 Mil. Outst.)	10.4
		Retained Earnings	<u>95.4</u>
		Total Common Equity	105.8
Total Assets	<u>286.9</u>	Total Liabilities and Equity	<u>286.9</u>

Figure 1.6: Balance Sheet

XYZ Corporation	
December 31, 1993 (In Millions \$)	
Net Sales	287.6
Cost and Expenses:	
Labor and Materials	249.3
Depreciation	8.9
Selling Expenses	1.6
General and Administrative	3.2
Lease Payments	<u>2.1</u>
Total Operating Costs	<u>265.1</u>
Net Operating Income (EBIT)	22.5
Interest Expenses	<u>6.0</u>
Earnings Before Taxes	16.5
Taxes (at 40%)	<u>6.6</u>
Net Income Before Preferred Dividend	9.9
Preferred Dividend	<u>3.7</u>
Net Income to Common	6.2
Common Dividend	<u>4.0</u>
Earnings Retained	<u>2.2</u>

Figure 1.7: Income Statement

Liquidity Ratios

Coverage of short-term obligations by assets

Liquidity Ratios address one of the first concerns of a firm: Will the company be able to meet its obligations? These ratios attempt to measure the extent to which the short-term creditors of the firm are covered by assets that are expected to be converted to cash in roughly the same time period. Liquidity Ratios include:

- Current Ratio
- Quick (Acid-Test) Ratio

Current Ratio

The **Current Ratio** is calculated by dividing CURRENT ASSETS by CURRENT LIABILITIES. In our XYZ Corporation example, the Current Ratio is:

$$\begin{aligned} \text{Current Ratio} &= (\text{Current Assets}) / (\text{Current Liabilities}) \\ &= (\$153.0) / (\$61.4) \\ &= 2.49 \text{ times} \end{aligned}$$

Sufficiency of assets to cover liabilities

Recall that CURRENT ASSET accounts include CASH, MARKETABLE SECURITIES, INVENTORIES, and PREPAID EXPENSES. The CURRENT LIABILITIES include ACCOUNTS PAYABLE, NOTES PAYABLE, and ACCRUED WAGES AND TAXES. The Current Ratio means that, if necessary, the company could use its current assets to pay off its current liabilities 2.49 times. If a company is experiencing financial difficulty, it may begin to pay its bills more slowly. This causes an increase in bank loans and similar activities. If CURRENT LIABILITIES are rising more quickly than CURRENT ASSETS, the Current Ratio will fall. This could indicate trouble in the company.

This ratio means little as a single value. However, when it is compared to similar companies in the industry or used in a trend analysis, the ratio becomes much more meaningful. For example, if the industry average Current Ratio is 1.82 times, the analyst may conclude that XYZ Corporation's policies are effective in assuring that it will be able to satisfy its current liabilities. If XYZ's Current Ratio is 2.67 times and, in the prior year, was 2.94 times, an analyst may begin to investigate reasons why the company has been less effective this year compared to the previous year.

Quick (Acid-Test) Ratio

Coverage of current liabilities by most liquid current assets

The **Quick (Acid-Test) Ratio** is computed by subtracting INVENTORIES from CURRENT ASSETS, then dividing the remainder by CURRENT LIABILITIES. XYZ Corporation's Quick Ratio can be calculated as follows:

$$\begin{aligned} \text{Quick Ratio} &= (\text{Current Assets} - \text{Inventories}) / (\text{Current Liabilities}) \\ &= (\$153.0 - \$88.7) / (\$61.4) \\ &= 1.05 \text{ times} \end{aligned}$$

Inventories usually are the least liquid of the current assets. They are the most difficult to convert to cash and most likely to incur losses in the course of a liquidation. The Quick Ratio gives an indication of the firm's ability to meet short-term obligations without relying on the sale of inventories. XYZ Corporation can use its most liquid current assets to pay off the current liabilities 1.05 times.

Asset Management Ratios

Assess appropriateness of amounts in asset accounts

These ratios attempt to measure how effectively the company is managing its assets. They are designed to tell the analyst if the amounts of each type of asset reported on the Balance Sheet are reasonable, given current and anticipated operating levels of the firm. Asset Management Ratios include:

- Inventory Turnover Ratio
- Average Collection Period
- Fixed Assets Turnover Ratio
- Total Assets Turnover Ratio

Inventory Turnover Ratio

The **Inventory Turnover (Inventory Utilization) Ratio** is calculated by dividing the Net SALES of the firm by its INVENTORIES.

$$\begin{aligned} \text{Inventory Turnover} &= (\text{Net Sales}) / (\text{Inventories}) \\ &= (\$287.6) / (\$88.7) \\ &= 3.24 \text{ times} \end{aligned}$$

Number of turnover times per year This means that the firm's inventory is roughly sold out and restocked, or "turned over," a little more than three times per year. Obsolete, unnecessary, or excess products held in inventories cause the Asset Turnover Ratio to fall, which may indicate a need for management action.

Avoid overstating turnover rate The analyst should consider two weaknesses of the Inventory Turnover Ratio. The first concern is that sales are stated at market prices, whereas inventories are usually carried at cost. In an environment with rapidly changing prices, the ratio would overstate the inventory turnover rate. When market prices are volatile, a more accurate calculation may be made using Cost of Goods Sold in the numerator.

Allow for seasonal trends The other weakness is that sales occur over the entire year, whereas the inventory is valued at a point of time. A business with highly seasonal trends may calculate the ratio using an average inventory figure.

Average Collection Period

Appraise collection of accounts receivable The **Average Collection Period (ACP)** is often used to appraise ACCOUNTS RECEIVABLE. It is computed by dividing average daily sales by the ACCOUNTS RECEIVABLE. Average daily sales are found by dividing the total year's SALES by 360 (the number of days in the year). For XYZ Corporation, the calculation is:

$$\begin{aligned} \text{ACP} &= (\text{Accounts Receivable}) / (\text{Average Daily Sales}) \\ &= (\$50.9) / (\$287.6/360 \text{ days}) \\ &= 63.7 \text{ days} \end{aligned}$$

The Average Collection Period represents the number of days the company must wait after a sale is made before receiving cash. If XYZ Corporation gives its customers credit terms of 30 days, this ratio indicates that the company is inefficient in collecting its receivables.

Fixed Assets Turnover Ratio

To measure the utilization of the firm's plant and equipment, the **Fixed Assets Turnover (Fixed Asset Utilization) Ratio** can be used. It is the firm's SALES divided by its FIXED ASSETS.

$$\begin{aligned}\text{Fixed Assets Turnover} &= (\text{Sales}) / (\text{Net Fixed Assets}) \\ &= (\$287.6) / (\$133.9) \\ &= 2.15 \text{ times}\end{aligned}$$

Generation of sales by fixed assets

This gives the analyst an idea of how well the fixed assets are being utilized. XYZ Corporation's fixed assets are generating slightly more than two times their value in sales for the company. Unnecessary or underutilized fixed assets that do not increase sales cause this ratio to become lower. Once again, consider that in a period of rapidly changing prices, the value of fixed assets on the Balance Sheet may be seriously understated. This causes a firm with older equipment to report a higher turnover than a firm with more recently purchased plants and equipment.

If the industry average for Fixed Assets Turnover is 2.98 times, a manager of XYZ Corporation may begin to investigate how other companies in the industry are able to generate more sales from their fixed assets.

Total Assets Turnover Ratio

The **Total Assets Turnover Ratio** measures the utilization of the company's assets. To compute the Total Assets Turnover Ratio, divide SALES by TOTAL ASSETS. For XYZ Corporation, it is:

$$\begin{aligned}\text{Total Assets Turnover} &= (\text{Sales}) / (\text{Total Assets}) \\ &= (\$287.6) / (\$286.9) \\ &= 1.00 \text{ times}\end{aligned}$$

Generation of sales by total asset value

Like the Fixed Assets Turnover Ratio, this ratio gives the analyst an indication of how well a company is utilizing its assets. The Total Assets Turnover Ratio indicates how many times the value of all ASSETS is being generated in SALES. The same concerns about understated assets also are applicable to the Total Assets Turnover Ratio.

Debt Management Ratios

Capital structure – mix of debt and equity financing

As discussed earlier, a company can choose to raise part of its capital in the form of equity (money from investors in the company) or in the form of debt (money borrowed from creditors). Debt is generally less expensive than equity, especially when tax issues are relevant (interest payments often are tax-deductible, but dividends usually are not). A company's "**capital structure**" refers to the mix of debt and equity that is used to finance assets. Most companies seek an ideal capital structure, one that maximizes the total value of the company.

*Financial leverage:
Use of debt to capitalize*

Financial leverage is the use of debt financing to raise capital for operations and growth. The concept of financial leverage can be explained with a short example.

Example

Consider a company wishing to grow and expand its operations. The company believes that, with an additional \$10 million in capital, it can generate \$40 million in net income in the coming year. The company can find investors to invest \$10 million in the company. However, at the end of the year, the new investors will be entitled to a share of the income, thus diluting the return to the original shareholders. If the company borrows the \$10 million, the company will simply pay the interest on the loan (and the principal if the loan is due). The entire net income, less the interest payment, will be available for the original shareholders.

***Implications of
using debt to
capitalize***

This use of debt as a source of capital has three important implications:

1. By using debt, control of the firm is achieved with limited equity investment.
2. If only a small percentage of the total funds are invested by equity holders, then creditors bear the risks of the company. For this reason, creditors often require a certain percentage of equity-invested funds as a margin of safety.
3. By leveraging the owners' (equity investors') capital, return on investment is increased if the firm earns more on the use of the borrowed funds than it pays in interest.

***Use of debt to
finance assets***

Analysts have developed ratios to determine the extent of the use of borrowed funds to finance assets and to determine how many times the income generated by those assets can be used to make interest payments. These ratios include:

- Total Debt to Total Assets Ratio
- Times Interest Earned (TIE) Ratio
- Fixed Charge Coverage Ratio

Total Debt to Total Assets Ratio (Debt Ratio)

***Percentage of
total funds from
debt***

The **Total Debt to Total Assets Ratio** measures the percentage of total funds provided by the use of debt. It is calculated by dividing TOTAL DEBT (LIABILITIES) by TOTAL ASSETS.

$$\begin{aligned}
 \text{Debt Ratio} &= (\text{Total Debt}) / (\text{Total Assets}) \\
 &= (\$61.4 + \$107.4) / (\$286.9) \\
 &= 58.8\%
 \end{aligned}$$

Notice that TOTAL DEBT includes CURRENT LIABILITIES and LONG-TERM DEBT. This **Debt Ratio** is used by creditors to help decide if they will loan money to the company.

Times Interest Earned (TIE) Ratio

The **Times Interest Earned Ratio** gives the analyst an idea of how far operating income can decline before the company is unable to meet its interest payments on currently held debt. The TIE Ratio for XYZ Corporation is computed by dividing EARNINGS BEFORE INTEREST AND TAXES (EBIT) by the INTEREST CHARGES.

$$\begin{aligned}\text{Times Interest Earned} &= (\text{EBIT}) / (\text{Interest Charges}) \\ &= (\$22.5) / (\$6.0) \\ &= 3.8 \text{ times}\end{aligned}$$

*Coverage of
interest payments
by earnings*

The calculation uses earnings before interest and taxes in the numerator because interest payments are tax deductible in many countries, and the ability to pay current interest is not affected by taxes. The TIE Ratio indicates to the analyst how many times the company can make interest payments with the earnings generated by the firm.

Fixed Charge Coverage Ratio

*Coverage of
interest payments
plus long-term
lease payments*

The **Fixed Charge Coverage Ratio** has one important difference from the TIE Ratio. Many companies enter long-term lease agreements for assets. This ratio recognizes those leases as obligations and includes the LEASE PAYMENTS as fixed charges along with INTEREST PAYMENTS on loans. The Fixed Charge Coverage Ratio is computed by dividing EBIT plus LEASE PAYMENTS by INTEREST CHARGES plus LEASE PAYMENTS.

$$\begin{aligned}\text{Fixed Charge} &= (\text{EBIT} + \text{Leases}) / (\text{Interest Charges} + \text{Leases}) \\ &= (\$22.5 + \$2.1) / (\$6.0 + \$2.1) \\ &= 3.0 \text{ times}\end{aligned}$$

This ratio is used more often than the TIE ratio, especially in industries where leasing of assets is common. It tells how many times all fixed payments incurred by the company can be made by using all the earnings of the firm.

Profitability Ratios

All policies and decisions made by a company are driven by the company's profitability goal. The previous ratios were designed to provide information about the operations of a company. Another group of ratios, Profitability Ratios, highlight the combined effects of liquidity, asset management, and debt management. These ratios include:

- Profit Margin Ratio
- Basic Earnings Power Ratio
- Return on Total Assets Ratio
- Return on Common Equity Ratio

Profit Margin Ratio

Percentage of sales for common shareholders

The **Profit Margin Ratio** shows the percentage of sales that is left for distribution to the common shareholders. The calculation is NET INCOME AVAILABLE TO COMMON SHAREHOLDERS divided by SALES for the period. For our XYZ Corporation example, the calculation is:

$$\begin{aligned} \text{Profit Margin} &= (\text{Net Income to Common}) / (\text{Sales}) \\ &= (\$6.2) / (\$287.6) \\ &= 2.16\% \end{aligned}$$

The Profit Margin Ratio reveals to the analyst how much profit is being generated by the company for each dollar of sales.

Basic Earnings Power Ratio

Effective use of assets to generate earnings

The **Basic Earnings Power Ratio** is used to help compare firms with different degrees of financial leverage and in different tax situations. It provides the analyst with an idea of how effectively the assets are used to generate earnings. The computation is EARNINGS BEFORE INTEREST and TAXES (EBIT) divided by TOTAL ASSETS.

$$\begin{aligned} \text{Basic Earnings Power} &= (\text{EBIT}) / (\text{Total Assets}) \\ &= (\$22.5) / (\$286.9) \\ &= 7.84\% \text{ of Total Assets} \end{aligned}$$

EBIT is used in the Basic Earnings Power Ratio to eliminate any interest payments or tax considerations of the firm. The Basic Earnings Power Ratio indicates the percentage of TOTAL ASSETS generated as EARNINGS.

Return on Total Assets Ratio

Profitability of company's assets

By taking NET INCOME AVAILABLE TO COMMON SHAREHOLDERS and dividing it by the value of all the ASSETS, the analyst calculates the return on those assets. The **Return on Total Assets (ROA) Ratio** is used to determine the return generated by the company on its assets. For XYZ Corporation, the calculation is:

$$\begin{aligned}\text{Return on Total Assets} &= (\text{Net Income to Common}) / (\text{Total Assets}) \\ &= (\$6.2) / (\$286.9) \\ &= 2.16\% \text{ of Total Assets}\end{aligned}$$

ROA is a common measure of the profitability of a company's ASSETS.

Return on Common Equity Ratio

Return on stockholder investment

The **Return on Common Equity (ROE) Ratio** is a measure of the rate of return on stockholders' investments. It is calculated by dividing NET INCOME AVAILABLE TO COMMON SHAREHOLDERS by the TOTAL COMMON EQUITY capital in the firm.

$$\begin{aligned}\text{Return on Common Equity} &= (\text{Net Income to Common}) / \\ &\quad (\text{Common Equity}) \\ &= (\$6.2) / (\$105.8) \\ &= 5.86\% \text{ of Common Equity}\end{aligned}$$

The ROE Ratio tells the analyst the return that common shareholders had on their investments.

Market Value Ratios

Market view of past and future performance

Market Value Ratios relate the company's stock price data with the earnings and capital structure of the company. This information gives the analyst an idea of the view investors have of the company's past performance and also their view of the firm's future prospects. These ratios include:

- Price / Earnings Ratio
- Market / Book Ratio

Price / Earnings Ratio

Amount investors will pay per dollar of profit

Price / Earnings (P/E) Ratio shows how much investors are willing to pay for every dollar of the company's reported profits. It is calculated by dividing the market price per share of Common Stock by EARNINGS PER SHARE (EPS). The **earnings per share** calculation is NET INCOME AVAILABLE TO COMMON divided by the Number of Shares Outstanding. The Number of Shares can be found next to the COMMON EQUITY figure on the Balance Sheet. Be sure to check the units on the Number of Shares; they may not always be the same as the other figures on the Balance Sheet. On XYZ Corporation's Balance Sheet, all figures are in millions.

$$\begin{aligned} \text{Earnings per share} &= (\text{Net Income to Common}) / (\text{Number of shares}) \\ &= (\$6.2) / (8.0 \text{ shares}) \\ &= \$0.775 \text{ per share} \end{aligned}$$

For our XYZ Corporation example, if the price of XYZ stock is \$12.70 per share, the P/E Ratio is:

$$\begin{aligned} \text{Price / Earnings} &= (\text{Market price per share}) / (\text{Earnings per share}) \\ &= (\$12.70) / (\$0.775) \\ &= 16.4 \text{ times} \end{aligned}$$

The P/E Ratio indicates that the common shares are selling for 16.4 times the EARNINGS of XYZ Corporation.

Market / Book Ratio

Amount investors will pay relative to value on books

The **Market / Book Ratio** shows how much investors are willing to pay relative to the value of the company as shown on its books. The total market value of a company is the value that investors in the stock market (where the shares are being traded) think the company is worth. This value is derived from the stock price at which the shares are trading. For example, the market value for XYZ Corporation is \$101.6 million (8 million shares times \$12.70 per share).

To calculate the Market / Book Ratio, the analyst must first compute the **book value per share**. This computation is the value of COMMON EQUITY divided by the Number of Shares.

$$\begin{aligned}\text{Book value per share} &= (\text{Common Equity}) / (\text{Number of shares}) \\ &= (\$105.8) / (8.0 \text{ shares}) \\ &= \$13.225 \text{ per share}\end{aligned}$$

The Market / Book Ratio is calculated by dividing the market price per share by the book value per share.

$$\begin{aligned}\text{Market / Book} &= (\text{Market price per share}) / (\text{Book value per share}) \\ &= (\$12.70) / (\$13.225) \\ &= 0.96 \text{ times}\end{aligned}$$

The Market / Book Ratio indicates how many times above (or below) the book value of the company investors are paying for an equity position. In our XYZ Corporation example, investors are not quite willing to pay the book value for the equity of the company. (1.00 times means that the market price and the book price are the same.)

Using Ratios

The ratios we have just studied provide the analyst with information about a company's liquidity, asset management, debt management, and profitability. They also indicate how market investors value the company's efforts. These ratios also provide additional insights when compared to the ratios of other companies and when trends are mapped over a period of time.

Industry comparisons

An astute analyst will first calculate a company's ratios and then make comparisons with other similar companies in that industry or with the industry as a whole. Any significant discrepancies will signal the analyst that closer inspection may be needed.

For example, if most of the industries competing with XYZ Corporation have Profit Margin Ratios of over 4% and XYZ has a Profit Margin Ratio of 2.16%, the analyst will begin to look for reasons why XYZ is performing so poorly. It is important to compare companies within the same industry to gain useful observations. An automobile manufacturer will have a much different structure than a consulting company.

Trend analysis

Trend analysis can also provide insights into the conditions of a company. By calculating ratios over a period of several years, an analyst can uncover potential problems within the firm.

For example, if XYZ Corporation has an Inventory Turnover Ratio of 3.24 times in 1993, 3.86 times in 1992, and 4.56 times in 1991, an analyst will begin to look for reasons why it is taking longer for XYZ Corporation to sell its products. These trends give clues as to whether the financial condition is improving or deteriorating. By using industry comparisons and trend analysis, the analyst can focus the investigation on areas of a company that may need attention.

UNIT SUMMARY

In this unit, we discussed the three types of financial statements:

- Balance Sheet – a snapshot of conditions at a specific point in time
- Income Statement – a summary of the operations of the company and its profitability over a given period of time
- Cash Flow Statement – a summary of the sources and uses of funds

We also described the most common financial ratios:

- Liquidity Ratios
 - Current
 - Quick (Acid-Test)
- Asset Management Ratios
 - Inventory Turnover
 - Average Collection Period
 - Fixed Assets Turnover
 - Total Assets Turnover
- Debt Management Ratios
 - Total Debt to Total Assets
 - Times Interest Earned (TIE)
 - Fixed Charge Coverage
- Profitability Ratios
 - Profit Margin
 - Basic Earnings Power
 - Return on Total Assets
 - Return on Common Equity
- Market Value Ratios
 - Price / Earnings
 - Market / Book

Combining ratio analysis with financial statement analysis provides the analyst with a set of tools to begin to understand how a company operates, how it generates cash flow, and how the company compares to competitors in the industry. These tools will assist the analyst in finding the areas of a company that need attention and management consideration.

You have completed Unit One: *Financial Statement Analysis*. Please complete Progress Check 1.4, then continue to Unit Two: *Financial Markets and Interest Rates*. If you answer any questions incorrectly, please review the appropriate text.

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✓ PROGRESS CHECK 1.4

Directions: Use the following Balance Sheet and Income Statement to answer the questions about financial ratios.

Fruit Packing, Inc. – Balance Sheet			
December 31, 1993		(In Millions \$)	
ASSETS		LIABILITIES AND EQUITY	
Cash	7.4	Accounts Payable - Fruit	10.2
Marketable Securities	0.6	Accounts Payable - Material	9.6
Accounts Receivable	23.7	Notes Payable	23.0
Raw Goods Inventory	19.5	Accrued Wages and Taxes	1.7
Finished Goods Inventory	22.4	Other Current Liabilities	<u>0.6</u>
Prepaid Expenses	1.5	Total Current Liabilities	45.1
Other Current Assets	<u>0.8</u>	Long-term Debt	45.1
Total Current Assets	75.9	Preferred Stock	<u>7.8</u>
Gross Fixed Assets	101.2	Total Long-term Liabilities	98.0
Less Depreciation	<u>19.9</u>	Common Stock (6.5 Mil. Outst.)	15.1
Net Fixed Assets	81.3	Retained Earnings	<u>44.1</u>
		Total Common Equity	59.2
Total Assets	<u>157.2</u>	Total Liabilities and Equity	<u>157.2</u>

Fruit Packing, Inc. – Income Statement	
December 31, 1993 (In Millions \$)	
Net Sales	150.6
Operating Expenses	101.2
Depreciation	<u>3.1</u>
EBIT	46.3
Interest Expense	<u>11.5</u>
Earnings Before Taxes	34.8
Taxes (@ 40%)	<u>13.9</u>
Net Income to Shareholders	20.9
Preferred Dividends	<u>6.3</u>
Net Income to Common	14.6
Common Dividends	<u>12.0</u>
Earnings Retained	<u>2.6</u>

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PROGRESS CHECK 1.4*(Continued)*

For each question, decide which ratio is appropriate and use it to calculate the answer. Mark the correct answer and indicate which ratio you used. Check your solution with the Answer Key on the following page.

12. How many times can Fruit Packing, Inc. pay off CURRENT LIABILITIES without relying on INVENTORIES?

- _____ a) 0.75 times
_____ b) 1.19 times
_____ c) 1.25 times
_____ d) 1.68 times
_____ e) 1.80 times

Ratio: _____

13. How many times did Fruit Packing, Inc. sell out and restock its merchandise?

- _____ a) 2.00 times
_____ b) 2.29 times
_____ c) 3.59 times
_____ d) 6.72 times
_____ e) 7.72 times

Ratio: _____

ANSWER KEY

12. How many times can Fruit Packing, Inc. pay off CURRENT LIABILITIES without relying on INVENTORIES?

a) 0.75 times

$$\begin{aligned}\text{Quick Ratio} &= (\text{Current Assets} - \text{Inventories}) / (\text{Current Liabilities}) \\ &= (\$75.9 - \$19.5 - \$22.4) / (\$45.1) \\ &= 0.75 \text{ times}\end{aligned}$$

Subtract both Raw and Finished Goods Inventory

13. How many times did Fruit Packing, Inc. sell out and restock its merchandise?

c) 3.59 times

$$\begin{aligned}\text{Inventory Turnover} &= (\text{Sales}) / (\text{Inventories}) \\ &= (\$150.6) / (\$19.5 + \$22.4) \\ &= 3.59 \text{ times}\end{aligned}$$

PROGRESS CHECK 1.4*(Continued)*

14. What percent of Fruit Packing, Inc.'s capital structure is provided by debt?

_____ a) 27.23%

_____ b) 28.69%

_____ c) 43.32%

_____ d) 57.38%

_____ e) 62.34%

Ratio: _____

15. What percentage of each dollar from SALES is available for common shareholders?

_____ a) 4.18%

_____ b) 9.69%

_____ c) 13.21%

_____ d) 22.05%

_____ e) 29.68%

Ratio: _____

16. If Fruit Packing, Inc.'s stock price is \$15.24 per share and there are 6.5 million shares outstanding, how much are investors willing to pay for every dollar of reported profits?

_____ a) 2.09 times

_____ b) 2.34 times

_____ c) 4.98 times

_____ d) 6.78 times

_____ e) 8.12 times

Ratio: _____

ANSWER KEY

14. What percent of Fruit Packing, Inc.'s capital structure is provided by debt?

d) 57.38%

$$\begin{aligned} \text{Debt Ratio} &= (\text{Total Debt}) / (\text{Total Assets}) \\ &= (\$45.1 + \$45.1) / (\$157.2) \\ &= 57.38\% \end{aligned}$$

Include Current Liabilities and Long-term Debt for Total Debt

15. What percentage of each dollar from SALES is available for common shareholders?

b) 9.69%

$$\begin{aligned} \text{Profit Margin} &= (\text{Net Income to Common}) / (\text{Sales}) \\ &= (\$14.6) / (\$150.6) \\ &= 9.69\% \end{aligned}$$

16. If Fruit Packing, Inc.'s stock price is \$15.24 per share and there are 6.5 million shares outstanding, how much are investors willing to pay for every dollar of reported profits?

d) 6.78 times

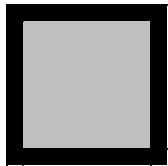
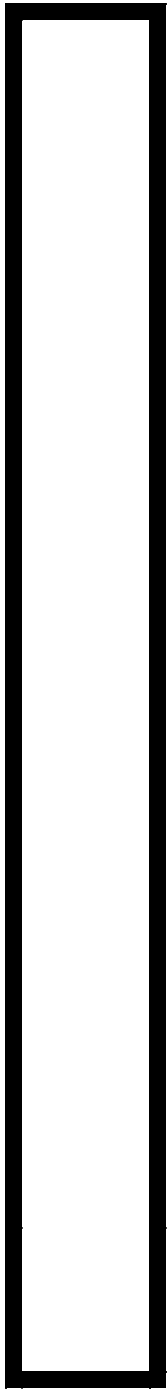
$$\text{Price / Earnings} = (\text{Market price per share}) / (\text{Earnings per share})$$

First we calculate the Earnings per share:

$$\begin{aligned} \text{Earnings / Share} &= (\text{Net Income to Common}) / (\text{Number of shares}) \\ &= (\$14.6) / (6.5 \text{ shares}) \\ &= \$2.2462 \text{ per share} \end{aligned}$$

Now we can calculate the Price / Earnings Ratio:

$$\begin{aligned} \text{Price / Earnings} &= (\$15.24 \text{ per share}) / (\$2.2462 \text{ per share}) \\ &= 6.78 \text{ times} \end{aligned}$$



Unit 2

UNIT 2: FINANCIAL MARKETS AND INTEREST RATES

INTRODUCTION

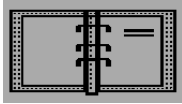
In Unit One, we introduced the basic financial operations and general financial structure of a company. In that context, we discussed the two types of capital that a company can access for its operations: debt and equity. We will now focus our discussion on the sources of capital and the process for calculating its value. In this unit, we look at the types of financial markets and the market participants. We introduce the factors that make up a quoted interest rate and the characteristics that affect the level of interest rates over time.



UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize the types of participants in the financial markets
- Differentiate between money markets and capital markets
- Differentiate between bond (debt) markets and equity markets
- Identify the factors that influence interest rates
- Understand the term structure of interest rates



OVERVIEW OF FINANCIAL MARKETS

Market Players

Investors / lenders and issuers / borrowers

As you know, a market is where buyers and sellers meet to exchange goods, services, money, or anything of value. In a financial market, the buyers are investors, or lenders: the sellers are issuers, or borrowers. An investor / lender is an individual, company, government, or any entity that owns more funds than it can use.

An issuer / borrower is an entity that has a need for capital. Each investor and issuer is active in a market that meets its needs. Needs are based on many factors, including a time horizon (short- or long-term), a cost / return preference, and type of capital (debt or equity).

Brokers and dealers

The third group of participants in the marketplace includes financial intermediaries called brokers and dealers. Brokers facilitate the buying and selling process by matching investors and issuers according to their needs. Dealers purchase securities from issuers and sell them to investors. Brokers and dealers may be referred to as investment bankers. Investment banking firms specialize in the financial markets.

Security is debt or equity IOU

Security is a generic term that refers to a debt or equity IOU issued by a borrower or issuer.

- Debt security or **bond** – an IOU promising periodic payments of interest and/or principal from a claim on the issuer's earnings
- Equity or stock – an IOU promising a share in the ownership and profits of the issuer

Types of Financial Markets

There are two general classifications of financial markets:

- Money markets
- Capital markets

Money Markets

Short-term, liquid securities

Money markets trade short-term, marketable, liquid, low-risk debt securities. These securities are often called "cash equivalents" because of their safety and liquidity. The liquidity of a market refers to the ease with which an investor can sell securities and receive cash. A market with many active investors and few ownership regulations usually is a liquid market; a market with relatively few investors, only a few securities, and many regulations concerning security ownership is probably less liquid.

Capital Markets

Long-term, higher risk securities

Capital markets trade in longer-term, more risky securities. There are three general subsets of capital markets: bond (or debt) markets, equity markets, and **derivative** markets. In this course, we will discuss debt markets and equity markets in some detail, and we will briefly introduce the derivative markets.

Bond (debt) markets

Debt markets specialize in the buying and selling of debt securities. To illustrate how debt markets look, we will analyze a short example.

Example

Suppose that HT Manufacturing Company needs new equipment for its operations. Most companies try to match assets and liabilities according to **maturity** (time left before the item is no longer useful). The company expects the new equipment to have a useful life of about 10 years and, therefore, after consulting with its financial advisors, HT Manufacturing decides to issue 10-year bonds to pay for the equipment. HT Manufacturing consults with investment bankers to find out what types of bonds investors are buying and to decide what interest rate the bonds will pay. The object is to make them attractive to investors, yet cost-efficient for HT Manufacturing.

After completing all of the details, HT Manufacturing issues the bonds to investors using two methods.

- 1) The investment banks use their brokers to find buyers for the bonds, and HT Manufacturing sells the bonds directly to the investors.

- 2) The investment banks also act as dealers by buying some of the bonds themselves, then selling them to investors.

Primary and secondary markets

The original issue of bonds (or bills, or any other debt security) is called the primary market issue. A **secondary market** also exists where debt securities are bought and sold by investors. For example, suppose an investor who bought HT Manufacturing bonds one year ago has a change in investment plans and no longer needs 10-year bonds. S/he can sell the bonds in the secondary market (usually with the assistance of a broker) to another investor who wants 10-year bonds. Because this transaction has no effect on HT Manufacturing's finances or operations, it is considered a secondary market transaction.

Equity markets

Equity markets, also called stock markets, specialize in the buying and selling of equity securities (stocks) of companies. As in the debt markets, the equity markets have a primary and secondary market.

The primary market is where companies originally issue stock in their companies, a process known as an initial public offering — or "taking the company public." Investment bankers advise a company on the process and can also act as brokers and dealers for new stock issues.

The secondary market is where investors buy and sell stocks at prices that reflect the investors' collective view of the future prospects of each individual firm.

Derivative markets

A derivative instrument is a security that derives its value from an underlying asset, including financial assets such as stocks and bonds or other assets such as commodities and precious metals. Derivative instruments include future and forward contracts and **options**. These instruments are bought and sold in the market by investors needing to hedge risk exposure.

Swaps are also used to hedge risk, but they differ from true derivative instruments in that they are essentially an exchange of financial assets. For this reason, they are not really traded among investors, but they are often included in a discussion of derivatives. We will briefly introduce options and swaps in Unit 10. For a more thorough discussion of these instruments and their uses, we recommend that you refer to the specific workbook covering each instrument.

INTEREST RATES

Price paid for use of capital

In a free economy, capital is allocated through the price system. An interest rate is the price paid by a borrower for the use of an investor's capital.

Interest rates provide the vehicle for allocating capital among firms. In a perfect free-market economy, firms with the most profitable investment opportunities attract capital away from companies with less inviting investment opportunities arising from problems such as inefficiency, low demand for products, poor management, etc. However, a perfect free-market economy doesn't exist. There are imperfections, usually introduced by governments, that lead to the allocation of capital to firms that do not necessarily have the most profitable investment opportunities.

An investor purchasing a debt security, such as a bond, receives an interest payment for the period that the borrower uses the investor's capital. In an equity investment, return to the investor takes two forms: dividend payments and appreciation of the investment, also called a capital gain.

Characteristics affecting the cost of money

There are two basic investor characteristics that affect interest rates, or the cost of money:

- 1) The rate of return investors expect to realize on their investment opportunities

Those who borrow money are unlikely to pay a higher rate than the expected return on the investments they plan to make with the borrowed money.

- 2) The preference of consumers (those who have money to either save or spend) to forego current consumption in favor of future consumption

Those who save will defer consumption if the rates of interest offered by borrowers are attractive. High offered rates for savings lead to higher levels of savings and lower levels of spending.

Let's look at some of the factors that influence the level of interest rates.

Calculating Interest Rates**Offered rate of interest**

The stated or offered rate of interest (r) reflects three factors:

- Pure rate of interest (r^*)
- Premium that reflects expected inflation (IP)
- Premium for risk (RP)

Each of these factors increases the stated interest rate. The resulting interest rate calculation is:

$$r = r^* + IP + RP$$

Let's examine the significance of each of these factors.

Pure Interest Rate

Risk-free security in non-inflationary environment

The *pure interest rate* (r^*) is the rate for a risk-free security when no inflation is expected. This pure rate is also called the real, **risk-free rate**. Most investors consider the interest rate on short-term U.S. Treasury securities to be a close approximation for the real, risk-free rate in a non-inflationary environment.

The pure rate constantly changes over time, depending on economic conditions. The two most important influences on the risk-free rate are the ones discussed above – the expected rate of return corporations and other borrowers can earn on their productive assets (investment opportunities) and the time preference of consumers for current-versus-future consumption. The real, risk-free rate is difficult to calculate, but most researchers believe that in recent years it has fluctuated between 2% and 4%.

Inflation

Erodes purchasing power over time

Inflation has a great influence on interest rates because it erodes the purchasing power of money over time. Investors build in an inflation premium to compensate for this loss of value. For example, if the real, risk-free rate is 3%, and 8% inflation is expected for the next year, the risk-free interest rate on one year U.S. Treasury bills is 11%.

$$\begin{aligned}r_{\text{T-Bill}} &= r^* + \text{IP} \\ &= 3\% + 8\% \\ &= 11\%\end{aligned}$$

Note: The 8% inflation rate is the *expected future rate*, not the past inflation rate. It is also important to know that, for longer time periods, the inflation premium is the expected average rate over that period. For example, the inflation premium for a ten year security would be the expected average annual inflation for the next ten years. As with the risk-free rate, the inflation premium is not constant; it is always changing based on investors' expectations of the future level of inflation.

Risk

There are several types of risk that can be included in the **risk premium**. Usually, it is difficult to quantify what percentage of the risk premium is associated with each type of risk. Let's look at three risks that affect the amount of the risk premium: counterparty risk, liquidity risk, and interest rate risk.

Counterparty (default) risk

Counterparty (default) risk is the chance that the borrower will not be able to pay the interest or pay off the principal of a loan. This risk can influence the level of interest rates. It is generally considered that U.S. Treasury securities have no default risk – the U.S. government will always pay interest and will repay the principal of its borrowings. Therefore, the difference in price between a U.S. Treasury security and another corporate bond with similar maturity, liquidity, etc. may be the risk premium for assuming counterparty risk.

Several ratings companies identify and classify the creditworthiness of corporations and governments to determine how large the risk premium should be. They apply specific ratings such as AAA or BBB to indicate the likelihood of default by the borrowing entity.

A corporation with a AAA rating will have a smaller default risk than a firm with a BBB rating and, therefore, will be able to borrow capital at a lower interest rate (other factors being equal).

Liquidity risk

Liquidity refers to the marketability of assets – the ease with which assets can be sold for cash on short notice at a fair price. Investors may require a premium return on an asset to compensate for a lack of liquidity. For example, there are active world markets in which investors holding bonds issued by Ford Motor Company can easily sell the bonds and receive cash within a matter of seconds. On the other hand, an investor who has loaned money to a small Honduran trading company will have a much more difficult time finding another investor in the secondary market to purchase the loan.

The Honduran trading company pays a higher interest rate for its borrowings than does Ford (all other factors being equal) because its securities are harder to sell in the secondary market.

Interest rate risk

As interest rates fluctuate, so do the prices of bonds in the secondary markets. We will discuss this relationship between interest rates and bonds in more detail later in the course. For now, you need to know how a change in rates affects bond prices.

- **As interest rates increase, bond prices decrease.**
- **As interest rates decrease, bond prices increase.**

After an interest rate increase, newly issued bonds yield larger interest payments than older bonds. The price of old bonds falls to attractive levels so that investors will continue to buy them. Older bonds are priced in the market so that the rate of return paid on the investment is approximately the same as other investment opportunities with similar risk and maturity.

All long-term bonds carry interest rate risk because there is more opportunity for interest rates to rise. For this reason, investors often require a risk premium in the stated interest rate. The net effect of this premium is to raise rates on long-term bonds relative to the rates on short-term bonds.

Measuring risk premium

The risk premium is difficult to measure, but researchers have identified two characteristics that serve as a reference:

1. Rate **volatility** – Premiums are higher when interest rates are volatile and uncertain.
2. Historical data – The premium on 30-year U.S. Treasury bonds generally has fluctuated between 1% and 2%, with an average of about 1.7% for the last 60 years.

Short-term bonds are also subject to risk as a result of changing interest rates. Most investors reinvest their funds as short-term bonds mature and are paid off. If interest rates are falling, the new bonds will pay less interest than the previously held bonds. This reinvestment risk is of particular concern to investors living off the proceeds of their investment.

Term Structure of Interest Rates**Relationship between long- and short-term rates**

The term structure of interest rates refers to the relationship between long-term and short-term interest rates. Investors are concerned with the term structure because it affects their investment preferences for long- or short-term bonds. Corporate treasurers also must decide whether to borrow by issuing long- or short-term bonds. We will discuss how long- and short-term rates are related and what causes changes in their relative values.

Yield Curve**Rates of return plotted on a graph**

Interest rates on bonds of differing maturities are plotted on a graph to illustrate their relationship. A graphic representation of the term structure of interest rates is called a **yield curve**. "Yield" refers to the rate of return on an investment. For our term structure study, the yield of a security is the rate of interest paid on the security. The graph in Figure 2.1 shows yield curves for two different U.S. Treasury securities for two different dates. Remember, interest rates are always changing.

The vertical axis represents the interest rate; the horizontal axis represents the number of years to maturity for the security. For instance, the interest rate paid to an investor on a one-year security in March of 1980 is 14% and the rate for a 20-year bond in January of 1987 is 7.6%. We have not plotted every point exactly, but we have plotted several common maturities and smoothed the curves to illustrate the trends.

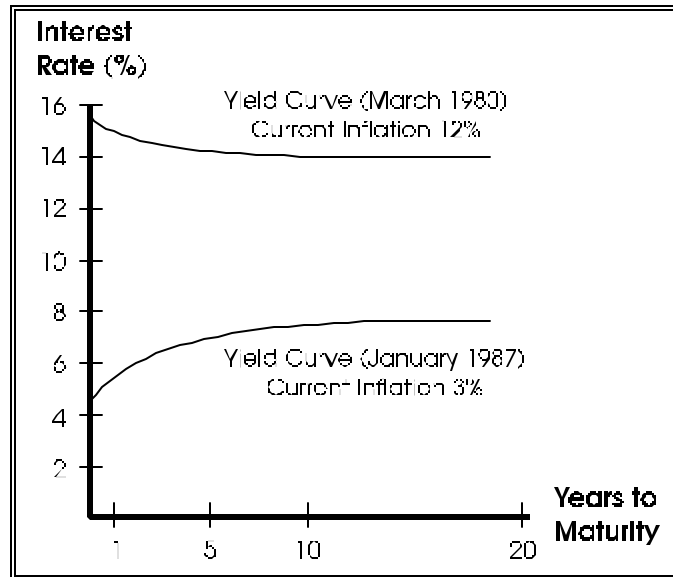


Fig. 2.1: Yield Curves for Two Different Maturities

Normal / abnormal yield curves

Let's examine the characteristics of the two curves. In March of 1980, all interest rates were at relatively high levels — with long-term rates lower than short-term rates. The January 1987 curve has overall lower rates, with long-term rates slightly higher than short-term rates. An upward sloping yield curve is often called a *normal yield curve*, whereas a downward sloping yield curve is called an *abnormal (inverted) yield curve*.

Normal Yield Curve Theories

Upward curve considered normal

There are three prevailing theories that explain why the upward sloping yield curve is considered normal. These three theories are: the expectations theory, the market segmentation theory, and the liquidity preference theory.

1. Expectations Theory

The basis of the expectations theory is that the yield curve reflects lenders' and borrowers' expectations of inflation. Changes in these expectations cause changes in the shape of the yield curve. Remember, the inflation premium is a major component of interest rates.

To illustrate the expectations theory, suppose that inflation is expected to be 4% this coming year, 6% in the year following, and 8% in the third year. The inflation premium (average expected yearly inflation) will be 4% for the first year, 5% for the second year, and 6% for the third year. If the real, risk-free rate is 3%, then the resulting Treasury Bill interest rates (risk-free rate plus inflation premium) will be 7% for a one-year T-bill, 8% for a two-year T-Bill, and 9% for a three-year T-bill. A change in these expectations will cause a shift in the yield curve for the T-Bills.

	1st yr	2nd yr	3rd yr
Expected inflation	4%	6%	8%
Average inflation premium	4%	5%	6%
+ Real risk-free interest rate	3%	3%	3%
= T-bill interest rate	7%	8%	9%

2. Market Segmentation Theory

Each lender and each borrower has a preferred maturity. Some lend / borrow for long-term needs and others lend / borrow for short-term needs. The market segmentation theory states that the slope of the yield curve depends on supply / demand conditions in the short-term and long-term markets.

An upward sloping curve results from a large supply of funds in the short-term market relative to demand and a shortage of long-term funds. A downward sloping curve indicates strong demand in the short-term market relative to the long-term market. A fairly horizontal yield curve indicates that supply and demand for funds is roughly balanced both in the short-term and long-term markets.

3. **Liquidity Preference Theory**

The liquidity preference theory is based on the observation that long-term securities often yield more than short-term securities. Two reasons are given to explain this:

- Investors generally prefer short-term securities, which are more liquid and less expensive to buy and sell. Investors require higher yield on long-term instruments to compensate for the higher cost.
- Borrowers dislike short-term debt because it exposes them to the risk of having to roll over the debt or raise new principal under adverse conditions (such as a rise in rates). Borrowers will pay a higher rate for long-term debt than for short-term debt, all other factors being held constant.

Under these "normal" conditions, there is a positive correlation between risk premium increases and maturity. Therefore, the "normal" yield curve slopes upward.

There is no evidence that any of these theories clearly explains the shape of the yield curve and its changes. However, each theory has some merit, and all three theories are discussed by participants in the markets as if they were valid.

Effect on Stock Prices

Impact on corporate profits

Interest rates indirectly impact stock prices through their effect on corporate profits. The payment of interest is a cost to companies – the higher the level of interest rates, the lower the level of corporate profits (other factors held constant). Interest rates affect the level of economic activity which, in turn, affects corporate profits.

Even more directly, interest rates affect the competition for funds (capital) between stocks and bonds. Higher interest rates mean that investors receive a higher return on their bond investments. High bond yields induce investors to sell their stock holdings and invest in more bonds. The reverse occurs when interest rates fall.

The effect of interest rates on corporate profits is more important for individual companies, especially those with high debt levels. The direct competition for capital has a more widespread effect on the general stock price level for the entire economy.

UNIT SUMMARY

In this unit, we discussed the types of financial markets — money markets and capital markets. There are two subsets of capital markets — bond (debt) and equity markets. Players in the markets include investors / lenders, issuers / borrowers, brokers, and dealers. We said that investors' expectations of return on investment and consumers' preferences for saving versus spending are two market characteristics that affect the level of interest rates.

Interest rates required by investors are the sum of three factors:

Risk-free real rate	(r^*)
+ Premium for inflation	(IP)
+ Premium for risk	(RP)
= Interest rate	(r)

$$r = r^* + IP + RP$$

Each of these factors fluctuates, given current and expected conditions of the economy, the company, and the market in which the company conducts business. Interest rates compensate investors for loaning money, for anticipated inflation, and for assuming counterparty, liquidity, and interest rate risk.

The term structure of interest rates is represented by a yield curve and refers to the relationship between long-term and short-term interest rates.

You have completed Unit Two: *Financial Markets and Interest Rates*. Please complete Progress Check 2 and then continue to Unit Three: *Time Value of Money*. If you answer any questions incorrectly, we suggest that you review the appropriate text.

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✓ PROGRESS CHECK 2

Directions: Select the one best answer for each question, unless otherwise indicated. Check your responses with the Answer Key on the next page.

1. Match each market player with a description of his/her role. Write the letter of the role next to the title of the market player.

<u>Market Players</u>	<u>Roles</u>
_____ Investor	a) Financial intermediary that buys and sells securities
_____ Issuer	
_____ Broker	b) Entity with more money than it needs
_____ Dealer	c) Entity in need of money
	d) Financial intermediary that facilitates the buying and selling of securities

2. The most liquid, lower-risk securities are traded in the:

- _____ a) bond market.
- _____ b) equity market.
- _____ c) money market.
- _____ d) derivative market.

3. QRS Manufacturing Company decides to "take the company public." This means it will:

- _____ a) sell bonds to the public through the debt market.
- _____ b) issue stock in the primary equity market.
- _____ c) offer certificates of deposit in the money markets.
- _____ d) derive its funding from the derivatives market.

ANSWER KEY

1. Match each market player with a description of his/her role. Write the letter of the role next to the title of the market player.

Market Players b Investor c Issuer d Broker a DealerRoles

a) Financial intermediary that buys and sells securities

b) Entity with more money than it needs

c) Entity in need of money

d) Financial intermediary that facilitates the buying and selling of securities

2. The most liquid, lower-risk securities are traded in the:

c) money market.

3. QRS Manufacturing Company decides to "take the company public." This means it will:

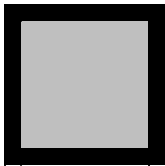
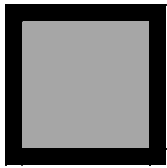
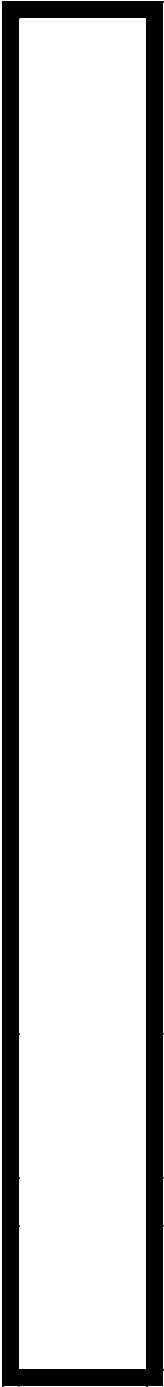
b) issue stock in the primary equity market.

PROGRESS CHECK 2*(Continued)*

4. Select **two** basic market characteristics that influence the level of interest rates.
- _____ a) Investors' expected rate of return
 - _____ b) Fees charged by brokers and dealers
 - _____ c) Government intervention and regulation
 - _____ d) Preference of potential investors for consumption versus savings
 - _____ e) Efficiency of the capital allocation price system
5. The offered rate of interest reflects the pure interest rate plus:
- _____ a) an investment premium (IP) to account for opportunity cost.
 - _____ b) a premium for investment costs plus a rate premium (RP) for profit.
 - _____ c) a premium for the expected future rate of inflation plus a premium for counterparty, liquidity, and interest rate risk.
 - _____ d) compensation for assuming risk plus a premium based on the current rate of inflation.
6. When planning their investment / borrowing strategies, investors and issuers look at the yield curve representing the term structure of interest rates to:
- _____ a) determine their preference for long-or short-term securities.
 - _____ b) predict future rate fluctuations.
 - _____ c) view the historical movement of interest rates over a given period of time.
 - _____ d) determine the shape of future market movements.

ANSWER KEY

4. Select **two** basic market characteristics that influence the level of interest rates.
- a) Investors' expected rate of return**
 - d) Preference of potential investors for consumption versus savings**
5. The offered rate of interest reflects the pure interest rate plus:
- c) a premium for the expected future rate of inflation plus a premium for counterparty, liquidity, and interest rate risk.**
6. When planning their investment / borrowing strategies, investors and issuers look at the yield curve representing the term structure of interest rates to:
- a) determine their preference for long-or short-term securities.**



Unit 3

UNIT 3: TIME VALUE OF MONEY

INTRODUCTION

We know that money has two general purposes: it serves as a store of value and as a medium of exchange. The term *time value of money* refers to all aspects of converting the value of cash flows at one point of time to their equivalent values at another time. It is important that we know the value of current and future cash flows to facilitate the financial planning process.

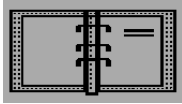
In Unit Two, we discussed the factors that are used to calculate interest rates and the market conditions that influence the level of interest rates. In this unit, we focus on the application of interest rates to specific debt instruments. You will learn how to calculate the future value of cash in hand, given different interest rates and payment schedules. You will also learn how to calculate the present value of future cash flows.



UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Explain how and why the value of money changes over time
- Calculate simple and compound interest to determine the future value of debt instruments
- Discount future cash flows to present value for comparison with other cash flows



FUTURE VALUE

Cash flow worth more today

The term **future value** (FV) refers to the value of a cash flow at a specified rate of interest at the end of a stated length of time. A cash flow that is received today is worth more than the same amount received in the future. Cash in hand today may be invested to earn a return over a period of time.

Example

For example, an investor receiving \$100 today may have the opportunity to invest the \$100 for one year at 10% per annum (p.a.). With a risk-free investment in an environment free from inflation, the investor will have \$110 value at the end of the one year. If the receipt of the \$100 is delayed for one year, the investor will have only \$100 at that time; the opportunity to earn the 10% will have been lost. The principal plus the return on the investment over time represents the future value of the investment.

Calculating Interest Payments

Simple / compound interest

There are two methods for calculating interest payments on debt instruments: **simple interest** and **compound interest**. In this section we will look at the formulas for deriving a simple interest payment and a simple interest rate. We also will see how interest is compounded discretely at regular intervals or continuously over a defined period of time.

In our discussions, we show formulas and calculations so that you may understand the process and become familiar with the terminology. However, a financial calculator is the most efficient and accurate tool for solving the types of problems in this course. We recommend that you become familiar with the functions on your financial calculator.

Simple Interest

Calculating simple interest payment

Simple interest refers to the payment of interest on the principal amount of an investment. Interest payments are made at a constant absolute rate. The equation for computing simple interest earned or paid is:

$$I = P \times R \times T$$

Where:

I = Interest payment

P = Principal

R = Rate of interest

T = Number of interest paying periods

Annual interest

Let's look at an example to illustrate the use of the formula. Suppose that you borrow \$160,000 for one year at a 12% annual rate. How much is the interest payment at the end of the one year period? (The interest rate is represented in decimal form for all of our calculations. For example, twelve percent is 0.12.)

$$\begin{aligned} I &= P \times R \times T \\ I &= (\$160,000) \times (0.12) \times (1) \\ I &= \$19,200 \end{aligned}$$

Interest for periods of less than one year

For periods of less than one year, the annual interest is adjusted with this formula:

$$I = P \times R \times (\text{days}/360)$$

This formula converts the time period to a fractional yearly equivalent. For example, suppose that you borrow \$100,000 for 90 days at a 10% annual rate. The calculation for interest is:

$$\begin{aligned} I &= P \times R \times (\text{days}/360) \\ I &= (\$100,000) \times (0.10) \times (90/360) \\ I &= \$2,500 \end{aligned}$$

The interest for 90 days based on an annual rate of 10% is \$2,500.

**Calculating
simple interest
rates**

Using simple algebra, the formula can be rearranged to calculate the rate. For example, suppose that you borrow \$120,000 for one year and you know that your interest payment is \$9,600. What is the simple interest rate on the loan?

$$\begin{aligned}R &= I / (P \times T) \\R &= \$9,600 / (\$120,000 \times 1) \\R &= 0.08 \text{ or } 8\%\end{aligned}$$

You have completed the section on simple interest. Please take a moment to complete the following practice exercise on calculating simple interest.

PRACTICE EXERCISE 3.1

Directions: Calculate the answer to each question and write the correct answer.
Check your solution with the Answer Key on the next page.

1. If you borrow \$300,000 for two years at a 9% simple annual interest rate, how much interest will you pay?

\$ _____

2. If you invest \$270,000 for one-half year at a 9.5% simple annual interest rate, how much will you earn on the investment?

\$ _____

3. If you borrow \$500,000 for one year and have to pay back \$540,000 at the end of the year (\$500,000 is the return of the principal; \$40,000 is the interest), what is the simple interest rate you are being charged for the loan?

_____ %

ANSWER KEY

1. If you borrow \$300,000 for two years at a 9% simple annual interest rate, how much interest will you pay?

\$54,000

$$\begin{aligned}I &= P \times R \times T \\I &= \$300,000 \times 0.09 \times 2 \\I &= \$54,000\end{aligned}$$

2. If you invest \$270,000 for one-half year at a 9.5% simple annual interest rate, how much will you earn on the investment?

\$12,825

$$\begin{aligned}I &= P \times R \times T & T &= 1/2 \text{ or } 0.5 \text{ or } (180 \text{ days}/360 \text{ days}) \\I &= \$270,000 \times 0.095 \times 0.5 \\I &= \$12,825\end{aligned}$$

3. If you borrow \$500,000 for one year and have to pay back \$540,000 at the end of the year (\$500,000 is the return of the principal; \$40,000 is the interest), what is the simple interest rate you are being charged for the loan?

8.0%

$$\begin{aligned}R &= I / (P \times T) \\R &= \$40,000 / (\$500,000 \times 1) \\R &= 0.08 \text{ or } 8.0\%\end{aligned}$$

PRACTICE EXERCISE 3.1*(Continued)*

4. If you borrow \$350,000 for two years and have to pay back \$413,000 in two years, what is the simple interest rate you are being charged for the loan per year?

_____ %

5. If the rate is 12% per annum (p.a.), what is the simple interest accrued over a 20 day period within the same month on a \$100 investment?

\$ _____

6. If the rate is 7.5% p.a. on a \$1,000 investment, what is the simple interest accrued over the period 13/08/91 to 20/09/91 (dd/mm/yy)?

\$ _____

ANSWER KEY

4. If you borrow \$350,000 for two years and have to pay back \$413,000 in two years, what is the simple interest rate you are being charged for the loan per year?

9.0%

$$\begin{aligned}R &= I / (P \times T) \\R &= \$63,000 / (\$350,000 \times 2) \\R &= 0.09 \text{ or } 9.0\%\end{aligned}$$

5. If the rate is 12% per annum (p.a.), what is the simple interest accrued over a 20 day period within the same month on a \$100 investment?

\$0.67

$$\begin{aligned}I &= P \times R \times (\text{days}/360) \\I &= \$100 \times 0.12 \times (20/360) \\I &= \$0.67\end{aligned}$$

6. If the rate is 7.5% p.a. on a \$1,000 investment, what is the simple interest accrued over the period 13/08/91 to 20/09/91 (dd/mm/yy)?

\$7.92

There are 38 days from Aug. 13, 1991 to Sep. 20, 1991.

$$\begin{aligned}I &= P \times R \times (\text{days}/360) \\I &= \$1,000 \times 0.075 \times (38/360) \\I &= \$7.92\end{aligned}$$

Compound Interest

Reinvestment of interest payment

It is common for investments to receive (pay) multiple interest payments rather than one payment at the end of the investment period. Usually, each interest payment is reinvested at the same rate as the original principal.

For example, interest is calculated on a savings account balance at the end of some period and the interest payment is credited to the account. At the end of the next period, interest is calculated on the new balance (the original principal plus the last interest payment) and credited to the account. The process of compounding interest continues for each period over the life of the investment. There are two ways to calculate compound interest: discretely and continuously.

Discrete Compounding for Annual Periods

Interest paid at regular intervals

Interest calculated at specified regular intervals is said to be **discretely compounded**. An example is a savings account on which interest is calculated monthly and deposited in the account. The formula for calculating discretely compounded interest is:

$$I = P(1 + R)^T - P$$

Where:

I = Interest earned or paid

P = Principal

R = Stated (nominal) interest rate for each interest paying period

T = Number of interest paying periods

Let's use the formula to calculate the interest on an investment. Suppose that you invest \$400,000 at 6% per annum (p.a.) for two years, compounded annually. At the end of the two years, the investment earns \$49,440 in interest.

$$\begin{aligned}
 I &= P (1 + R)^T - P \\
 I &= (\$400,000) (1 + 0.06)^2 - (\$400,000) \\
 I &= (\$400,000) (1.1236) - (\$400,000) \\
 I &= (\$449,440) - (\$400,000) \\
 I &= \$49,440
 \end{aligned}$$

Future value of principal

By adding the interest and principal, you can see that at the end of the two year period the investment will be worth \$449,440. This is the future value of the \$400,000 investment. Future value is the worth of an investment at a specified rate of interest at the end of a stated period of time. We can convert the interest payment formula to a future value formula.

$$FV_T = P (1 + R)^T$$

Where:

FV_T = Future value after T paying periods

P = Principal

R = Stated interest rate for each interest paying period

T = Number of interest paying periods

Future value interest factor

The expression $(1 + R)^T$ is referred to as the *future value interest factor*. The future value is the principal multiplied by the future value interest factor.

Example

To illustrate, suppose that we invest \$500 in an account earning 12% annual interest. At the end of one year, the future value of our investment is:

$$\begin{aligned}
 FV_T &= P (1 + R)^T \\
 FV_1 &= (\$500) (1 + 0.12)^1 \\
 FV_1 &= (\$500) (1.12) \\
 FV_1 &= \$560
 \end{aligned}$$

If we leave the \$560 dollars in the account for another year, at the same rate, the future value is:

$$\begin{aligned}
 FV_1 &= (\$560) (1 + 0.12)^1 \\
 FV_1 &= (\$560) (1.12) \\
 FV_1 &= \$627.20
 \end{aligned}$$

We can arrive at the same result by using the future value formula with a two year time frame.

$$\begin{aligned}FV_2 &= (\$500) (1 + 0.12)^2 \\FV_2 &= (\$500) (1.2544) \\FV_2 &= \$627.20\end{aligned}$$

The interest earned in the second year is \$67.20: \$60 dollars on the \$500 principal, and \$7.20 on the \$60 of interest earned during the first year. Compounded interest is interest earned on interest.

Discrete Compounding for Nonannual Periods

Adjust for number of periods

Many investments are compounded for periods other than annually. Some bonds are compounded semiannually or quarterly; many savings accounts are compounded monthly, weekly, or even daily.

We use the future value equation to calculate discrete compounding for nonannual periods with two slight modifications. First, we adjust the interest rate by dividing the annual rate (R) by the number of compounding periods during the year (M). For semiannual compounding, M is 2; for monthly compounding, M is 12. We also adjust the number of compounding periods by multiplying the time in years (T) by the number of compounding periods during the year (M). Our new future value formula then becomes:

$$FV_T = P (1 + R/M)^{T \times M}$$

Where:

FV_T = Future value after T years

P = Principal

R = Stated annual interest rate

M = Number of compounding periods within a year

T = Number of years

Example

To illustrate the use of this formula, suppose that you invest \$10,000 in an account that earns 8% annual interest compounded quarterly for five years. Assuming no other transactions, how much will be in your account at the end of five years? Using the future value formula, $P = \$10,000$, $R = 0.08$, $M = 4$, and $T = 5$:

$$\begin{aligned}FV_T &= P (1 + R/M)^{T \times M} \\FV_5 &= (\$10,000)(1 + 0.08/4)^{5 \times 4} \\FV_5 &= (\$10,000)(1.485947) \\FV_5 &= \$14,859.47\end{aligned}$$

One more example: What is the future value of the \$10,000 if it is invested for one year and compounded monthly at an annual rate of 8%?

$$\begin{aligned}FV_T &= P (1 + R/M)^{T \times M} \\FV_1 &= (\$10,000)(1 + 0.08/12)^{1 \times 12} \\FV_1 &= (\$10,000)(1.082995) \\FV_1 &= \$10,829.95\end{aligned}$$

Continuous Compounding**Infinite number of periods**

Continuous compounding is a special case of nonannual compounding. It extends the notion of compounding periods to a point where the number of periods becomes infinitely large and the length of each period is correspondingly small. In other words, interest is always being calculated and compounded. The future value interest factor for continuous compounding is represented by the term e^{RT} . For continuous compounding, the future value formula is:

$$FV_T = P \times e^{RT}$$

Where:

FV_T = Future value after T periods

P = Principal

e = Base of the natural logarithm (2.718282)

R = Stated (nominal) annual interest rate for each period

T = Number of periods

Example

For example, let's calculate the future value of a \$10,000 investment for one year at an 8% annual rate compounded continuously. We will use the values $P = \$10,000$, $R = 0.08$, and $T = 1$.

$$\begin{aligned}FV_T &= P \times e^{RT} \\FV_1 &= (\$10,000)(2.718282)^{(0.08)(1)} \\FV_1 &= (\$10,000)(1.083287) \\FV_1 &= \$10,832.87\end{aligned}$$

Discrete vs. continuous compounding

A comparison of the investment's future values discretely compounded for different periods, and continuously compounded, shows how the value changes — depending on the frequency of compounding.

Annually:	\$10,800.00
Monthly:	\$10,829.95
Weekly:	\$10,832.20
Continuously:	\$10,832.87

You can see that as the compounding period becomes smaller, the future value of the investment increases. The increase occurs because we are compounding the interest more frequently.

To further illustrate continuous compounding and compare it to discrete compounding, refer to the next three graphs (Figures 3.1, 3.2, and 3.3). The first graph shows the growth of \$100 of capital invested at 25% for three years compounded annually. The second graph shows \$100 invested at 25% for three years compounded semi-annually. The final graph represents the growth of \$100 invested at 25% for three years compounded continuously.

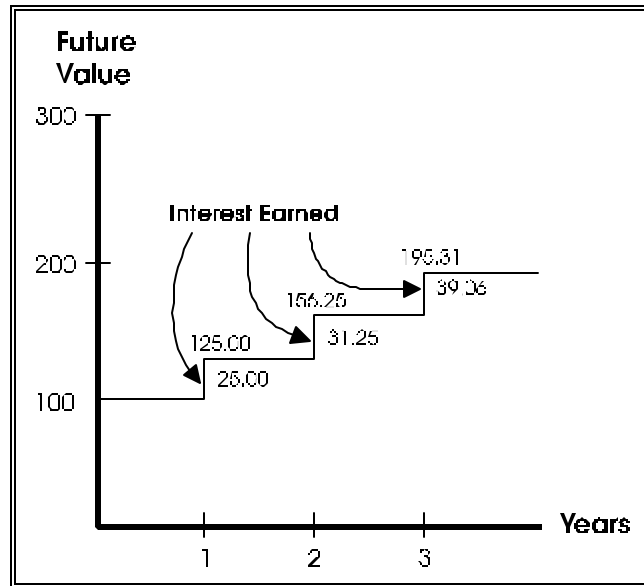


Figure 3.1: Annual Compounding: \$100 at 25% for 3 Years

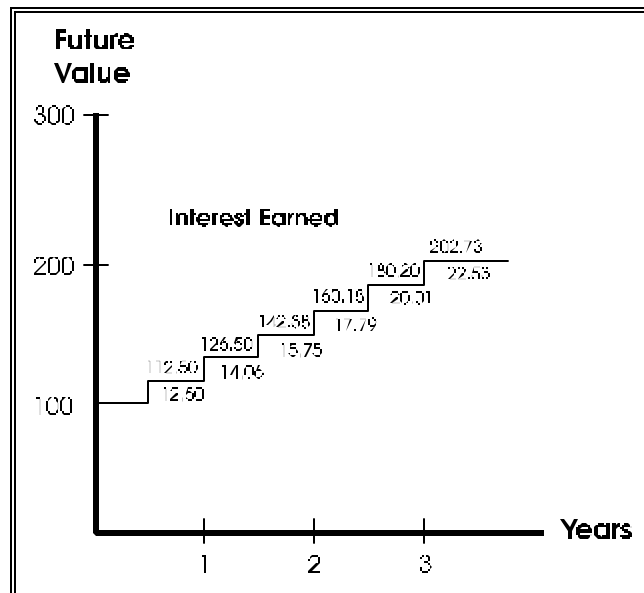


Figure 3.2: Semi-annual Compounding: \$100 at 25% for 3 Years

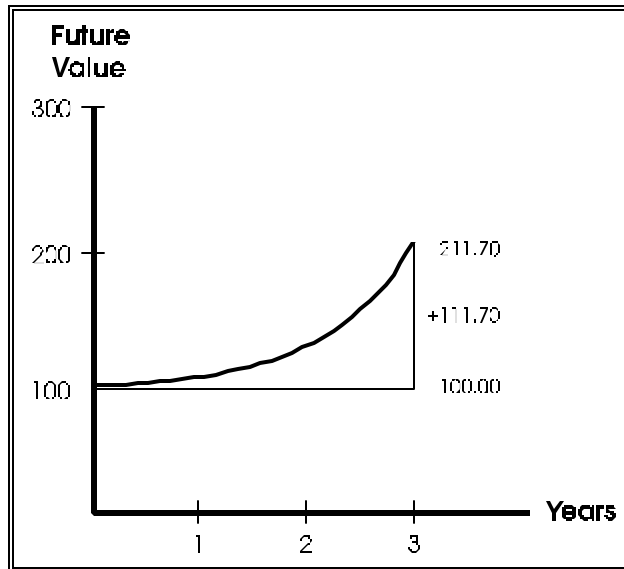


Figure 3.3: Continuous Compounding: \$100 at 25% for 3 Years

In the first two graphs, the interest is calculated and added to the account at discrete periods – at the end of each year (Figure 3.1) and after each six-month period (Figure 3.2). Notice in Figure 3.3 that when interest is calculated and added to the account continuously, the graph is a smooth curve.

We can summarize the results as follows:

<u>\$100 at 25% for three years</u>	
Annual compounding:	195.31
Semi-annual:	202.73
Continuous:	211.70

Summary

In this section, we discussed discrete and continuous compounding of interest. We looked at examples for calculating the interest payment on investments for annual and nonannual periods and the future value of these investments. The formulas for calculating interest payments and future value are:

Interest payment

$$I = P(1 + R)^T - P$$

I = Interest earned or paid

P = Principal

R = Stated (nominal) interest rate for each interest paying period

T = Number of interest paying periods

Future value of investment discretely compounded for annual periods

$$FV_T = P(1 + R)^T$$

FV_T = Future value after T paying periods

P = Principal

R = Stated (nominal) interest rate for each interest paying period

T = Number of interest paying periods

Future value of investment discretely compounded for non-annual periods

$$FV_T = P(1 + R/M)^{T \times M}$$

FV_T = Future value after T years

P = Principal

R = Stated (nominal) annual interest rate

M = Number of compounding periods within a year

T = Number of years

Future value of investment compounded continuously

$$FV_T = P \times e^{RT}$$

FV_T = Future value after T periods

P = Principal

e = Base of the natural logarithm (2.718282)

R = Stated (nominal) annual interest rate for each period

T = Number of periods

You can use these formulas or your financial calculator to answer the questions in the Practice Exercise that begins on page 3-17.

PRACTICE EXERCISE 3.2

Directions: Calculate the answer to each question and write the correct answer in the blank. Check your solution with the Answer Key on the next page.

7. A deposit of \$100 is made into an account on Jan. 1, 1950. If the account has a balance of \$480.10 on Dec. 31, 1989, what is the annual rate of interest earned on the account?

_____%

8. What is the future value of \$2,000 after 5 years if the interest rate is 30% p.a.?

\$_____

9. If you invest \$250,000 for one year at 8% compounded semi-annually, what is the total amount of interest you will receive during the year?

\$_____

ANSWER KEY

7. A deposit of \$100 is made into an account on Jan. 1, 1950. If the account has a balance of \$480.10 on Dec. 31, 1989, what is the annual rate of interest earned on the account?

4.0%

Use the future value formula and solve for R.

$$\begin{aligned}
 FV_T &= P(1+R)^T \\
 \$480.10 &= \$100(1+R)^{40} \\
 \$4.8010 &= (1+R)^{40} && \text{Divide both sides by \$100} \\
 (\$4.8010)^{1/40} &= (1+R) && \text{Raise each side of the equation by the exponent 1/40} \\
 1.0400 &= 1+R \\
 0.04 &= R \\
 R &= 4.0\%
 \end{aligned}$$

8. What is the future value of \$2,000 after 5 years if the interest rate is 30% p.a.?

\$7,425.86

$$\begin{aligned}
 FV_T &= P(1+R)^T \\
 FV_5 &= \$2,000(1+0.30)^5 \\
 FV_5 &= \$7,425.86
 \end{aligned}$$

9. If you invest \$250,000 for one year at 8% compounded semi-annually, what is the total amount of interest you will receive during the year?

\$20,400

$$\begin{aligned}
 I &= P(1+R/M)^{T \times M} - P \\
 I &= \$250,000(1+0.08/2)^{1 \times 2} - \$250,000 \\
 & \text{(If the annual rate is 8\%, the semi-annual rate is 4\% and the number of periods is 2)} \\
 I &= \$250,000(1.04)^2 - \$250,000 \\
 I &= \$250,000(1.0816) - \$250,000 \\
 I &= \$270,400 - \$250,000 \\
 I &= \$20,400
 \end{aligned}$$

PRACTICE EXERCISE 3.2*(Continued)*

10. If the interest in question 9 is compounded quarterly, how much interest will you receive during the year?

\$_____

11. If you borrow \$200,000 for three years at a 10% annual rate, compounded quarterly, but payable at the end of the loan, how much will you have to repay at the end of the three years?

\$_____

12. What is the future value of \$1,000 after 10 years, with an annual rate of 12% compounded continuously?

\$_____

ANSWER KEY

10. If the interest in question 9 is compounded quarterly, how much interest will you receive during the year?

\$20,608.04

$$\begin{aligned}
 I &= P (1 + R)^T - P \\
 I &= \$250,000 (1 + 0.02)^4 - \$250,000 \\
 I &= \$250,000 (1.082432) - \$250,000 \\
 I &= \$270,608.04 - \$250,000 \\
 I &= \$20,608.04
 \end{aligned}$$

11. If you borrow \$200,000 for three years at a 10% annual rate, compounded quarterly, but payable at the end of the loan, how much will you have to repay at the end of the three years?

\$268,977.76

$$\begin{aligned}
 FV_T &= P (1 + R/M)^{T \times M} \\
 FV_3 &= P (1 + 0.10/4)^{3 \times 4} \\
 FV_3 &= \$200,000 (1 + 0.025)^{12} \\
 FV_3 &= \$200,000 (1.34488) \\
 FV_3 &= \$268,977.76
 \end{aligned}$$

12. What is the future value of \$1,000 after 10 years, with an annual rate of 12% compounded continuously?

\$3,320.12

$$\begin{aligned}
 FV_T &= P \times e^{R \times T} \\
 FV_{10} &= \$1,000 \times e^{0.12 \times 10} \\
 FV_{10} &= \$1,000 \times (2.7183)^{1.2} \\
 FV_{10} &= \$1,000 \times (3.32012) \\
 FV_{10} &= \$3,320.12
 \end{aligned}$$

PRESENT VALUE

Discounting – reversal of compounding

Discounting is the process we use to equate a future cash flow to its **present value**. The calculation is the reversal of the compounding process and also may be done discretely or continuously.

To understand the discounting process, let's begin with the future value equation.

$$FV_T = PV (1 + R)^T$$

Notice that there is one slight difference in this future value equation — the variable P (principal) has been replaced by a PV variable. The PV stands for present value and refers to the value of the investment at the present time. To solve for PV, we divide both sides of the equation by the interest factor $(1 + R)^T$. The resulting present value equation is:

$$PV = \frac{FV_T}{(1 + R)^T}$$

Present value formula

The present value equation is often expressed in another format.

$$PV = FV_T [1/(1 + R)]^T$$

Where:

PV = Present value of the investment (value today)

FV_T = Future value of the investment at time T

T = Time in years until maturity

R = Annual discount rate

Present value interest factor

The term $1 / (1 + R)^T$ is the *present value interest factor* at R percent for T periods. FV_T is the future value of the investment at time T.

**Opportunity
cost of money**

Notice that we now refer to R as the "**discount**" rate. The discount rate represents the opportunity cost of money, which is the rate of return that could have been earned on the best alternative investment. In finance, analysts often use the concept of opportunity cost to evaluate investment alternatives. The logic is to estimate the next best return scenario if a different investment decision had been made. Applying this concept to discount rates, analysts say that, given approximately equal time frames and levels of risk, the appropriate rate to discount cash flows is the rate of return that represents the next best alternative in the list of investment choices. The discount rate or opportunity cost is also called the investor's required rate of return.

Discount rate = Opportunity cost = Investor's required rate of return

Present value terminology is similar to the future value terminology we studied in the last section. You can use most financial calculators for quick and easy present value calculations.

Discrete Discounting

A present value interest factor is the reciprocal of the future value interest factor. The value of the present value interest factor decreases as the time period or interest rate increases. Let's apply the present value formula to a few examples.

Example

Suppose that you expect to receive four future cash flows of \$1,000:

- 1) In one year at a discount rate of 10%
- 2) In two years at a discount rate of 10%
- 3) In two years at a discount rate of 12%
- 4) In three years at a discount rate of 10%

What is the present value of each of these \$1,000 cash flows?

Time	Discount Rate	Present Value
1 year	10%	$PV = FV_T [1 / (1 + R)]^T$ $PV = \$1,000 [1 / (1 + 0.10)]^1$ $PV = \$1,000 [0.90909]$ PV = \$909.09
2 years	10%	$PV = FV_T [1 / (1 + R)]^T$ $PV = \$1,000 [1 / (1 + 0.10)]^2$ $PV = \$1,000 [0.82645]$ PV = \$826.45
2 years	12%	$PV = FV_T [1 / (1 + R)]^T$ $PV = \$1,000 [1 / (1 + 0.12)]^2$ $PV = \$1,000 [0.79719]$ PV = \$797.19
3 years	10%	$PV = FV_T [1 / (1 + R)]^T$ $PV = \$1,000 [1 / (1 + 0.10)]^3$ $PV = \$1,000 [0.75131]$ PV = \$751.31

Look at the three cash flows that are discounted at 10% p.a. You can see that the present value of those three cash flows decreases as the length of time increases. Also, for the two cash flows expected in two years, the cash flow discounted at an annual rate of 10% has a greater present value (\$826.45) than the cash flow discounted at an annual rate of 12% (\$797.19).

Present value versus future value

If an investor has the opportunity to invest funds today at the same rate as the discount rate, it doesn't matter if the cash flow is received today for the present value amount, or later for the future value amount. For example, the \$1,000 cash flow expected in three years, given an annual discount rate of 10%, equals \$751.31 today. An investor with opportunities to invest at 10% compounded annually would be indifferent between receiving \$751.31 today or \$1,000 in three years, all other factors being equal.

Zero-coupon bond

The zero-coupon bond concept has a very practical application in the valuing of securities. For example, it is a security that pays no interest; it is priced at a discount from the **face value** to give the investor a specific rate of return. The face value is the amount the issuer will pay at the time of maturity. The issuer prices the bond at a discount so that investors receive a return on their investments.

Example

Suppose that a company is issuing zero-coupon bonds to raise funds for its operation. The bonds have a face value of \$10,000 and a maturity of one year. What would an investor requiring a 9% return be willing to pay for one of the bonds today? Let's apply our present value formula with $FV_T = \$10,000$, $T = 1$ year, and $R = 0.09$.

$$\begin{aligned} PV &= FV_T [1 / (1 + R)]^T \\ PV &= \$10,000 [1 / (1 + 0.09)]^1 \\ PV &= \$10,000 [0.917431] \\ PV &= \$9,174.31 \end{aligned}$$

An investor requiring a 9% return on investment would pay \$9,174.31 for one of the bonds. As an alternative, s/he could invest the \$9,174.31 in an account earning 9% annually; at the end of one year, the account would be worth \$10,000. As you can see, discounting to present value and compounding to future value are simply inverse calculations.

Continuous Discounting**Reverse of continuous compounding**

We can also reverse the calculation of continuous compounding to calculate a present value that results from **continuous discounting**. Recall that the formula for continuous compounding is:

$$FV_T = PV \times e^{RT}$$

Formula for continuous discounting

To discount continuously, we rearrange the continuous compounding formula and use present value (PV) in the place of principal (P). Solving for PV, we arrive at the continuous discounting formula.

$$PV = FV_T [1 / (e^{RT})]$$

Where:

PV = Present value of the investment (value today)

FV_T = Future value of the investment at time T

e = Base of the natural logarithm (2.718282)

T = Time in years until maturity

R = Annual discount rate (opportunity cost of money)

As you can see, the present value interest factor for continuous discounting $1 / (e^{RT})$ is the reciprocal of the future value interest factor for continuous compounding e^{RT} . A more common notation for this present value interest factor is e^{-RT} ; the formula is:

$$PV = FV_T [e^{-RT}]$$

Example

As an example, let's calculate the present value of a \$1,500 cash flow to be received in 4 years, discounted continually at an annual rate of 10%.

$$PV = FV_T [e^{-RT}]$$

$$PV = \$1,500 [2.718282^{-0.10 \times 4}]$$

$$PV = \$1,500 [0.6703200]$$

$$PV = \$1,005.48$$

Use the the formula for discrete discounting ($PV = FV_T [1/(1 + R)^T]$) or continuous discounting ($PV = FV_T [e^{-RT}]$) or your financial calculator to solve the discounting problems in the Practice Exercise that follows.

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PRACTICE EXERCISE 3.3

Directions: Calculate the answer to each question and enter the correct answer.
Check your solution with the Answer Key on the next page.

13. If the annual discount rate is 8.5%, how much will an investor pay to receive \$100,000 at the end of 1 year?

\$ _____

14. What is the present value of \$250, to be received in two years time, using an annual discount rate of 9%?

\$ _____

15. What is the present value of \$10,000, due five years from now, using a discount rate of 10%?

\$ _____

ANSWER KEY

13. If the annual discount rate is 8.5%, how much will an investor pay to receive \$100,000 at the end of 1 year?

\$92,166

$$\begin{aligned}PV &= FV_T [1 / (1 + R)]^T \\PV &= \$100,000 [1 / (1 + 0.085)]^1 \\PV &= \$100,000 [1 / 1.085] \\PV &= \$100,000 [0.92166] \\PV &= \$92,166\end{aligned}$$

14. What is the present value of \$250, to be received in two years time, using an annual discount rate of 9%?

\$210.42

$$\begin{aligned}PV &= FV_T [1 / (1 + R)]^T \\PV &= \$250 [1 / (1 + 0.09)]^2 \\PV &= \$250 [1 / (1.09)]^2 \\PV &= \$250 [0.84170] \\PV &= \$210.42\end{aligned}$$

15. What is the present value of \$10,000, due five years from now, using a discount rate of 10%?

\$6,209.20

$$\begin{aligned}PV &= FV_T [1 / (1 + R)]^T \\PV &= \$10,000 [1 / (1 + 0.10)]^5 \\PV &= \$10,000 [1 / (1.10)]^5 \\PV &= \$10,000 [0.62092] \\PV &= \$6,209.20\end{aligned}$$

PRACTICE EXERCISE 3.3*(Continued)*

16. What is the present value of \$2,000 to be received after 10 years, given an annual discount rate of 12% discounted continuously?

\$ _____

17. Discounting continuously, what is the present value of \$2,000, received after 10 years, given an annual discount rate of 10%?

\$ _____

ANSWER KEY

16. What is the present value of \$2,000 to be received after 10 years, given an annual discount rate of 12% discounted continuously?

\$602.38

$$\begin{aligned}PV &= FV_T [e^{-RT}] \\PV &= \$2,000 [e^{-0.12 \times 10}] \\PV &= \$2,000 [e^{-1.2}] \\PV &= \$2,000 [0.3011918] \\PV &= \$602.38\end{aligned}$$

17. Discounting continuously, what is the present value of \$2,000, received after 10 years, given an annual discount rate of 10%?

\$735.75

$$\begin{aligned}PV &= FV_T [e^{-RT}] \\PV &= \$2,000 [e^{-0.10 \times 10}] \\PV &= \$2,000 [e^{-1.0}] \\PV &= \$2,000 [0.367877] \\PV &= \$735.75\end{aligned}$$

ANNUITIES

Series of evenly spaced, equal payments

Many financial market securities involve more than one cash flow. An **annuity** is a security with a series of equal payments (or receipts) spaced evenly over a determined period. There are two types of annuities:

- Ordinary annuity (also called deferred annuity)
- Annuity due

An **ordinary annuity** payment is received at the *end* of the payment period, while an **annuity due** requires payment at the *beginning* of the period. In this section, we focus on the **valuation** of ordinary annuities, which are the most common. The calculations for an annuity due are almost the same, but they include an adjustment for the timing of cash payments.

Future Value of an Annuity

Compounding annuity payments

Calculating the future value of an annuity is a fairly straight-forward exercise. However, it can be tedious as the following example illustrates. Suppose that an investor invests \$100 each December 31st for the next ten years. The account pays an annual interest rate of 5%. What will be the value of the account at the end of the ten years? Let's build a table that outlines the compounding of the cash flows.

Year	Deposit	x	FVIF	=	Payments
1	\$100	x	$(1.05)^9$	=	155.14
2	\$100	x	$(1.05)^8$	=	147.75
3	\$100	x	$(1.05)^7$	=	140.71
4	\$100	x	$(1.05)^6$	=	134.01
5	\$100	x	$(1.05)^5$	=	127.63
6	\$100	x	$(1.05)^4$	=	121.55
7	\$100	x	$(1.05)^3$	=	115.76
8	\$100	x	$(1.05)^2$	=	110.25
9	\$100	x	$(1.05)^1$	=	105.00
10	\$100	x	$(1.05)^0$	=	<u>100.00</u>
			FV	=	\$1,257.80

Figure 3.4: Future Value of an Annuity

As you can see from the table, the interest on the deposit at the end of Year One is compounded for nine years, interest on the deposit at the end of Year Two for eight years, and so forth. The deposit at the end of Year Ten earns no interest. Thus, at the end of ten years, the total future value of this annuity is \$1,257.80.

Using a financial calculator

Most financial calculators can perform this type of calculation. The basic idea is to input the interest rate, the number of payments and the size of the payments, and push the *future value* key to calculate. Many calculators allow you to specify how many payments per year and whether the payment is made at the beginning of the period (annuity due) or at the end of the period (ordinary annuity).

Present Value of an Annuity

Discounting annuity payments

The calculation of the present value of an annuity is the reversal of the compounding process for the future value. The idea is to discount each individual payment from the time of its expected receipt to the present. We can illustrate by building another table to calculate the present value of the annuity in Figure 3.4: ten payments of \$100, made on December 31st of each year, discounted at a rate of 5%.

Year	Payment	x	PVIF	=	Present Value
10	\$100	x	$1/(1.05)^{10}$	=	61.39
9	\$100	x	$1/(1.05)^9$	=	64.46
8	\$100	x	$1/(1.05)^8$	=	67.68
7	\$100	x	$1/(1.05)^7$	=	71.07
6	\$100	x	$1/(1.05)^6$	=	74.62
5	\$100	x	$1/(1.05)^5$	=	78.35
4	\$100	x	$1/(1.05)^4$	=	82.27
3	\$100	x	$1/(1.05)^3$	=	86.39
2	\$100	x	$1/(1.05)^2$	=	90.70
1	\$100	x	$1/(1.05)^1$	=	<u>95.24</u>
			PV	=	\$772.17

Figure 3.5: Present Value of an Annuity

The present value of this annuity is \$772.17. This means that an investor with opportunities to invest in other investments of equal risk at 5% would be willing to pay \$772.17 for this annuity. Notice that because the payments are received at the end of each period, we discount them for the entire period.

Formula for present value of annuity

The formula for calculating the present value of an annuity is:

$$PV_a = A[1/(1+R)]^1 + A[1/(1+R)]^2 + \dots + A[1/(1+R)]^T$$

Where:

PV_a = Present value of the annuity

A = Amount of each annuity payment

R = Discount rate

T = Number of periods that the annuity is to be received

The idea is to discount each individual payment from the year of its expected receipt to the present. After discounting each cash flow, add up the discounted cash flows to arrive at the present value. The table in Figure 3.5 is a good way to organize the calculations.

Using a financial calculator

The quickest method for calculating the present value of an annuity is to use a financial calculator. Input the number of payments, the size of the payment, and the interest rate. Push the *present value* key to derive the present value.

Perpetuities

Infinite number of payments

A **perpetuity** is special type of annuity where the time period for the payments is infinity. Some examples of perpetuities are payments from an education endowment fund or the dividends paid on preferred stock.

Formula for present value of a perpetuity

The future value of a perpetuity is always infinity and, therefore, pointless to calculate. The present value of a perpetuity is easily calculated. The formula is:

$PV_p = A \times (1 / R)$ <p>Where:</p> <p>PV_p = Present value of the perpetuity</p> <p>A = Amount of each payment</p> <p>R = Discount rate (required rate of return)</p>
--

Example

Let's use the formula with an example. An investor requires 16% return on investment. What will the investor be willing to pay for a share of preferred stock with a \$10 annual dividend? To calculate, use these values: A = \$10 and R = 16%.

$$PV_p = A \times (1 / R)$$

$$PV_p = \$10 \times (1 / 0.16)$$

$$PV_p = \$62.50$$

If the investor pays \$62.50 per share, she will receive a \$10 dividend per year, which equals a 16% return on her investment.

UNEVEN PAYMENT STREAMS

Unequal size or spacing

Sometimes an investment has payments of unequal size or spacing. The present and future value formulas can be used to accommodate these irregularities. For instance, suppose that you deposit \$100 in an investment at the end of Year One, \$350 at the end of Year Three, \$250 at the end of Year Four, and \$200 at the end of Year Five. If the investment earns 6% compounded annually, what is the value of this investment at the end of Year Six? Let's build a table to illustrate the cash flows and their future values.

Year	Deposit	x	FVIF	=	Payments
1	\$100	x	(1.06) ⁵	=	133.82
2	\$000	x	(1.06) ⁴	=	0.00
3	\$350	x	(1.06) ³	=	416.86
4	\$250	x	(1.06) ²	=	280.90
5	\$200	x	(1.06) ¹	=	212.00
6	\$000	x	(1.06) ⁰	=	<u>0.00</u>
			FV	=	\$1,043.58

Figure 3.6: Future Value of Annuity with Unequal Payments

You can see that the first deposit is in the account for five years, earning compounded interest of \$33.82 during that time; no deposit is made in the second year; the third deposit earns compound interest for three years in the amount of \$66.86; and so on. The total interest the account will earn for the six years is \$1,043.58 – \$900.00 (the total amount of all the deposits) = \$143.58.

Formula for present value of unequal payments

The formula for calculating the present value of unequal payments is a modification of the formula for the present value for an annuity.

$$PV = CF_1[1/(1+R)]^1 + CF_2[1/(1+R)]^2 + \dots + CF_T[1/(1+R)]^T$$

Where:

PV = Present value of the stream of cash flows

CF_i = Cash flow (payment) in period i

R = Discount rate

T = Number of periods in the stream of cash flows

The idea is to discount each payment or cash flow from the time of receipt back to the present time. The present value is the sum of all of the discounted cash flows.

Example

To illustrate, we will calculate the present value of an investment with expected payments of \$250 at the end of Year One, \$500 at the end of Year Two, \$400 at the end of Year Four, and \$100 at the end of Year Five. The investor's required rate of return is 8%.

Year	Payment	x	PVIF	=	Present Value
5	\$100	x	$1/(1.08)^5$	=	68.06
4	\$400	x	$1/(1.08)^4$	=	294.01
3	\$000	x	$1/(1.08)^3$	=	0.00
2	\$500	x	$1/(1.08)^2$	=	428.67
1	\$250	x	$1/(1.08)^1$	=	<u>231.48</u>
			PV	=	\$1,022.22

Figure 3.7: Present Value of Unequal Discounted Cash Flows

Using a financial calculator

The present value of this investment is \$1,022.22. Another approach to solving this problem is to use a financial calculator that allows the input of individual cash flows. The process is to input each cash flow and the appropriate discount rate. Then, push the *present value* key to derive the present value of the investment as \$1,022.22.

Many calculators can handle this type of calculation. The logic is to input each payment for the period and the interest rate the investment is earning. Then push the *future value* key to derive the future value.

You have completed the section on calculating the present value and future value of annuities, perpetuities, and uneven payment streams. Please check your understanding of these calculations by completing the Practice Exercise that begins on page 3-37. Check your answers with the Answer Key.

Following the Practice Exercise is a summary of the concepts we have presented in this unit. Please read the summary, then answer the questions in Progress Check 3. It is important that you master these basic concepts, as they form the foundation for understanding the material in succeeding units of this course. If you answer any of the questions incorrectly, we recommend that you reread the appropriate section.

PRACTICE EXERCISE 3.4

Directions: Calculate the answer to each question and write the correct answer in the blank. Check your solution with the Answer Key on the next page.

18. A security is structured to pay cash flows of \$100 per year for ten years. If an investor's required rate of return is 5%, what price would the investor be willing to pay for the security?

\$ _____

19. You have agreed to pay five installments of \$100 for a television, the first payment being due immediately. If the discount rate is 10%, what is the equivalent cash price for the television?

\$ _____

20. PQR Inc. sets up a fund into which it will make annual payments that will accumulate at 5% per annum. The fund is designed to repay a bond issue at the end of 25 years. If the annual payment into the fund is fixed at \$5,000,000, how large is the bond issue?

\$ _____

ANSWER KEY

18. A security is structured to pay cash flows of \$100 per year for ten years. If an investor's required rate of return is 5%, what price would the investor be willing to pay for the security?

\$772

Building the table and adding the present value interest factors for the ten cash flows.

$$PV_a = \$100 (7.7217)$$

$$PV_a = \$772.17$$

Using a financial calculator:

Number of payments = 10

Interest rate = 5%

Payment = \$100

Press the present value key and you should get \$772.17

19. You have agreed to pay five installments of \$100 for a television, the first payment being due immediately. If the discount rate is 10%, what is the equivalent cash price for the television?

\$417

With the formula, calculate the present value of the last four payments and add the total to the deposit.

$$PV_a = \$100 (3.1699)$$

$$PV_a = \$317$$

$$\text{Cash Price} = \$317 + \$100 = \$417$$

Using a financial calculator:

Number of payments = 4

Interest rate = 10%

Payment = \$100

Press the present value key and you should get \$316.99. Then add the deposit of \$100 to get a cash price of \$417.

20. PQR Inc. sets up a fund into which it will make annual payments that will accumulate at 5% per annum. The fund is designed to repay a bond issue at the end of 25 years. If the annual payment into the fund is fixed at \$5,000,000, how large is the bond issue?

\$238,635,494

Using a financial calculator:

Number of payments = 25

Interest rate = 5%

Payment = \$5,000,000

Press the future value key and you should get \$238,635,494.

PRACTICE EXERCISE 3.4*(Continued)*

21. You are offered a contract that will pay \$50.00 per year indefinitely. If your discount rate is 10%, what would you be willing to pay for the offered contract?

\$ _____

22. If your required return is 10%, what would you be willing to pay for a perpetuity cash flow of \$5.00 per year?

\$ _____

ANSWER KEY

21. You are offered a contract that will pay \$50.00 per year indefinitely. If your discount rate is 10%, what would you be willing to pay for the offered contract?

\$500.00

$$PV_p = A \times (1/R)$$

$$PV_p = \$50.00 (1 / 0.10)$$

$$PV_p = \$500.00$$

22. If your required return is 10%, what would you be willing to pay for a perpetuity cash flow of \$5.00 per year?

\$50.00

$$PV_p = A \times (1/R)$$

$$PV_p = \$5.00 (1 / 0.10)$$

$$PV_p = \$50.00$$

UNIT SUMMARY

In this unit, we focused on the changes in the value of money over time.

Future value — simple / compound interest

The future value of money at the end of a defined period of time results from adding the amount of interest that may be earned on an investment to the principal amount.

- Simple interest is paid on the principal at the end of the payment period at a defined rate.
- Interest also may be compounded, which means that interest is reinvested after each payment period and earns interest in each subsequent period. Interest may be compounded at the end of discrete annual or non-annual periods, or continuously throughout the life of the investment.

Present value – discount

Sometimes we want to know the current value of a cash flow that will occur at some future date. Discrete or continuous discounting is the process for equating a future cash flow with its present value. The discount rate may be referred to as the opportunity cost of an investment or the investor's required rate of return.

Annuity

An annuity is a security that involves a series of equal payments made at regular time intervals. To derive the future value of an annuity, each payment is compounded for the remaining life of the security. A payment made at the end of the first year of a ten-year annuity is compounded for nine years, the second payment is compounded for eight years, etc. All compounded payments added together equal the future value.

Calculating the present value of an annuity is the reverse process of compounding. The payment due at the end of the tenth year is discounted over the ten year period, the ninth payment is discounted over nine years, etc. The present values of all payments added together equal the present value of the annuity.

Perpetuity

A perpetuity is a special type of annuity with an infinite number of payment periods. The future value of a perpetuity is infinite — and of little consequence. The present value is easy to calculate and provides the investor with information to facilitate a comparison of investment prices.

The present and future values of investments with payments of uneven size or spacing also may be calculated by totaling the compounded or discounted values of the cash flows.

You can see that the present / future value of an investment depends on the discount / interest rate, the number of payments, the length of time until maturity, and the amount of each cash flow. By manipulating these factors, we can discover the future value of an interest-bearing investment or the discounted present value of future cash flows.

You have completed Unit Three: *Time Value of Money*. Before proceeding to Unit Four: *Valuing Financial Assets*, please answer the following questions. They will check your comprehension of the concepts presented in this unit. If you answer any questions incorrectly, we suggest that you review the appropriate text.

✓ PROGRESS CHECK 3

Directions: Select the one best answer for each question, unless otherwise indicated. Check your responses with the Answer Key on the next page.

1. ABC Bank receives a loan payment of \$1,000 principal and interest from a customer. The bank loans the \$1,000 to another customer for one year at the rate of 6.25% per annum. The second customer will pay \$1,062.50 to the bank. This amount represents the:
 a) loan made to the second customer discounted to its present value on the maturity date.
 b) compounded value of the first customer's payment after it was loaned to customer two.
 c) future value of the loan payment received from the first customer calculated on a simple interest basis.
 d) simple interest future value of the amount of the loan made to the first customer.

2. In the future value formula $FV_T = P(1 + R/M)^{T \times M}$, if:
 a) M equals two, interest is compounded semi-annually for each year in the period.
 b) T equals one and M equals twelve, interest is compounded once during each twelve month period.
 c) T equals three, interest is compounded every four months.
 d) T equals five and M equals one, simple interest is calculated once each year for five years.

3. In the formula $FV_T = P \times e^{RT}$, e^{RT} represents:
 a) the future value interest factor for a discrete number of compounding periods.
 b) the future value interest factor for an infinitely large number of compounding periods.
 c) simple interest calculated on a continuous basis for a specified period of time.
 d) interest factor compounded for a discrete period (e) at a certain rate (R) for a specific amount of time (T).

ANSWER KEY

1. ABC Bank receives a loan payment of \$1,000 principal and interest from a customer. The bank loans the \$1,000 to another customer for one year at the rate of 6.25% per annum. The second customer will pay \$1,062.50 to the bank. This amount represents the:
 - c) **future value of the loan payment received from the first customer calculated on a simple interest basis.**

2. In the future value formula $FV_T = P(1 + R/M)^{T \times M}$; if:
 - a) **M equals two, interest is compounded semi-annually for each year in the period.**

3. In the formula $FV_T = P \times e^{RT}$, e^{RT} represents:
 - b) **the future value interest factor for an infinitely large number of compounding periods.**

PROGRESS CHECK*(Continued)*

4. If you borrow \$150,000 for five years at an 11.5% annual rate, compounded monthly, payable at the end of the loan, how much will you have to pay at the end of five years?

\$_____

5. To calculate the present value of a bond portfolio, we apply a discount rate that represents the (select **two**):

- _____ a) rate of return on the next best alternative investment.
_____ b) reciprocal value of an interest rate on a comparable investment.
_____ c) investor's required rate of return.
_____ d) investor's return on the bond portfolio.
_____ e) risk factor associated with the bond portfolio.

6. XYZ Company is issuing zero-coupon bonds to fund its expansion plans. The bonds have a face value of \$100,000 and a maturity of five years. How much would an investor requiring a 10% return be willing to pay today for one of these bonds?

\$_____

ANSWER KEY

4. If you borrow \$150,000 for five years at an 11.5% annual rate, compounded monthly, payable at the end of the loan, how much will you have to pay at the end of five years?

$$\mathbf{FV = \$265,840.78}$$

Using a financial calculator:

$$PV = \$150,000$$

$$\%_i = 11.5, 12$$

$$N = 60$$

Compute FV

5. To calculate the present value of a bond portfolio, we apply a discount rate that represents the (select **two**):

- a) **rate of return on the next best alternative investment.**
- c) **investor's required rate of return.**

6. XYZ Company is issuing zero-coupon bonds to fund its expansion plans. The bonds have a face value of \$100,000 and a maturity of five years. How much would an investor requiring a 10% return be willing to pay today for one of these bonds?

$$\mathbf{\$62,092.13}$$

Using a financial calculator:

$$FV = \$100,000$$

$$\%_i = 10$$

$$N = 5$$

Compute PV

PROGRESS CHECK*(Continued)*

7. Assuming the investor can purchase the bond at the price you calculated in question 6, compare the XYZ bond to a \$62,000 investment in ABC Company at 10% compounded annually for five years.

What is the return on this investment?

\$ _____

Which investment will yield more to the investor?

- _____ a) Loan to ABC Company
_____ b) Purchase of bond from XYZ Company

8. A college graduate with a new job and a promising career devises a ten-year savings plan. On December 31 of each year she will make the following deposits:

Years 1-3: \$1,000 each year

Years 4-6: \$2,000 each year

Years 7-10: \$5,000 each year

If the interest on the account is locked in at 8% per annum, how much will be in the account at the end of ten years?

\$ _____

ANSWER KEY

7. Assuming the investor can purchase the bond at the price you calculated in question 6, compare the XYZ bond to a \$62,000 investment in ABC Company at 10% compounded annually for five years.

What is the return on this investment?

\$37,851.62

Using a financial calculator:

$$PV = \$62,000$$

$$\%i = 10$$

$$N = 5$$

$$\text{Compute } FV = \$99,851.62 - 62,000 = 37,851.62$$

Which investment will yield more to the investor?

b) Purchase of bond from XYZ Company

$$\text{ABC Company Loan: } FV = \$99,851.62 - 62,000 = 37,851.62$$

$$\text{XYZ Bond: } FV = \$100,000 - 62,092.13 = 37,907.87$$

8. A college graduate with a new job and a promising career devises a ten-year savings plan. On December 31 of each year she will make the following deposits:

Years 1-3: \$1,000 each year

Years 4-6: \$2,000 each year

Years 7-10: \$5,000 each year

If the interest on the account is locked in at 8% per annum, how much will be in the account at the end of ten years?

\$36,927.70

Using a financial calculator (first deposit):

$$PV = \$1,000$$

$$\%i = 8$$

$$N = 9$$

$$\text{Compute } FV = 1,999.00$$

Year	Deposit	x	FVIF	=	Payments
1	\$1000	x	(1.08) ⁹	=	1,999.00
2	\$1000	x	(1.08) ⁸	=	1,850.93
3	\$1000	x	(1.08) ⁷	=	1,713.82
4	\$2000	x	(1.08) ⁶	=	3,173.75
5	\$2000	x	(1.08) ⁵	=	2,938.66
6	\$2000	x	(1.08) ⁴	=	2,720.98
7	\$5000	x	(1.08) ³	=	6,298.56
8	\$5000	x	(1.08) ²	=	5,832.00
9	\$5000	x	(1.08) ¹	=	5,400.00
10	\$5000	x	(1.08) ⁰	=	<u>5,000.00</u>
			FV	=	\$36,927.70

PROGRESS CHECK*(Continued)*

9. If the investor wishes to have \$40,000 at the end of ten years, and plans to invest an equal amount on December 31 of each year at 8% p.a., what will be the amount of the annuity payment?

\$_____

10. An investor intends to purchase, and to hold for an indefinite period of time, shares of Mega Company preferred stock. The stock pays an \$8 annual dividend. The investor requires a 14% return on investment. How much will the investor be willing to pay per share for Mega Company preferred stock?

\$_____

ANSWER KEY

9. If the investor wishes to have \$40,000 at the end of ten years, and plans to invest an equal amount on December 31 of each year at 8% p.a., what will be the amount of the annuity payment?

\$2,761.18

Using a financial calculator:

$$FV = \$40,000$$

$$\%i = 8$$

$$N = 10$$

$$\text{Compute PMT} = \$2,761.18$$

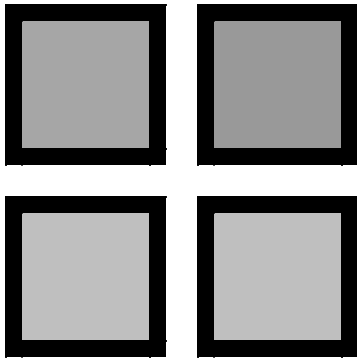
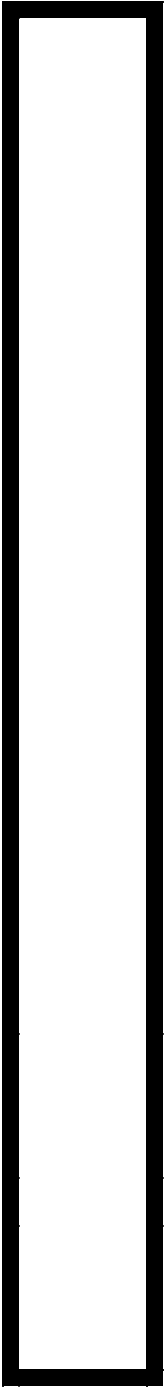
10. An investor intends to purchase, and to hold for an indefinite period of time, shares of Mega Company preferred stock. The stock pays an \$8 annual dividend. The investor requires a 14% return on investment. How much will the investor be willing to pay per share for Mega Company preferred stock?

\$57.14

$$PV_p = A \times (1 / R)$$

$$PV_p = \$8 \times (1 / 0.14)$$

$$PV_p = \$57.14$$



Unit 4

UNIT 4: VALUING FINANCIAL ASSETS

INTRODUCTION

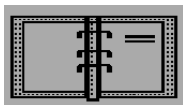
In Units One, Two, and Three, you learned some of the basics of financial markets, interest rates, and the time value of money. In this unit, you will apply these concepts to calculating the value of common financial assets. You will begin to learn how the market prices bonds, common stock, and preferred stock securities.



UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize some terminology associated with bonds
- Calculate the present value of a bond
- Apply the dividend valuation model to find the present value of common stock
- Calculate the present value of preferred stock



BONDS

Debt financing

There are two major types of debt financing:

- Bank loans
- Bonds

**Bank loan:
agreement
between lender
and borrower**

A bank loan is an agreement between a company and its bank. The bank provides a line of credit for the company to use, and the company pays the bank a rate of interest when it uses the credit line to borrow funds. Bank loans are priced so that the lending bank covers the cost of the funds and makes a profit.

**Bond:
agreement
between issuer
and investor**

Bonds represent loans by investors to a company. In a bond contract, the investor purchases a certificate from the issuer in exchange for a stream of interest payments and the return of a principal amount at the end of the contract. In this section we will discuss the terminology of the bond market and the methodology for calculating the price (present value) of a bond.

Bond Terminology

There are several terms that are commonly used by investors and issuers when dealing with bonds.

Coupon

The periodic interest payment made by the issuer. When bonds were first developed, the bond certificate had detachable coupons that the investor would send to the issuer to receive each interest payment. The term still applies to payments, even though coupons are no longer used to redeem them.

Coupon rate

The interest rate used to calculate the coupon amount the bond will pay. This rate is multiplied by the face value of the bond to arrive at the coupon amount.

Face (par) value

The amount printed on the certificate. The face value represents the principal in the loan agreement, which is the amount the issuer pays at maturity of the bond.

Maturity date	The date the loan contract ends. At this time, the issuer pays the face value to the investor who owns the bond.
Zero-coupon	A type of bond where the company pays no periodic interest payments. The bond is priced at a discount so that interest is imputed throughout the life of the bond. At maturity, the issuer pays the face value and the investor receives all of the return in the form of capital gain.

Bonds are often referred to as *fixed income securities* because they have a fixed payout to the investor. Since the coupon rate is set before the sale of the bond, the investor knows the amount of the interest payments.

Process for Issuing Bonds

A simple example will illustrate the process for issuing bonds.

Example

ABC Company needs capital to purchase a new piece of equipment for its operations. The company meets with financial advisors and investment bankers to discuss the possibilities of raising the necessary capital. They decide that a bond issue is the least expensive method for the company. The process is as follows:

Terms

1. ABC Company sets the maturity date and face value of the bonds.

The bonds will have a maturity date of ten years from the date of issue and a face value of \$1,000. The company will issue as many bonds as it needs for the equipment purchase – if the equipment costs \$10,000,000 fully installed, then the company will issue 10,000 bonds.

Coupon rate

2. Investment bankers set the coupon rate for the bonds.

The investment bankers attempt to gauge the interest rate environment and set the coupon rate commensurate with other bonds with similar risk and maturity. The coupon rate dictates whether the bonds will be sold in the secondary market at face value or at a discount or premium. If the coupon rate is higher than the prevailing interest rate, the bonds will sell at a *premium*; if the coupon rate is lower than the prevailing interest rate, the bonds will sell at a *discount*.

Primary issue

3. Investment bankers find investors for the bonds and issue them in the **primary market**.

The investment bankers use their system of brokers and dealers to find investors to buy the bonds. When investment bankers complete the sale of the bonds to investors, they turn over the proceeds of the sale (less the fees for performing their services) to the company to use for the purchase of equipment. The total face value of the bonds appears as a liability on the company's balance sheet.

Secondary market

4. The bonds become available in the secondary market.

Once the bonds are sold in the primary market to investors, they become available for purchase or sale in the secondary market. These transactions usually take place between two investors – one investor who owns bonds that are no longer needed for his/her investment **portfolio** and another investor who needs those same bonds.

Pricing Bonds

This section focuses on the calculations investors make to determine the price at which they will sell or buy bonds in the secondary markets.

Present value of cash flows

Pricing a bond involves finding the present value of the cash flows from the bond throughout its life. The formula for calculating the present value of a bond is:

$$V = C[1 / (1+R)]^1 + C[1 / (1+R)]^2 + \dots + C[1 / (1+R)]^T + F[1 / (1+R)]^T$$

Where:

V = Present value of the bond

C = Coupon payment (coupon rate multiplied by face value)

R = Discount rate (current prevailing rate)

F = Face value of the bond

T = Number of compounding periods until maturity

Similar to present value of an annuity

You may notice that this formula is similar to the formula for calculating the present value of an annuity. In fact, it is the same except the coupon payment (C) replaces the annuity payment (A).

There is also one additional term, $F[1/(1+R)]^T$, at the end of the formula. This term represents the discounting of the face value amount that is received at the maturity of the bond. The formula can be shortened to:

$$V = C \times PVIFA(R,T) + F \times PVIF(R,T)$$

Using this formula, we multiply the present value interest factor of an annuity by the coupon payment and add the product of the face value and the present value interest factor. Let's look at two examples to see how this formula works.

Example one: present value of bond

What is the present value of a bond with a two-year maturity date, a face value of \$1,000, and a coupon rate of 6%? The current prevailing rate for similar issues is 5%. To apply the formula, C = \$60 ($\$1,000 \times 0.06$), R = 0.05, T = 2, and F = \$1,000.

$$V = C[1 / (1+R)]^1 + C[1 / (1+R)]^2 + F[1 / (1+R)]^2$$

$$V = \$60[1 / (1+0.05)] + \$60[1 / (1+0.05)]^2 + \$1,000[1 / (1+0.05)]^2$$

$$V = \$60[0.95238] + \$60[0.90703] + \$1,000[0.90703]$$

$$V = \$57.14 + \$54.42 + \$907.03$$

$$V = \$1,018.59$$

The present value of \$1,018.59 is the price that the bond will trade for in the secondary bond market. You will notice that the price is higher than the face value of \$1,000. In the time since these bonds were issued, interest rates have fallen from 6% to 5%. Investors are willing to pay more for the \$60 interest payments when compared with new bond issues that are only paying \$50 in interest per \$1,000 face value. This inverse relationship is important.

**As interest rates fall, bond prices rise;
as interest rates rise, bond prices fall.**

A bond with a coupon rate that is the same as the market rate sells for face value. A bond with a coupon rate that is higher than the prevailing interest rate sells at a premium to par value; a bond with a lower rate sells at a discount. Investment bankers attempt to set the coupon rate for a newly issued bond at the market interest rate so that the bond sale will yield the amount of capital their corporate clients need for operations.

Using the financial calculator

You can use a financial calculator to find the present value of the bond in the example. With most financial calculators you input the number of interest payments, the size of the payments, the face value, and the discount rate. The *present value* key gives you the appropriate answer. Check your owner's manual for the specifics on how to calculate the present value of a bond.

Example two: present value of a bond

Let's look at one more example. If the current interest rate is 9%, how much would an investor be willing to pay for a \$10,000 face value bond with 5 coupon payments of 7.5% left until maturity? Use these values in the formula: $C = \$750$, $R = 0.09$, $T = 5$, and $F = \$10,000$.

$$\begin{aligned}
 V &= C[1 / (1+R)]^1 + C[1 / (1+R)]^2 + \dots + C[1 / (1+R)]^5 + F[1 / (1+R)]^5 \\
 V &= \$750[1 / (1+0.09)]^1 + \$750[1 / (1+0.09)]^2 + \dots + \$750[1 / (1+0.09)]^5 + \\
 &\quad \$10,000[1/(1+0.09)]^5 \\
 V &= \$750[0.91743] + \$750[0.84168] + \dots + \$750[0.64993] + \$10,000[0.64933] \\
 V &= \$9,416.55
 \end{aligned}$$

As you can see, longer maturities require more calculations. Using your calculator's cash flow functions can save a lot of time.

In this section, we have discussed the pricing of one type of debt financing. You have seen that the present value of a bond is the price at which it will trade in the secondary market. Now, let's see how the market values equity financing.

COMMON STOCK

Ownership claim on assets

In Unit One we defined common stock as a form of capital that represents an ownership claim on the assets of a company. Common stockholders have dividend and voting rights.

A common stock certificate may have a face (par) value printed on it that represents the price of the stock at the time it was originally issued. The current value of the stock may be greater than or less than the face value. Since common stock has no maturity date, it is often treated as a perpetuity when attempting to place a value on each share.

Valuing Common Stock

Dividend valuation model

One method of valuing common stock is the dividend valuation model. Remember, this is only one method of valuing stock. We will discuss some of the other methods later in this course.

Stock with constant dividend

The most straight-forward application of the dividend valuation model is to find the present value of stock that pays a constant dividend over time and is held indefinitely by the shareholder. This is really a simple perpetuity. As you learned in Unit Three, the formula for the present value of a perpetuity is:

$PV_p = A \times (1 / R)$	
Where:	
PV_p	= Present value of the perpetuity
A	= Amount of each payment
R	= Discount rate (required rate of return)

Dividend valuation formula

For the dividend valuation formula, we replace **A** with **D** to represent the dividend payment (the perpetuity) and use **V** to represent the value of the stock. The dividend valuation formula is:

$$V = D \times (1 / R)$$

Example

Let's look at an example. ABC Company's last common stock dividend was \$1.50, which is not expected to change in the foreseeable future. If the investor has a required return of 10%, what is the present value of this stock?

$$\begin{aligned} V &= D \times (1 / R) \\ V &= \$1.50 \times (1 / 0.10) \\ V &= \$15.00 \end{aligned}$$

Dividend with constant growth rate

The next application of the dividend valuation model is one in which the dividend grows at some constant rate. The formula is:

$$V = D_0 [(1+G) / (1+R)]^1 + D_0 [(1+G) / (1+R)]^2 + \dots + D_0 [(1+G) / (1+R)]^{\infty}$$

Where:

- V** = Present value of the stock per share
- D₀** = Most recent dividend per share
- R** = Discount rate (investor's required return)
- G** = Growth rate of the dividend

Condensed formula

You may remember from our discussion on the present value of a perpetuity, the sum of an infinite series can be reduced to a more workable form. For the present value of a stock with constant dividend growth, the condensed formula is:

$$V = D_0 [(1+G) / (R - G)]$$

Sometimes the formula is presented in a slightly different format. The expression **D₁** replaces **D₀ x (1+G)** and represents the value of the next dividend. The formula is:

$$V = D_1 / (R - G)$$

Both formulas arrive at the same result.

Three assumptions to consider

You must be aware of three important assumptions when using these formulas.

1. The growth rate must be constant (G cannot change).

The formulas will not work if the analyst expects different growth rates for the investment over its life.

2. The investor's required rate of return must be larger than the growth rate.

As you can see by looking at the formula, if the growth rate is larger than the discount rate, the present value of the stock will be negative – not an acceptable solution.

3. The dividends earned on the stock are reinvested at the same historical rate of return.

If the rate of return changes, the formulas must be adjusted to allow for the differences.

Lets look at two examples. In the first example, we use the dividend valuation model to solve for the present value of the stock, given the investor's required rate of return. In the second example, we solve for the investor's required rate of return, given the present value of the stock.

Example one: solve for present value

The last common stock dividend of ABC Company was \$1.00 and it is expected to grow at an annual rate of 5%. If the required rate of return is 10%, what is the present value of the common stock? We use our dividend valuation model formula, with the following values: $D_0 = \$1.00$, $G = 0.05$, and $R = 0.10$.

$$\begin{aligned} V &= D_0 [(1+G) / (R - G)] \\ V &= \$1.00 [(1 + 0.05) / (0.10 - 0.05)] \\ V &= \$1.00[(1.05) / (0.05)] \\ V &= \$1.00[21.00] \\ V &= \$21.00 \end{aligned}$$

The present value of a share of ABC Company's common stock is \$21.00. We could have arrived at the same answer using the shorter formula.

$$\begin{aligned} V &= D_1 / (R - G) \\ V &= \$1.05 / (0.10 - 0.05) \\ V &= \$1.05 / (0.05) \\ V &= \$21.00 \end{aligned}$$

As you may be able to infer from the formula, a larger growth rate translates to a larger present value of the stock and a smaller growth rate means a smaller present value.

**Example two:
solve for
required rate of
return**

Suppose that you buy a share of stock in a company for \$50, which you know is the appropriate price for the stock. The next dividend will be \$2, and you expect that the dividend will grow at a 10% annual rate. What is your required rate of return?

$$\begin{aligned} R &= (D_1 / V) + G \\ R &= (\$2 / \$50) + 0.10 \\ R &= (0.04) + 0.10 \\ R &= 0.14 \text{ or } 14\% \end{aligned}$$

This formula automatically splits the rate of return into two components: the return on the dividend (0.04) and the gain in the capital investment (0.10). The dividend return is expressed as D_1 / V , and the capital gain is expressed as the growth rate, G .

It is important to remember that this is only one approach to valuing common stock. Because of the three restrictive assumptions we discussed on page 4-9, this model can be used only in a limited number of situations. If a company does not pay any dividends, or if the growth rate is not constant or is higher than the discount rate, then we must use other valuation models. We will discuss some of these other approaches in Unit 7: *Corporate Valuation – Cost of Capital*.

PREFERRED STOCK

Common vs. preferred stock

Preferred stock also represents an ownership claim on the assets of a company. However, a company's preferred stockholders receive dividends before the common stockholders, they have no voting rights, and are given preference to company assets ahead of common stockholders when a company is liquidated.

Pricing Preferred Stock

Behaves like a perpetuity

We can take the same approach to valuing preferred stock as we did for common stock. Since preferred stock has no maturity, it also behaves like a perpetuity. Remember, the present value of a perpetuity is the sum of an infinite series. The shortened formula for calculating the present value of preferred stock is identical to the common stock valuation formula for constant dividends.

$$V = D \times (1 / R)$$

The characteristics of preferred stock are better suited for this formula. Most companies that issue preferred stock do pay a constant dividend. In fact, they have the obligation to pay the preferred dividends before paying the common dividends. If business is slow, the company may choose to reduce or eliminate the common dividend, but it usually will try to continue paying the preferred dividend.

Example: present value of preferred stock

Let's look at an example using the formula to value preferred stock. If Mega Company pays an annual dividend of \$5 to its preferred shareholders, and an investor's required rate of return is 10%, what is the present value of the stock?

$$\begin{aligned} V &= D \times (1 / R) \\ V &= \$5 \times (1 / 0.10) \\ V &= \$5 \times (10.00) \\ V &= \$50.00 \end{aligned}$$

An investor with a 10% required rate of return would be willing to pay \$50 for a share of Mega Company preferred stock that pays an annual dividend of \$5.

UNIT SUMMARY

In this unit, we discussed some bond terminology, the process for issuing bonds, and some ways to value simple debt and equity securities.

Calculating the present value of a bond is similar to calculating the present value of an annuity.

$$V = C \times PVIFA(R,T) + F \times PVIF(R,T)$$

The dividend valuation model is one method for valuing common stock. It is similar to the formula for calculating the present value of a perpetuity. The dividend valuation formula for finding the present value of stock that pays a constant dividend is:

$$V = D \times (1 / R)$$

Another application of the dividend valuation model is one in which the dividend grows at some constant rate. The formula is:

$$V = D_1 / (R - G)$$

There are three important assumptions to consider when using this formula.

1. The growth rate must be constant (G cannot change).
2. The investor's required rate of return must be larger than the growth rate.
3. The dividends earned on the stock are reinvested at the same historical rate of return.

The same formula that is used to value common stock with constant dividends is used to value preferred stock.

You have completed Unit Four: *Valuing Financial Assets*. Please complete the Progress Check that follows. If you answer any questions incorrectly or have difficulty with the calculations, please review the appropriate section in the text and then proceed to Unit Five: *Introduction to Capital Budgeting*.

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✓ PROGRESS CHECK 4

Directions: Answer each question by checking the correct answer(s) or by calculating the answer and entering it on the blank line.

1. Match each term in Column A with its correct definition in Column B.

<u>Column A</u>	<u>Column B</u>
_____ Coupon	a) Issuer pays face value to investor
_____ Coupon rate	b) Investor receives return at maturity
_____ Face value	c) Establishes amount issuer will pay
_____ Maturity date	d) Period interest payment
_____ Zero-coupon	e) Principal of loan agreement

2. Since the present value of a bond has an inverse relationship to market rates:

- _____ a) a bond with a coupon rate that is higher than the prevailing interest rate sells at a discount to the par value.
- _____ b) a bond with a coupon rate that is lower than the prevailing interest rate sells at a premium to the par value.
- _____ c) a bond with a coupon rate that is higher than the prevailing interest rate sells at a premium to the par value.
- _____ d) a bond with a coupon rate that is the same as the market rate only can be sold in the primary market.

ANSWER KEY

1. Match each term in Column A with its correct definition in Column B.

Column A

- d Coupon
c Coupon rate
e Face value
a Maturity date
b Zero-coupon

Column B

- a) Issuer pays face value to investor
b) Investor receives return at maturity
c) Establishes amount issuer will pay
d) Period interest payment
e) Principal of loan agreement

2. Since the present value of a bond has an inverse relationship to market rates:

c) a bond with a coupon rate that is higher than the prevailing interest rate sells at a premium to the par value.

PROGRESS CHECK*(Continued)*

3. A bond matures in one year and pays interest at maturity. If the face value of the bond is \$500,000, the coupon rate is 9%, and the required rate of return is 9%, what is the present value ?

\$ _____

4. The XYZ Corporation has issued a 10-year bond with a face value of \$1,000 and a coupon rate of 12%. Interest is paid annually, and the discount rate is 11%. What is the present value of the bond?

\$ _____

5. A \$100 bond carries an 8% coupon rate, matures in 10 years with interest paid annually, and is discounted at a rate of 7%. What is the bond's present value?

\$ _____

ANSWER KEY

3. A bond matures in one year and pays interest at maturity. If the face value of the bond is \$500,000, the coupon rate is 9%, and the required rate of return is 9%, what is the present value ?

\$500,000.00

Using a financial calculator:

Number of payments = 1
Payment = \$45,000
Discount rate = 9%
Face value = \$500,000.
Press the present value key and you should get \$500,000.

4. The XYZ Corporation has issued a 10-year bond with a face value of \$1,000 and a coupon rate of 12%. Interest is paid annually, and the discount rate is 11%. What is the present value of the bond?

\$1,058.89

Using a financial calculator:

Number of payments = 10
Payment = \$120
Discount rate = 11%
Face value = \$1,000
Press the present value key to get \$1,058.89.

5. A \$100 bond carries an 8% coupon rate, matures in 10 years with interest paid annually, and is discounted at a rate of 7%. What is the bond's present value?

\$107.02

Using a financial calculator:

Number of payments = 10
Payment = \$8
Discount rate = 7%
Face value = \$100
Press the present value key to get \$107.02.

PROGRESS CHECK*(Continued)*

6. ABC Industries' bonds have a coupon rate of 11%, a maturity of 10 years, and a face value of \$1,000. If the coupons are semi-annual, and the investor's required return is 12%, what is the value of the bond?

\$ _____

7. A company has been paying an annual common stock dividend of \$0.75 per share and is not expecting any increase in the future. If the appropriate discount rate is 13%, what is the present value of the common stock?

\$ _____

8. A company has been paying an annual common stock dividend of \$12 per share and is expecting to raise the dividend at a 5% annual rate. If the required rate of return is 11%, what would an investor expect the stock to sell for?

\$ _____

ANSWER KEY

6. ABC Industries' bonds have a coupon rate of 11%, a maturity of 10 years, and a face value of \$1,000. If the coupons are semi-annual, and the investor's required return is 12%, what is the value of the bond?

\$942.65

Since the bond pays interest semi-annually, we need to make a couple of adjustments to arrive at the proper solution. With a 11% semi-annual coupon, there are two \$55 interest payments per year ($\$110 / 2 = \55). There will be 20 payments over the life of the bond. The appropriate discount rate is a semi-annual rate of 6% ($12\% / 2 = 6\%$).

Using a financial calculator:

*Number of payments = 20
 Payment = \$55
 Discount rate = 6%
 Face value = \$1,000
 Press the present value key to get \$942.65.*

7. A company has been paying an annual common stock dividend of \$0.75 per share and is not expecting any increase in the future. If the appropriate discount rate is 13%, what is the present value of the common stock?

\$5.77

$$V = D / R$$

$$V = \$0.75 / 0.13$$

$$V = \$5.77$$

8. A company has been paying an annual common stock dividend of \$12 per share and is expecting to raise the dividend at a 5% annual rate. If the required rate of return is 11%, what would an investor expect the stock to sell for?

\$210

$$V = D_0 [(1 + G) / (1 + R)]$$

$$V = \$12[(1 + 0.05) / (0.11 - 0.05)]$$

$$V = \$12[1.05 / 0.06]$$

$$V = \$210$$

PROGRESS CHECK*(Continued)*

9. PPQ Company has paid a \$3.60 dividend on each share of common stock for the past three years and does not expect growth in the future. If the present value of the stock is \$40.00, what rate of return must investors be demanding?

\$ _____

10. A share of preferred stock pays an annual dividend of \$10. Calculate its present value to an investor whose required return is 12%.

\$ _____

11. A stock pays a 10% dividend on its \$50 par value and the investor's required rate of return is 8%. What is the present value of the preferred stock?

\$ _____

ANSWER KEY

9. PPQ Company has paid a \$3.60 dividend on each share of common stock for the past three years and does not expect growth in the future. If the present value of the stock is \$40.00, what rate of return must investors be demanding?

9%

$$\begin{aligned}R &= D / V \\R &= \$3.60 / 40 \\R &= 0.09 \text{ or } 9\%\end{aligned}$$

10. A share of preferred stock pays an annual dividend of \$10. Calculate its present value to an investor whose required return is 12%.

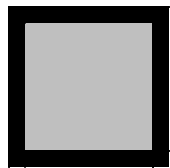
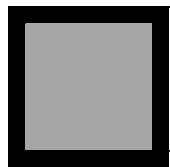
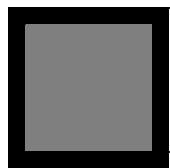
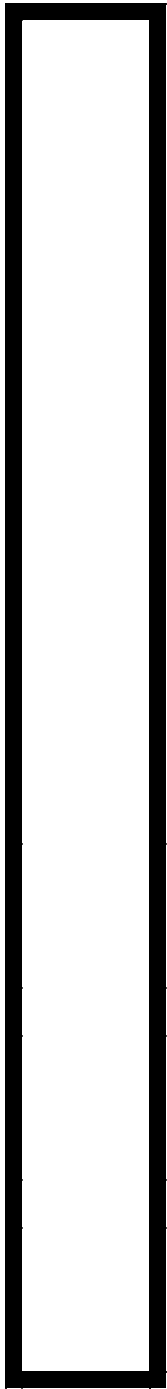
\$83.33

$$\begin{aligned}V &= D / R \\V &= \$10 / 0.12 \\V &= \$83.33\end{aligned}$$

11. A stock pays a 10% dividend on its \$50 par value and the investor's required rate of return is 8%. What is the present value of the preferred stock?

\$62.50

$$\begin{aligned}V &= D / R \\V &= \$5 / 0.08 \quad \text{The dividend is } \$5 \text{ (} 0.10 \times \$50 = \$5 \text{)} \\V &= \$62.50\end{aligned}$$



Unit 5

UNIT 5: INTRODUCTION TO CAPITAL BUDGETING

INTRODUCTION

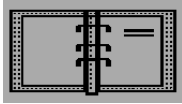
Capital budgeting refers to the techniques used by financial analysts to evaluate, compare, and select investment alternatives that will maximize the shareholder's equity in the company. In this unit, the formulas and calculations you have learned will be applied to new situations that require an analyst to make "invest / don't invest" decisions. We will also introduce some additional factors, including inflation, that the analyst must consider in making investment decisions.



UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize the difference between nominal and effective interest rates
- Define net present value (NPV) and internal rate of return (IRR)
- Calculate the net present value of a stream of cash flows
- Calculate the internal rate of return of a stream of cash flows
- Recognize other analytical techniques that may be applied to the capital budgeting process



NOMINAL AND EFFECTIVE RATES

Interest paid more than annually

In previous units, most of the securities we referred to in the examples paid annual interest (or dividend) payments. However, in the real world, there are many securities that pay interest more frequently. U.S. Treasury securities that pay interest semi-annually and corporate bonds that pay interest quarterly are some examples. For these types of securities, there is a difference between the quoted interest rate and the amount of interest that is actually paid.

Stated vs. actual rate of return

The **nominal interest rate** is the stated or contracted rate of return that a security is expected to pay. On a bond, the coupon rate is the nominal rate. The **effective interest rate** is the rate of return actually earned on the investment.

Calculating Effective Interest Rates

Example

An example will illustrate the difference between nominal and effective interest rates. Suppose that you invest \$1,000 in an account that pays a 10% nominal annual interest, compounded semi-annually. At the end of one year, how much will be in your account? Recall the future value formula:

$$FV_T = P (1 + R/M)^{T \times M}$$

Where:

P = Principal invested

R = Nominal annual interest rate

M = Number of compounding periods in the year

T = Number of years

The future value of your account at the end of the year is:

$$FV_1 = \$1,000 (1 + (0.10/2))^{1 \times 2}$$

$$FV_1 = \$1,102.50$$

The effective interest rate is the rate of return actually earned on an investment. This is calculated by dividing the total amount of interest earned over the year by the original investment. In the example, the effective interest rate is:

$$\text{\$102.50} / \text{\$1,000.00} = 0.1025 \text{ or } 10.25\%$$

**Effective rate
higher than
nominal rate**

As you can see, the effective annual rate (10.25%) is higher than the nominal annual rate (10%). This is always the case when the quoted annual interest is compounded more frequently than at annual intervals.

There is a more direct formula for comparing nominal and effective interest rates:

$$R^* = [1 + (R / M)]^M - 1$$

Where:

R* = Effective interest rate

R = Nominal interest rate

M = Number of compounding periods in the year

Example

Using this formula, find the effective annual interest rate of an investment that pays a quoted annual rate of 11% per annum (p.a.), compounded monthly. Use the values $R = 0.11$ and $M = 12$.

$$R^* = [1 + (R / M)]^M - 1$$

$$R^* = [1 + (0.11 / 12)]^{12} - 1$$

$$R^* = [1.009167]^{12} - 1$$

$$R^* = [1.115719] - 1$$

$$R^* = 0.115719 \text{ or } 11.57\%$$

Adjusting for Inflation

Relationship between nominal, real, and inflation rates

Inflation is one of the most influential factors in the determination of nominal interest rates. You may recall that when we discussed inflation in Unit Two, we said that the quoted (nominal) interest rate is equal to the risk-free **real interest rate** plus an inflation premium. A more precise formula should be used to find the effective interest rate when making adjustments for longer-term securities or in environments of high inflation. The relationship between nominal, real, and inflation rates can be calculated with this formula:

$$R_R = [(1 + R_N) / (1 + h)] - 1$$

Where:

R_R = Real interest or real discount rate

R_N = Nominal interest or discount rate

h = Expected inflation rate

Effective interest rate after adjusting for inflation

Let's look at an example to see how we apply the formula to find out the effective interest rate on an investment after adjusting for inflation.

Suppose that a Central Treasury Bank issues one-year bonds at a nominal interest rate of 25% p.a. Economists expect that the average inflation for the coming year will be 14%. What is the real interest rate that an investor can expect to earn by buying the bonds? We use the values $R_N = 0.25$ and $h = 0.14$.

$$R_R = [(1 + R_N) / (1 + h)] - 1$$

$$R_R = [(1 + 0.25) / (1 + 0.14)] - 1$$

$$R_R = [(1.25) / (1.14)] - 1$$

$$R_R = [1.0965] - 1$$

$$R_R = 0.0965 \text{ or } 9.65\%$$

This means that although the stated interest rate on the bonds is 25%, after adjusting for inflation, an investor actually earns only 9.65% on the investment. The difference between the two rates represents the loss of purchasing power resulting from inflation. As we mentioned before, this concept is particularly important in economies experiencing high levels of inflation.

Solving for nominal rate

By adjusting the formula, we can solve for the nominal rate.

$$R_N = R_R + h + (h \times R_R)$$

The following example illustrates how to find the appropriate nominal rate at which to invest in an inflationary environment when the targeted real rate of return is known.

Example

An investor would like to earn a real interest rate of 7% p.a. on an investment. If the expected inflation rate is 8% for the coming year, at what nominal rate should s/he invest so that the real rate of return is 7%? Use the second formula with the values $R_R = 0.07$ and $h = 0.08$.

$$\begin{aligned} R_N &= R_R + h + (h \times R_R) \\ R_N &= 0.07 + 0.08 + [(0.08)(0.07)] \\ R_N &= 0.1556 \text{ or } 15.56\% \end{aligned}$$

Given the expected inflation rate of 8%, the investor should invest at a 15.56% nominal rate to earn a 7% real rate on the investment.

In the Practice Exercise that follows, you will have an opportunity to practice what you have learned about calculating effective interest rates for multiple compounding periods and annual interest rates adjusted for inflation. Please complete the practice exercises and check your answers with the Answer Key before continuing with the next section on "Net Present Value."

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PRACTICE EXERCISE 5.1

Directions: Calculate the answer to each question, then enter the correct answer. Check your solution with the Answer Key on the next page.

1. An investor buys \$1M of 8% Treasury notes which pay interest after six months and are redeemed after 1 year. What is the effective interest rate earned by the investor, assuming that he can reinvest the first coupon payment at 8% for the next six months?

\$ _____

2. An account pays 1.5% interest on the balance at the end of each month. What is the effective annual interest rate earned on the account?

\$ _____

3. Treasury bonds are issued at a nominal interest rate of 30% for the following year. Inflation is expected to be 15% for the same period. What is the real interest rate earned on the bonds?

\$ _____

ANSWER KEY

1. An investor buys \$1M of 8% Treasury notes which pay interest after six months and are redeemed after 1 year. What is the effective interest rate earned by the investor, assuming that he can reinvest the first coupon payment at 8% for the next six months?

8.16%

$$R^* = [1 + (R/M)]^M - 1$$

$$R^* = [1 + (0.08/2)]^2 - 1$$

$$R^* = [1.0816] - 1$$

$$R^* = 0.0816 \text{ or } 8.16\%$$

2. An account pays 1.5% interest on the balance at the end of each month. What is the effective annual interest rate earned on the account?

19.56%

$$R^* = [1 + (R/M)]^M - 1$$

$$R^* = [1 + (0.015)]^{12} - 1$$

$$R^* = [1.1956] - 1$$

$$R^* = 0.1956 \text{ or } 19.56\%$$

3. Treasury bonds are issued at a nominal interest rate of 30% for the following year. Inflation is expected to be 15% for the same period. What is the real interest rate earned on the bonds?

13.04%

$$R_R = [(1 + R_N) / (1 + h)] - 1$$

$$R_R = [(1 + 0.30) / (1 + 0.15)] - 1$$

$$R_R = [(1.30) / (1.15)] - 1$$

$$R_R = [1.1304] - 1$$

$$R_R = 0.1304 \text{ or } 13.04\%$$

NET PRESENT VALUE

The concept of present value and its calculation should be familiar to you from our discussion of the time value of money in Unit Three. In this section, we extend the concept to show another way it is used: to calculate the **net present value** of an investment.

Present Value of Cash Flows

Let's begin by reviewing the formula for calculating the present value of a series of cash flows.

$$PV = CF_1 [1 / (1+R)]^1 + CF_2 [1 / (1+R)]^2 + \dots + CF_T [1 / (1+R)]^T$$

Sum of a series of discounted cash flows

We discount the cash flow in the first period (CF_1) for one period, discount the cash flow received in the second period (CF_2) for two periods, and so forth until we have discounted all of the cash flows. The present value is the sum of the discounted cash flows. We can shorten this formula using the summation symbol Σ (pronounced sigma) to represent the sum of a series.

$$PV = \sum_{t=1}^T CF_t [1 / (1 + R)]^t$$

These two equations are equal; they both represent the sum of a series of discounted cash flows. In the shortened version, the T at the top of the Σ means that we end with the T (last) cash flow, and the $t = 1$ at the bottom means that we start the summation with the first cash flow.

Net Present Value of Cash Flows

The calculation for the present value of a series of cash flows may be used to find out how much an investor will be willing to pay for an investment. Because the investor has a specific required rate of return, it is unlikely that a rational investor will pay more than the present value for an investment.

Present value minus initial investment

The term *net present value* refers to an investor's position after making an investment. To calculate the net present value of an investment, we modify the present value formula by subtracting the initial investment from the present value calculation.

$$NPV = \sum_{t=1}^T CF_t [1/(1+R)]^t - CF_0$$

Where:

- NPV = Net present value of the project or investment
- T = Number of cash flows generated by the project
- CF_t = Cash flow in period t
- CF₀ = Initial cash investment
- R = Discount rate (required rate of return)

The original capital investment is often called the cash flow at time 0 (or present time) and is represented by the symbol CF₀. The net present value (NPV) is equal to the sum of all the discounted cash flows minus the original payment made in order to receive the cash flows.

Positive / Negative NPV

A positive NPV means that the investor paid less than the present value for the stream of cash flows. A negative NPV means that the investor paid more than the present value for the stream of cash flows. An NPV equal to zero means that the investor paid the same amount as the present value for the stream of cash flows.

Finding NPV with a financial calculator

Most financial calculators may be used to calculate NPV. The basic idea is to input each cash flow in its proper order. Remember to input the original capital investment (CF_0) and any other cash outflows as negative numbers. After inputting the positive and negative cash flows, input the appropriate discount rate and then push the *NPV* key to find the solution. Check the owner's manual for specific details.

Analyzing a potential project

Companies often calculate NPV to evaluate potential **projects**. Analysts forecast the expected cash flows, discount the cash flows at the appropriate rate, and subtract the estimated initial capital investment. A project with a positive NPV creates value for the shareholders of the company, whereas a project with a negative NPV destroys value for the shareholders.

Independent projects

Companies with limited funds to invest, and several potential **independent projects** to evaluate, often rank the projects according to NPV and then invest in those projects with the highest NPV. The term "independent projects" means that a change in the cash flows in one project has no effect on the cash flows in any other project — each project is independent of all others being considered. The concept of independence is important for making NPV decisions. For a more thorough discussion on independent events, consult a probability textbook.

An example will illustrate the use of the net present value formula to evaluate a project.

Example

Project A requires a capital investment of \$2,000 and promises a payment of \$1,000 at the end of Years One, Two, and Three. If the investor's required rate of return is 12%, what is the NPV of the investment? We can use the NPV formula with the values CF_1 , CF_2 , and $CF_3 = \$1,000$, $CF_0 = \$2,000$, $T = 3$, and $R = 0.12$.

$$NPV = \sum_{t=1}^T CF_t [1 / (1 + R)]^t - CF_0$$

$$\begin{aligned} NPV &= \$1,000[1 / (1.12)]^1 + \$1,000[1 / (1.12)]^2 + \$1,000[1 / (1.12)]^3 - \$2,000 \\ NPV &= \$1,000[0.8929] + \$1,000[0.7972] + \$1,000[0.7118] - \$2,000 \\ NPV &= \$401.83 \end{aligned}$$

Using the financial calculator

To solve for NPV with your financial calculator, enter each of the cash flows and capital investments in their proper order. Enter the discount rate of 12% and push the *NPV* key to get \$401.83. Project A has a positive net present value.

Analyzing opportunity cost for use of funds

Some analysts use net present value to determine if the cash flows are sufficient to repay the capital investment plus an amount for the opportunity cost of using the company's funds. A positive NPV means that the project is able to repay the initial investment, pay the opportunity cost for the use of the funds, and generate additional funds that create value for the company; a negative NPV means that the project cannot generate enough funds to cover the original investment and the opportunity cost for the use of the funds.

Practice what you have learned about calculating the net present value of an investment by completing the Practice Exercise that begins on page 5-13; then continue to the section on "Internal Rate of Return."

PRACTICE EXERCISE 5.2

Directions: Calculate the answer to each question, then enter the correct answer.
Check your solution with the Answer Key on the next page.

4. A project has an initial investment of \$10,000 and a cash flow annuity of \$5,000 for three years. If the required rate of return is 10%, what is the NPV of the project?

\$_____

Should the project be accepted based on the NPV method of capital budgeting?

_____ a) Yes

_____ b) No

5. A project has an initial investment of \$20,000 and a cash flow annuity of \$5,000 for four years. If the required rate of return is 12%, what is the NPV of the project?

\$_____

Should the project be accepted based on the NPV method of capital budgeting?

_____ a) Yes

_____ b) No

ANSWER KEY

4. A project has an initial investment of \$10,000 and a cash flow annuity of \$5,000 for three years. If the required rate of return is 10%, what is the NPV of the project?

\$2,434.26

Enter -\$10,000 as CF_0 and three cash flows of \$5,000

Enter 10% as the discount rate

Press the NPV key and you should get \$2,434.26

Should the project be accepted based on the NPV method of capital budgeting?

Yes – *The project should be accepted because its NPV is positive.*

5. A project has an initial investment of \$20,000 and a cash flow annuity of \$5,000 for four years. If the required rate of return is 12%, what is the NPV of the project?

-\$4,813.25

Enter -\$20,000 as CF_0 and four cash flows of \$5,000

Enter 12% as the discount rate

Press the NPV key and you should get -\$4,813.25

Should the project be accepted based on the NPV method of capital budgeting?

No – *The project should be rejected because its NPV is negative.*

INTERNAL RATE OF RETURN

Actual rate of return on investment

The **internal rate of return (IRR)** is another calculation for evaluating the value to shareholders of potential project investments. The IRR represents the actual rate of return earned on an investment. The idea of IRR is to find the discount rate that will make the net present value of the cash flows equal to the initial investment. We start with the NPV formula.

$$NPV = \sum_{t=1}^T CF_t [1 / (1 + R)]^t - CF_0$$

To calculate the IRR, we set the NPV to 0 and solve for R (the internal rate of return for the investment). As you can see, this can be very difficult, especially if T (the number of cash flows) is quite large. The formula looks like this:

$$0 = \sum_{t=1}^T CF_t [1 / (1 + R)]^t - CF_0$$

Finding IRR with a financial calculator

Most financial calculators can handle this calculation. The method is to input the cash flows in the proper order (remember to enter CF_0 as a negative number) and push the *IRR* key. Check your owner's manual for the specifics.

Try the following example using your calculator.

Example

An investment requires an initial investment of \$2,000 and will pay \$1,000 at the end of the next three years. Calculate the investment's internal rate of return (IRR). Enter the cash flows into your calculator, $CF_0 = -\$2,000$ and $CF_1, CF_2,$ and $CF_3 = \$1,000$ each. Now push the *IRR* key and you should get $IRR = 23.38\%$.

Company's required rate of return

Many companies and analysts use IRR as an additional tool for making investment decisions. They refer to a hurdle rate, which is the required rate of return for a company or investor. A project that generates an IRR which is greater than the hurdle rate will create value for the shareholders. A project with an IRR that is less than the hurdle rate will destroy shareholder value and should not be accepted.

The diagram in Figure 5.1 illustrates the relationship of NPV to IRR.

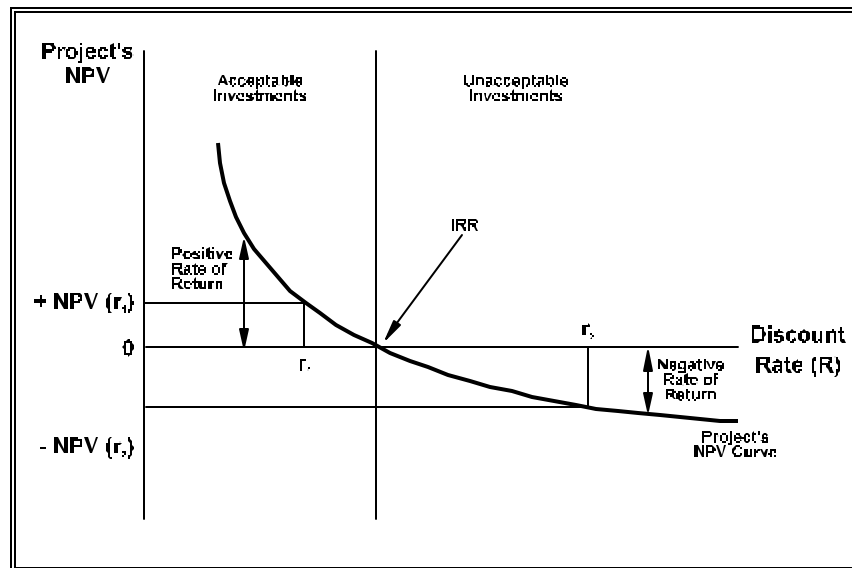


Figure 5.1: Relationship of NPV to IRR

The vertical axis represents the net present value of the project; the horizontal axis represents the discount rate (required rate of return) for the investor. The curved bold line represents the NPV of the project at different discount rates. As the discount rate increases, the NPV of the project decreases. The point where the NPV of the project equals zero (the curved line crosses the horizontal axis) is the IRR of the project. Acceptable investments have a positive NPV (above the horizontal axis) and unacceptable investments have a negative NPV (below the horizontal axis).

Likewise, an investor with a required rate of return at r_1 will invest in the project, because the IRR of the project is greater than the required return. However, an investor with a required rate of return at r_2 will not invest in the project because the IRR is less than the hurdle rate.

An investment may have a different NPV curve for each company doing the analysis, depending on each company's discount rate. This means that a project may be acceptable for one company, but unacceptable for another with a higher cost of funds.

Problems with using IRR for investment decisions

Usually, the NPV and the IRR calculations produce the same accept/reject decisions about investments. However, IRR does have some problems if the required rate of return (or cost of funds) varies from year to year. This situation requires a complicated weighted average IRR. Also, if the cash flows for the project change from positive to negative (meaning that additional investments are needed later in the project) or vice-versa, the IRR becomes complicated to use. It is conceivable that the NPV curve could cross the horizontal axis more than once producing more than one internal rate of return. For these reasons, net present value is considered superior to internal rate of return for making investment decisions.

ADJUSTING NPV AND IRR FOR INFLATION

Cash flows may change with inflation

Calculating the net present value and internal rate of return for a potential investment may require an adjustment for inflation. Actual cash inflows and outflows may change considerably as inflation changes. The variation may be large enough to change an invest/ don't invest decision, especially if the inflation rate is high. Earlier, we introduced a formula to convert a real, risk-free interest rate to a nominal interest rate, based on a specific rate of inflation.

$$R_N = R_R + h + (h \times R_R)$$

Most of the time, analysts estimate future cash flows in nominal terms and discount them with a nominal rate. The same result is achieved by estimating future cash flows in real terms and discounting them with a real discount rate. It is important to be consistent – discount nominal cash flows with a nominal rate and real cash flows with a real rate.

Another thing to consider is the effect of inflation on different types of costs and benefits. One inflation rate may be appropriate for one type of cost, but inappropriate for another type. Factor-specific inflation indices may be used to overcome this problem. We only mention this for your information; these types of considerations are beyond the scope of this workbook.

OTHER METHODS FOR MAKING INVESTMENT DECISIONS

Payback Period

Time until cash inflow equals investment

In addition to the NPV and IRR calculations, there are several other methods for gaining information that may be used when making capital budgeting decisions. One popular method is to calculate the **payback period**. The payback period is the time it takes for the cumulative cash flow of a project to equal the initial investment.

Example

We can illustrate the concept of a payback period with an example. The cash flows for a potential investment are listed in the table below:

	Initial Investment	Cash Flow	Cumulative Cash Flow
Year 0	\$25,000	0	0
Year 1		\$ 7,000	\$ 7,000
Year 2		\$ 9,000	\$16,000
Year 3		\$10,000	\$26,000
Year 4		\$13,000	\$39,000

The investor expects to recover the initial investment of \$25,000 sometime before the end of Year Three. The cumulative cash flow at the end of Year Two is \$16,000 and at the end of Year Three, \$26,000. Therefore, the payback period is more than two years but less than three years.

We calculate the payback period by dividing the difference between the initial investment and the cash flow in Year Two (\$9,000) by the cash flow in Year Three (\$10,000) and adding the result to two years.

$$2 \text{ years} + (\$9,000/\$10,000) = 2 + 0.9 = 2.9 \text{ years}$$

Hurdle time for payback period

The 2.9 years may also be expressed as 2 years and 47 weeks, by multiplying 0.9 years by 52 weeks/year. Those companies that analyze the payback period may have a hurdle time that must be met to invest in the project. For example, if the investor in the example has a payback hurdle of three years, s/he may consider investing in this project.

Problems with payback period methodology

The payback period methodology has some weaknesses. First, it does not take into account the time value of money. Cash flows that will be received in ten years are weighted the same as cash flows that will be received in one year. Second, the payback period does not consider any cash inflows that may occur after the hurdle time. A profitable investment may be rejected because the cash flows will be received after the hurdle time. Finally, it is often difficult to set an appropriate hurdle time, especially if the cash flow pattern is not well-known to the analyst.

The payback period methodology is a good beginning for analyzing a potential investment, but it is not recommended as an analyst's key tool for making investment decisions.

Profitability Index (PI)

**Cash inflow
PV / Cash
outflow PV**

Another common tool for making investment decisions is the **profitability index (PI)**. The PI is the ratio between the present value of the cash inflows and the present value of the cash outflows. The formula is:

$$PI = PV / PV_{CO}$$

Where:

PI = Profitability index of an investment

PV = Present value of the cash inflows generated by the investment

PV_{CO} = Present value of the cash outflows required by the investment

If there is only one cash outflow at the beginning of the investment (CF₀), the formula may be expressed as:

$$PI = PV / CF_0$$

Example

For example, if an investment generates discounted cash flows of \$3,500 and requires an investment of \$2,500, what is the profitability index of the investment?

$$\begin{aligned} PI &= PV / CF_0 \\ PI &= \$3,500 / \$2,500 \\ PI &= 1.40 \end{aligned}$$

**PI greater than
one is
acceptable**

An investment with a profitability index greater than one creates shareholder value and should be considered. Investments with PIs less than one should not be accepted. You can probably see the relationship between the NPV and the PI – both methods use the same information.

Often a company will analyze several possible projects and rank the projects according to their profitability indices. This information tells the analyst which investment will give the company the most return for its investment. In an ideal world, all projects with PIs greater than one would be accepted. However, limited funds and resources require most companies to choose the most profitable investment opportunities.

Can't compare projects of different size

The weakness of PI is that it cannot measure the investment value for projects of differing sizes. This concern is especially important when considering mutually exclusive projects – the acceptance of one project may not allow for the acceptance of any other projects.

Example

For example, consider a \$20 million dollar project with a profitability index of 1.3 and an \$8 million project with a PI of 1.5. If the company had only \$25 million with which to invest in projects, a decision based only on PI would indicate the company should invest in the \$8 million project. However, due to its size, the \$20 million dollar project would actually create more value for shareholders.

Practice what you have learned about calculating the internal rate of return, the payback period, and the profitability index of an investment by completing the Practice Exercise that begins on page 5-23; then continue to the "Unit Summary" and "Progress Checks."

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PRACTICE EXERCISE 5.3

Directions: Calculate the answer to each question, then enter the correct answer.
Check your solution with the Answer Key on the next page.

6. An investment generates the following cash flows:

Year 1	\$10,000
Year 2	\$12,000
Year 3	\$13,000
Year 4	\$14,000
Year 5	\$14,000

If the investment requires an initial cash flow of \$35,000 and the investor's required rate of return is 12%, what is the NPV of the project?

\$ _____

What is the profitability index of the project?

Should the investment be accepted using the profitability index method of capital budgeting?

_____ a) Yes

_____ b) No

ANSWER KEY

6. An investment generates the following cash flows:

Year 1	\$10,000
Year 2	\$12,000
Year 3	\$13,000
Year 4	\$14,000
Year 5	\$14,000

If the investment requires an initial cash flow of \$35,000 and the investor's required rate of return is 12%, what is the NPV of the project?

\$9,589.27

Enter -\$35,000 as CF_0 and the other cash flows as listed in the table.

Enter 12% as the discount rate.

Press the NPV key and you should get \$9,589.27.

What is the profitability index of the project?

1.274

Calculate the PV of the cash flows – you should get \$44,589.27.

You could also take the NPV you calculated in the last problem and add back the initial investment of \$35,000 to arrive at the present value of the future cash flows.

$$PI = PV / CF_0$$

$$PI = \$44,589.27 / 35,000$$

$$PI = 1.274$$

Should the investment be accepted using the profitability index method of capital budgeting?

Yes – *The investment should be accepted, because the PI is greater than one.*

PRACTICE EXERCISE*(Continued)*

7. What is the IRR of an investment paying \$4,000 annually for three years when the initial investment is \$10,000?

_____ %

8. Compute the payback period for the following cash flows, assuming an initial investment of \$40,000.

Year 1	\$10,000
Year 2	\$14,000
Year 3	\$18,000
Year 4	\$22,000
Year 5	\$25,000

ANSWER KEY

7. What is the IRR of an investment paying \$4,000 annually for three years when the initial investment is \$10,000?

9.7%

Enter -\$10,000 as CF_0 and three cash flows of \$4,000.

Press the IRR key and you should get 9.7%.

8. Compute the payback period for the following cash flows, assuming an initial investment of \$40,000.

2 years, 46 weeks

After 2 years: $\$10,000 + \$14,000 = \$24,000$

3rd year: $\$40,000 - \$24,000 = \$16,000$

The initial investment was \$40,000; we need to figure out how much of Year Three's cash flow is needed to reach \$40,000.

$\$16,000 / \$18,000 = 0.88888$

0.88888 of Year Three's cash flow is needed.

$0.88888 \times 52 \text{ weeks} = 46 \text{ weeks}$

2 years and 46 weeks is the payback period.

UNIT SUMMARY

In this unit we have looked at some methodologies for making capital budgeting decisions. Analysts use these techniques to evaluate and select the best investment alternatives in order to maximize shareholders' equity.

Nominal vs. effective rates

Securities that pay interest more frequently than annually have a stated interest rate that may differ from the rate that is actually earned from the investment. The stated rate is referred to as the *nominal rate*, while the actual interest earned translates into the *effective rate*. The nominal rate includes an adjustment to the *real interest rate* to account for the rate of inflation.

Net present value

The *net present value* of an investment is the sum of all the discounted cash flows minus the original investment.

$$NPV = \sum_{t=1}^T CF_t [1 / (1 + R)]^t - CF_0$$

A positive NPV means that the investor paid less than the present value for the stream of cash flows. A negative NPV means that the investor paid more than the present value.

Internal rate of return

The *internal rate of return* is the actual rate of return that is earned on an investment. The objective of this methodology is to find the discount rate (required rate of return) that will equate the present value of the cash flows with the amount of the original investment.

$$0 = \sum_{t=1}^T CF_t [1 / (1 + R)]^t - CF_0$$

Inflation

The amount of cash inflows and outflows may change as a result of inflation. Therefore, inflation should be considered when calculating the NPV and the IRR.

Payback period The payback period is the length of time it takes to recover the initial investment in a project. Some companies set a hurdle time for a project and will invest only if the payback will occur within that time frame.

Profitability index The ratio between the present value of cash inflows and the present value of cash outflows is the profitability index (PI) of a project.

$$PI = PV / PV_{CO}$$

Comparing projects according to the size of return on investment is a good indicator of where to invest – as long as all projects in the comparison are approximately the same size.

You have completed Unit Five: *Introduction to Capital Budgeting*. Please complete Progress Check 5 that begins on the next page to check your understanding of the concepts presented in this unit. If you answer any questions incorrectly, we suggest that you return to the text and review the appropriate section before continuing to Unit Six: *Corporate Valuation – Risk*.

✓ PROGRESS CHECK 5

Directions: Select the one best answer for each question. Check your responses with the Answer Key on the next page.

1. If an analyst compares the net present value of several projects, s/he will know which project:
 - _____ a) will yield the most profit.
 - _____ b) has cash flows that will exceed the initial investment by the greatest amount.
 - _____ c) will recover the initial investment by the hurdle date.
 - _____ d) will yield a return on investment equal to the hurdle rate.

2. An investor with a one-year bond is interested in the number of compounding periods in the year because:
 - _____ a) if the effective interest rate is compounded more frequently than on an annual basis, the nominal interest rate will exceed the quoted interest rate.
 - _____ b) the present value of the bond equals the face value plus the compounded quoted interest rate.
 - _____ c) the nominal rate is a floating rate that may change from period to period.
 - _____ d) the more frequently the nominal interest rate is compounded, the greater will be the effective interest rate.

3. If the internal rate of return (IRR) of a project is 35% and the hurdle rate is 42%, then:
 - _____ a) the project should be considered. The hurdle rate includes a percentage for risk, and anything below it is less risky.
 - _____ b) the profit is 7%.
 - _____ c) the project should be rejected because it will reduce the shareholder's value.
 - _____ d) we calculate the cost of funds before making an "invest / don't invest" decision.

ANSWER KEY

1. If an analyst compares the net present value of several projects, s/he will know which project:
 - b) has cash flows that will exceed the initial investment by the greatest amount.**

2. An investor with a one-year bond is interested in the number of compounding periods in the year because:
 - d) the more frequently the nominal interest rate is compounded, the greater will be the effective interest rate.**

3. If the internal rate of return (IRR) of a project is 35% and the hurdle rate is 42%, then:
 - c) the project should be rejected because it will reduce the shareholder's value.**

PROGRESS CHECK*(Continued)*

4. An investor with a payback hurdle time of three years is evaluating two potential projects. The cash flows from Project A will equal the initial investment after two years and nine months; the cash flows from Project B will equal the initial investment after three years and two months.

The internal rate of return for Project A is less than the required rate of return. Project B has a positive NPV. Ignoring other factors that may influence the decision, what is the best choice for the investor?

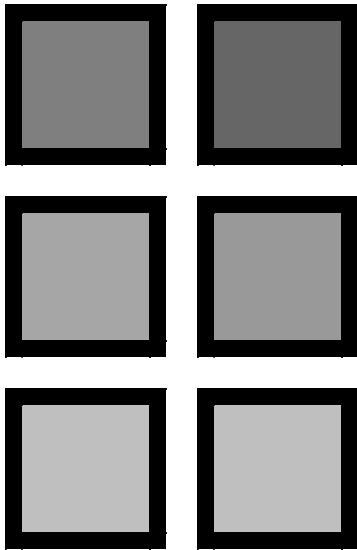
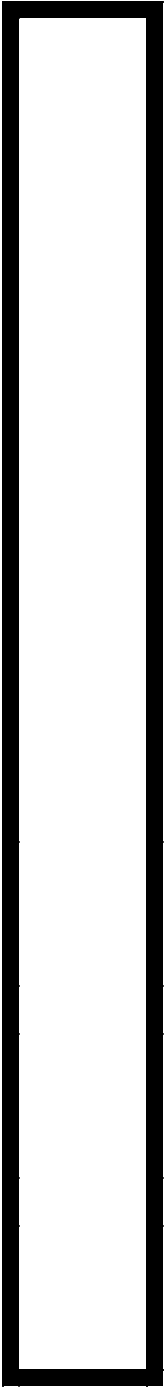
- _____ a) Project A, because the required rate of return exceeds the IRR and the payback will occur before the hurdle time.
- _____ b) Project B, because even though the payback period exceeds the hurdle time by a few months, the project has a positive NPV.
- _____ c) The wise investor would reject both Project A, because of the IRR, and Project B, because of the payback period.

ANSWER KEY

4. An investor with a payback hurdle time of three years is evaluating two potential projects. The cash flows from Project A will equal the initial investment after two years and nine months; the cash flows from Project B will equal the initial investment after three years and two months.

The internal rate of return for Project A is less than the required rate of return. Project B has a positive NPV. Ignoring other factors that may influence the decision, what is the best choice for the investor?

- b) Project B, because even though the payback period exceeds the hurdle time by a few months, the project has a positive NPV.**



Unit 6

UNIT 6: CORPORATE VALUATION – RISK

INTRODUCTION

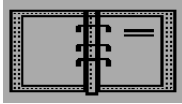
In your study of interest rates and their role in corporate finance, you focused on securities with well-defined, predictable cash flows. In the real world of finance, there are many securities or projects with uncertain returns. This uncertainty translates into an exposure to risk. In this unit we will see how risk is required for individual securities and for portfolios of securities. We will also see how to calculate the rate of return that is required by an investor for investing in a security.



UNIT OBJECTIVES

After you successfully complete this unit, you will be able to:

- Calculate the expected rate of return for an individual security and a portfolio of investments
- Determine the variance and standard deviation of possible returns on investment
- Analyze and compare payoff matrices to determine best expected rate of return or least amount of risk
- Differentiate between systematic and unsystematic risk
- Recognize the beta coefficient as a measure of volatility



CALCULATING EXPECTED CASH FLOWS

The dictionary defines risk as "a hazard; a peril; exposure to loss or injury." The concept of risk is very important in the study of corporate finance.

Investor expectations

A company that invests in a project and an investor that buys stock in a company expect to earn a return on their investments. However, there are no guarantees; it is possible that the project or investment may lose money for the company or investor. Exposure to a possible loss occurs at the time an investment is made. To compensate for this exposure to risk, an investor expects a higher possible return on investment.

In Unit Two, you learned about risk and identified it as a component of interest rates — investors require that a premium be added to the risk-free interest rate to compensate for risk. In this section, you will be introduced to the methods that are used to measure and quantify risk. Many of these concepts have been derived from statistical theory. Don't let that worry you; the concepts and applications are usually straightforward.

Expected Return

Probability distribution

The term "probability" refers to the chance that an event will occur. Suppose that you flip a coin. The probability of the coin landing tails up on one toss is 0.50, or 1/2, or 50%. (To be consistent, we will use decimal numbers (0.50) to represent probabilities throughout this discussion.) A **probability distribution** is a list of all possible outcomes and the chance of each outcome occurring.

Example

A simple example will illustrate a probability distribution. Suppose that economists predict a 30% chance for a boom economy in the coming year, a 40% chance for a normal economy, and a 30% chance for a recession. Figure 6.1 illustrates what the probability distribution for the state of the economy in the next year would look like.

(1) State of Economy	(2) Probability
Boom	0.30
Normal	0.40
Recession	0.30

Figure 6.1: Probability Distribution for the Economy

Each possible outcome for the economy is listed, along with the chance or probability of its occurrence.

Possible rates of return

Let's expand this example to include the financial data of a corporation. Suppose that in a boom economy, XYZ Corporation is expected to earn a 100% annual rate of return on investment. They think that XYZ will earn a 15% return in a normal economy and will have a negative 70% return in a recession. The probability distribution can now be revised to include the possible expected rates of return for XYZ Corporation (Figure 6.2).

(1) State of Economy Return	(2) Probability	(3) Possible
Boom	0.30	100%
Normal	0.40	15%
Recession	0.30	-70%

Figure 6.2: Possible Expected Rates of Return

Payoff matrix

The question now becomes, what is the expected rate of return for XYZ Corporation in the coming year? To calculate the expected rate of return $E(k)$, we calculate a weighted average of the possible returns that XYZ could earn. The concept of a weighted average is that each value has a different contribution to the total based on the probability of its occurrence. A value with a higher probability of occurrence weighs more in the calculation. When the data is arranged in a table, it is often referred to as a payoff matrix (Figure 6.3).

(1) State of Economy	(2) Probability	(3) Possible Return	(4) Weighted Possible Return = (2) x (3)
Boom	0.30	100%	30%
Normal	0.40	15%	6%
Recession	0.30	-70%	-21%
Expected Return $E(k)$			= 15%

Figure 6.3: Payoff Matrix for XYZ Corporation

To calculate the weighted average of the possible returns, multiply each possible return by its probability (Column 2 x Column 3). The sum of all of these products equals XYZ's expected return for the coming year. The mathematical formula to calculate the expected return $E(k)$ is:

$$\text{Expected Return} = E(k) = \sum_{i=1}^n P_i k_i$$

Where:

- $E(k)$ = Expected rate of return
- \sum = Sum of the series of all possible occurrences
- $i = 1$ = Series begins at the first possible occurrence
- n = Number of possible occurrences
- P_i = Probability of each return
- k_i = Each possible return

The expected return is the sum of these products. Using the formula, XYZ's expected annual return is:

$$E(k) = 0.30 \times (100\%) + 0.40 \times (15\%) + 0.30 \times (-70\%)$$

$$E(k) = 30\% + 6\% + -21\%$$

$$E(k) = 15\%$$

Expected Cash Flows

Although we used stock returns to illustrate the concept of expected return, we just as easily could have calculated an expected cash flow.

Projects A and B payoff matrix

Suppose, for example, that analysts want to forecast the expected cash flow for a given project. (Let's call it Project A.) They try to anticipate future conditions and their effect on the cash flow. The analysts decide that there are five possible cash flow payoffs for the project and estimate the probability of each cash flow occurring. Their analysis of the project's expected cash flow is summarized in a payoff matrix (Figure 6.4).

(1) Estimate Number	(2) Probability	(3) Possible Cash Flow	(4) Weighted Cash Flow (2) x (3)
1	0.15	\$31,000	\$4,650
2	0.15	32,000	4,800
3	0.40	33,000	13,200
4	0.15	34,000	5,100
5	0.15	35,000	5,250
		E(CF) =	\$33,000

Figure 6.4: Project A — Payoff Matrix

The analysts think that there is a 15% chance that the project will generate \$31,000 in cash flow, a 15% chance it will generate \$32,000, and so forth. The calculation is done the same way as the calculation for the expected rate of return — by multiplying each possible cash flow by the probability of its occurrence (Column 2 and Column 3). The generic formula for calculating the expected cash flow $E(\text{CF})$ of an investment or project is the same one we use for calculating an expected return.

$$\text{Expected Cash Flow} = E(\text{CF}) = \sum_{i=1}^n P_i \text{CF}_i$$

Suppose there is another project for which analysts have computed the expected cash flow. Let's call it Project B. The payoff matrix for Project B is shown in Figure 6.5.

(1)	(2)	(3)	(4)
<u>Estimate Number</u>	<u>Probability</u>	<u>Possible Cash Flow</u>	<u>Weighted Cash Flow (2) x (3)</u>
1	0.10	\$10,000	\$1,000
2	0.25	30,000	7,500
3	0.50	35,000	17,500
4	0.10	45,000	4,500
5	0.05	50,000	2,500
		E(CF) =	\$33,000

Figure 6.5: Project B — Payoff Matrix

You can see that the cash flows for the projects are different — but the expected cash flow for both projects is exactly the same.

Before we compare the merits of Project A and Project B, practice what you know about calculating the expected cash flow by completing the exercise on the next page.

PRACTICE EXERCISE 6.1

Directions: For Projects X and Y, calculate the weighted cash flows in Column 4 and the expected cash flow E(CF). Compare your calculations to the correct ones on the next page.

Project X

(1)	(2)	(3)	(4)
<u>Estimate Number</u>	<u>Probability</u>	<u>Possible Cash Flow</u>	<u>Weighted Cash Flow (2) x (3)</u>
1	0.20	\$16,000	\$ _____
2	0.50	22,000	_____
3	0.30	26,000	_____
		E(CF) =	\$ _____

Project Y

(1)	(2)	(3)	(4)
<u>Estimate Number</u>	<u>Probability</u>	<u>Possible Cash Flow</u>	<u>Weighted Cash Flow (2) x (3)</u>
1	0.20	\$15,000	\$ _____
2	0.50	22,000	_____
3	0.30	40,000	_____
		E(CF) =	\$ _____

ANSWER KEY*Project X – Payoff Matrix*

(1)	(2)	(3)	(4)
<u>Estimate Number</u>	<u>Probability</u>	<u>Possible Cash Flow</u>	<u>Weighted Cash Flow (2) x (3)</u>
1	0.20	\$16,000	\$3,200
2	0.50	22,000	11,000
3	0.30	26,000	<u>7,800</u>
		E(CF) =	\$22,000

Project Y – Payoff Matrix

(1)	(2)	(3)	(4)
<u>Estimate Number</u>	<u>Probability</u>	<u>Possible Cash Flow</u>	<u>Weighted Cash Flow (2) x (3)</u>
1	0.20	\$15,000	\$3,000
2	0.50	22,000	11,000
3	0.30	40,000	<u>12,000</u>
		E(CF) =	\$26,000

If you did not get the correct solutions, we suggest that you review the appropriate text. If you solved the problems correctly, go on to the next section on "Measuring Risk."

MEASURING RISK

The payoff matrices for Project A and Project B on pages 6-7 and 6-8 show that the expected cash flow for each project is \$33,000. If you could only invest in one project, which would you choose – Project A or Project B?

Payoff matrix analysis

Before you answer that question, let's analyze the payoff matrices for both projects. In Figures 6.6 and 6.7, you can see graphs of the possible cash flows from the payoff matrices of both projects. Along the vertical axis are the probabilities of each possible cash flow. Along the horizontal axis are the calculated amounts of each possible cash flow. By plotting each possible cash flow along the two axes, the differences between them are clearer.

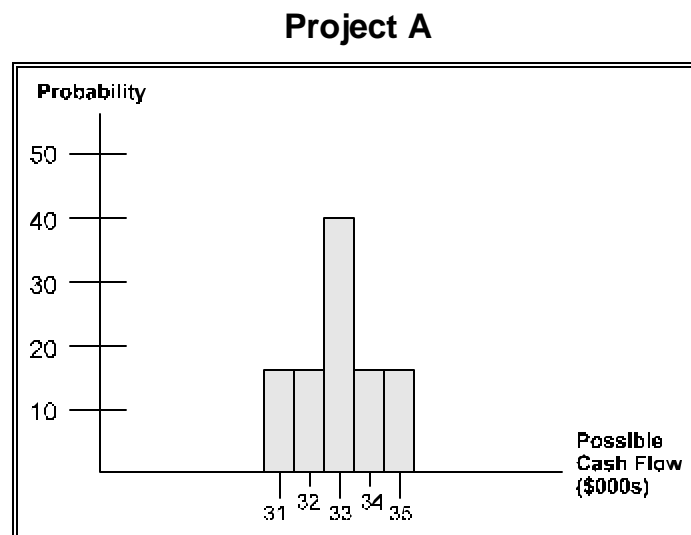


Figure 6.6: Project A – Possible Cash Flows

Project A:

- 1) The possible cash flows are concentrated into a narrow range.
- 2) In the worst case scenario, the project will generate a possible cash flow of \$31,000.
- 3) In the best case scenario, the project will generate a possible cash flow of \$35,000.
- 4) The expected cash flow is \$33,000.

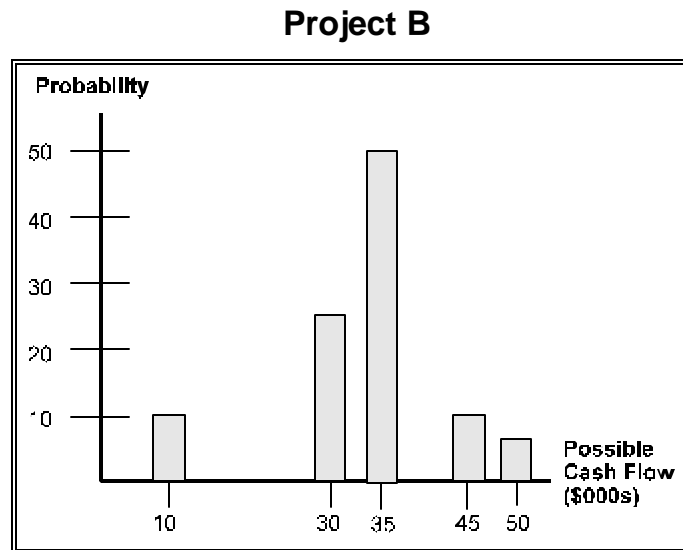


Figure 6.7: Project B – Possible Cash Flows

Project B:

- 1) The range of possible cash flows is broad.
- 2) In the worst case scenario, the project will generate a possible cash flow of \$10,000.
- 3) In the best case scenario, the project will generate a possible cash flow of \$50,000.
- 4) The expected cash flow is \$33,000.

Indicator of risk

This "spread" between possible cash flows is one indicator of risk for a potential investment. Project B has a greater spread between possible cash flows than Project A and, therefore, is considered the more risky investment.

Project A is probably a better investment because the expected cash flow is equal to that of Project B, but with much less risk.

We can quantify the risk associated with expected cash flows by using two statistical computations: **variance** and **standard deviation**.

Variance

Variance for Project A

The variance is an average of the squared deviations from the possible cash flows that have been weighted by the probability of each deviation's occurrence. The deviation is the difference between each *possible* cash flow and the *expected* cash flow. We can modify the payoff matrix to illustrate the calculation of the variance for Project A (Figure 6.8).

(1)	(2)	(3)	(4)	(5)	(6)
Estimate Number	Probability P_1	Possible Cash Flow CF_1	Deviation $CF_1 - E(CF)$	Deviation Squared $[CF_1 - E(CF)]^2$	(2) x (5) Weighted Deviation
1	0.15	\$31,000	-2,000	4,000,000	600,000
2	0.15	32,000	-1,000	1,000,000	150,000
3	0.40	33,000	0	0	0
4	0.15	34,000	1,000	1,000,000	150,000
5	0.15	35,000	2,000	4,000,000	600,000
Variance =					1,500,000

Figure 6.8: Variance for Project A

Calculation of variance

So that you can fully understand the variance calculation, let's look at each column of the Project A pay off matrix.

Columns 1, 2, and 3 = The estimate number, probability, and amount of possible cash flow – data that is taken straight from the original payoff matrix.

Column 4 = The deviation, which is computed by subtracting the expected cash flow for the project (\$33,000) from each possible cash flow (Column 3). It represents the difference between the possible cash flow and the expected cash flow.

Column 5 = Deviation squared (raised to the second power). The deviations in Column 4 are squared so that the negative deviations will not cancel out the positive deviations when a sum is calculated.

Column 6 = The probability (Column 2) multiplied by the squared deviation (Column 5). This weights each squared deviation according to the probability of its occurrence.

Variance = Sum of the weighted deviations.

Statistical formula for variance

The statistical formula used to calculate variance is shorthand for the calculations in Figure 6.8.

$$\text{Variance} = \sigma^2 = \sum_{i=1}^n [CF_i - E(CF)]^2 P_i$$

Where:

- σ^2 = Variance
- \sum = Sum of the series of all possible occurrences
- $i = 1$ = Series begins at the first possible occurrence
- n = Number of possible occurrences
- $[CF_i - E(CF)]^2$ = Each possible cash flow minus the expected cash flow, squared
- P_i = Probability of occurrence

Let's summarize the steps for calculating the variance:

- 1) Calculate the expected cash flow.
- 2) Subtract the expected cash flow from each possible cash flow to find the deviations.
- 3) Square each of these deviations.
- 4) Multiply the squared deviations by their respective probabilities (to weight them).
- 5) Total the weighted deviations to arrive at the variance.

Variance for Project B

The variance calculation for Project B is shown in Figure 6.9.

(1)	(2)	(3)	(4)	(5)	(6)
Estimate Number	Probability P_1	Possible Cash Flow CF_1	Deviation $CF_1 - E(CF)$	Deviation Squared $[CF_1 - E(CF)]^2$	(2) x (5) Weighted Deviation
1	0.10	\$10,000	-23,000	529,000,000	52,900,000
2	0.25	30,000	-3,000	9,000,000	2,250,000
3	0.50	35,000	2,000	4,000,000	2,000,000
4	0.10	45,000	12,000	144,000,000	14,400,000
5	0.05	50,000	17,000	289,000,000	14,450,000
Variance =					86,000,000

Figure 6.9: Variance for Project B

Greater variance indicates riskier project

Now we can compare the variance for Project A (1,500,000) with the variance for Project B (86,000,000). As you can see, the variance of Project B's possible cash flows is greater than the variance of Project A's possible cash flows. Therefore, we consider Project B to be riskier than Project A.

Variance of possible returns

Let's return to the XYZ Corporation example (page 6-5) to demonstrate how to calculate the variance of a set of possible returns. Remember that the payoff matrix of XYZ Corporation looked like this:

(1)	(2)	(3)	(4)
State of Economy	Probability	Possible Return	Weighted Possible Return = (2) x (3)
Boom	0.30	100%	30%
Normal	0.40	15%	6%
Recession	0.30	-70%	-21%
Expected Return E(k) =			15%

Let's continue to build the table to demonstrate how to calculate the variance of XYZ Corporation. In Figure 6.10, the calculations for Columns 4, 5, and 6 have been added to the first three columns of the payoff matrix. The deviation is the difference between the possible return (k_1) and the expected return $[E(k)]$.

(1)	(2)	(3)	(4)	(5)	(6)
State of Economy	Probability	Possible Return	Deviation $k_1 - E(k)$	Deviation Squared $[CF_1 - E(CF)]^2$	(2) x (5) Weighted Deviation
Boom	0.30	100%	85	7,225	2,167.5
Normal	0.40	15%	0	0	0.0
Recession	0.30	-70%	-85	7,225	2,167.5
Variance =					4,335.0

Figure 6.10: Variance Calculation for XYZ Corporation

The variance of the possible stock returns for the XYZ Corporation is 4,335.0. You can see that the variance method used for calculating possible returns is identical to the variance used for possible cash flows.

Standard Deviation

Because variance is expressed in terms of squared units, it is difficult to interpret. Analysts often will convert variance to an expression of standard deviation as a more convenient measure of risk.

Square root of the variance

Standard deviation is the square root of the variance. The units of measure for standard deviation are the same as the units of measure for the data that is being analyzed (e.g. dollars, percentages). The mathematical formula for the standard deviation of expected cash flows is:

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^n [CF_i - E(CF)]^2 P_i}$$

Where:

♦	=	The square root of the variance, or the variance raised to the ½ power
♠	=	Sum of the series of all possible occurrences
n	=	Number of possible occurrences
i = 1	=	Series begins at the first possible occurrence
$[CF_i - E(CF)]^2 P_i$	=	Each possible cash flow minus the expected cash flow, squared, and then multiplied by each probability of occurrence
P_i	=	Probability of occurrence

To calculate the standard deviation, first find the variance and then take the square root of the variance. We interpret the standard deviation by thinking in terms of the average deviation from the expected cash flow (or return).

Standard deviations of Project A and B cash flows

Let's compare the standard deviations of Projects A and B. Notice that taking the square root of a number and raising that number to the ½ power are identical calculations.

Project A

- Variance = 1,500,000
- Standard deviation = $(1,500,000)^{1/2}$ or \$1,224.74

Project B

- Variance = 86,000,000
- Standard deviation = $(86,000,000)^{1/2}$ or \$9,273.61

As you can see, the standard deviation of Project A's possible cash flows is considerably less than the standard deviation of Project B's. In other words, Project A's possible cash flows, on average, deviate less from the expected cash flow.

Lower standard deviation means less volatility

Project A has the same expected cash flow as Project B, but with considerably less volatility (or spread) among the possible cash flows. Project A is considered less risky than Project B and, therefore, the better investment.

Standard deviation of expected returns

The method for calculating the standard deviation of a set of possible returns is to take the square root of the variance of that set of returns. For example, the standard deviation for the possible stock returns of XYZ Corporation would be $\sigma = (4,335)^{1/2} = 65.84\%$.

The formula for the standard deviation of expected returns is:

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^n [k_i - E(k)]^2 P_i}$$

Summary

In this section, we discussed the use of the statistical measures of variance and standard deviation to measure the risk associated with a project or investment. The variance is the sum of the squared deviations from the possible cash flows or expected returns that have been weighted by the probability of each deviation's occurrence. Standard deviation (the square root of the variance) is preferred by analysts because it is easier to use as a measure of risk. A lower standard deviation indicates less volatility among possible cash flows or returns and, therefore, a lower amount of risk.

Before continuing to the section on "Portfolio Risk," please complete the Practice Exercise which follows.

PRACTICE EXERCISE 6.2

Directions: For Project X and Project Y, use the information provided in the payoff matrices to calculate the variation and the standard deviation. After you have completed every exercise on this page, compare your answers to the correct ones on the next page.

1. For Project X, the expected cash flow is \$22,000.

(1)	(2)	(3)	(4)	(5)	(6)
Estimate Number	Probability P_1	Possible Cash Flow CF_1	Deviation $CF_1 - E(CF)$	Deviation Squared $[CF_1 - E(CF)]^2$	(2) x (5) Weighted Deviation
1	0.20	\$16,000	_____	_____	_____
2	0.50	22,000	_____	_____	_____
3	0.30	26,000	_____	_____	_____
				Variance =	_____

For **Project X** the standard deviation is _____

2. For Project Y, the expected cash flow is \$26,000.

(1)	(2)	(3)	(4)	(5)	(6)
Estimate Number	Probability P_1	Possible Cash Flow CF_1	Deviation $CF_1 - E(CF)$	Deviation Squared $[CF_1 - E(CF)]^2$	(2) x (5) Weighted Deviation
1	0.20	\$15,000	_____	_____	_____
2	0.50	22,000	_____	_____	_____
3	0.30	40,000	_____	_____	_____
				Variance =	_____

For **Project Y** the standard deviation is _____

ANSWER KEY

1. For Project X, the expected cash flow is \$22,000.

(1)	(2)	(3)	(4)	(5)	(6)
Estimate Number	Probability P_1	Possible Cash Flow CF_1	Deviation $CF_1 - E(CF)$	Deviation Squared $[CF_1 - E(CF)]^2$	(2) x (5) Weighted Deviation
1	0.20	\$16,000	-6,000	36,000,000	7,200,000
2	0.50	22,000	0	0	0
3	0.30	26,000	4,000	16,000,000	4,800,000
				Variance =	12,000,000

For **Project X** the standard deviation = (variance)^{1/2}
 = (12,000,000)^{1/2}
 = **3,464.10**

2. For Project Y, the expected cash flow is \$26,000.

(1)	(2)	(3)	(4)	(5)	(6)
Estimate Number	Probability P_1	Possible Cash Flow CF_1	Deviation $CF_1 - E(CF)$	Deviation Squared $[CF_1 - E(CF)]^2$	(2) x (5) Weighted Deviation
1	0.20	\$15,000	-11,000	121,000,000	24,200,000
2	0.50	22,000	-4,000	16,000,000	8,000,000
3	0.30	40,000	14,000	196,000,000	58,800,000
				Variance =	91,000,000

For **Project Y** the standard deviation = (variance)^{1/2}
 = (91,000,000)^{1/2}
 = **9,539.4**

PORTFOLIO RISK

Up to now, the discussion has focused on the methods used to identify and measure risk associated with individual investment securities. Large investors such as mutual funds, pension funds, insurance companies, and other financial institutions own stocks of many different companies — government regulations require that they do so. Individual investors also purchase the stock of several different firms rather than invest with one company. In this section, we explain the reasons why investors prefer to vary their stock holdings.

Portfolio Diversification

The collection of investment securities held by an investor is referred to as a portfolio. **Diversification** is the strategy of investing with several different industries. If an investor maintains a portfolio that holds Ford Motor Company, Exxon, and IBM stocks, that investor is said to have a **portfolio diversification**.

Focus on expected return of a portfolio

An investor with a diversified portfolio is not overly concerned when an individual security in the portfolio moves up or down. The main concern is the riskiness and overall return of the portfolio. Therefore, the risk and return of each security needs to be analyzed in terms of how that security affects the risk and return of the entire portfolio.

The method for calculating the expected rate of return of a portfolio of stocks can be illustrated with an example.

Example

An investor has invested \$10,000 in a portfolio of stocks. The amount invested and the expected rate of return for each stock in that portfolio is as follows:

- \$2,000 invested in General Motors, which is expected to earn a 14% rate of return

- \$5,000 invested in Ford Motor Company, which is expected to generate a 17% return
- \$3,000 invested in Chrysler Motors, which is expected to earn an 18% return

Now, let's arrange the portfolio in a payoff matrix.

Stock	Amount	(W_i) Weight	(k_i) Expected Return	($W_i \times k_i$) Weighted Return
GM	\$2,000	0.20	14%	2.80
Ford	5,000	0.50	17%	8.50
Chrysler	3,000	0.30	18%	5.40
Expected Return $E(k_p) =$				16.70%

Figure 6.11: Payoff Matrix for Stock Portfolio

The expected return of the portfolio is calculated using almost the same method as we use to calculate the expected return for a range of possible returns of an individual stock. The only difference is that the weights in a portfolio are derived from the percentage of an individual investment relative to the total amount invested in the portfolio. In this example, the total portfolio investment is \$10,000; the investment in General Motors stock is \$2,000 or 20% of the portfolio. Therefore, the **portfolio weight** of GM stock in the payoff matrix is .20. *The weights always total 1.0.*

Calculating risk for portfolio of stocks

Unlike the portfolio's expected return, the riskiness of the portfolio is not a weighted average of the standard deviations of the individual stocks in the portfolio. The portfolio's risk will be smaller than the weighted average of the standard deviations because the variations in the individual stocks will be offsetting to some degree.

Example

Let's consider an example to illustrate the concept of offsetting variations. The rates of return over a five year period for two stocks (W and M) are listed in Figure 6.12.

	Stock W k_W (%)	Stock M k_M (%)
1988	40	-10
1989	-10	40
1990	35	-5
1991	-5	35
1992	<u>15</u>	<u>15</u>
Average Return	15	15
Standard Deviation	22.6	22.6

Figure 6.12: Rates of Return for Stocks W and M over Five Years

In Figure 6.13 you can see the rates of return for both stocks plotted on graphs. Notice the patterns of rate variation.

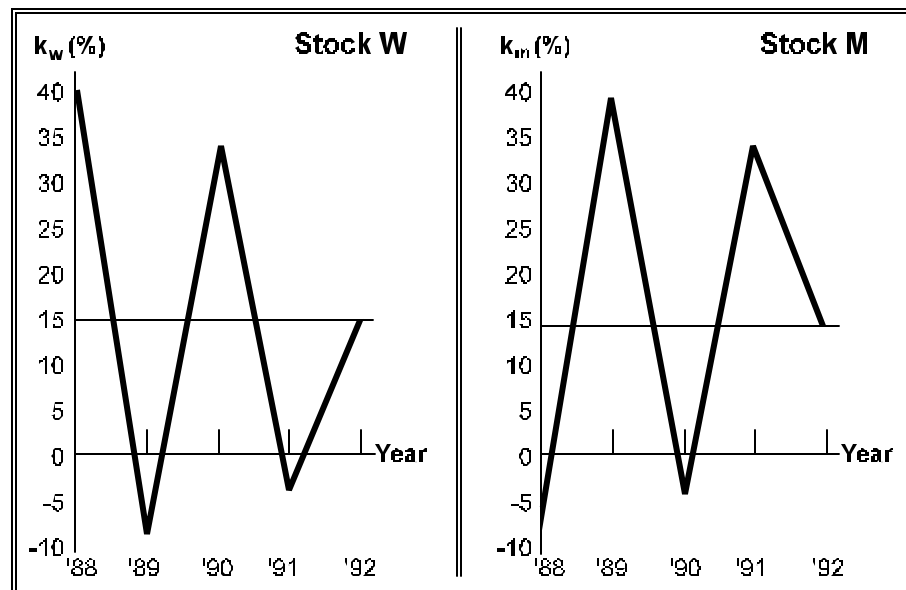


Figure 6.13: Rates of Return for Stock W and Stock M

Offsetting variations

If held individually, both stock W and stock M would have a high level of risk. As you can see from the graphs, a rise in the rate of return in stock W is offset by an equal fall in the rate of return of stock M. Imagine the profile of a portfolio where each of these stocks represents 50% of the total investment.

The rate of return distribution for portfolio WM is shown in Figure 6.14.

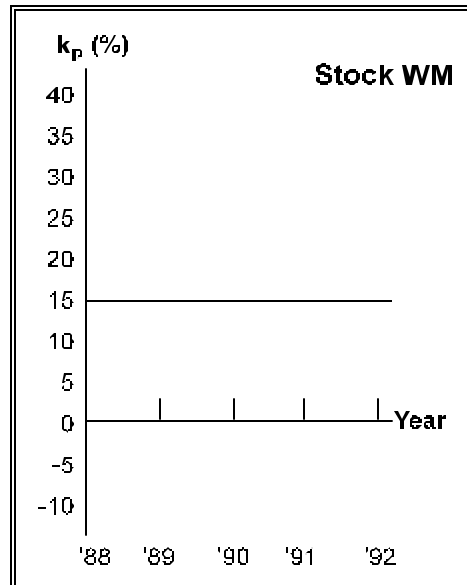


Figure 6.14: Rate of Return Distribution for Portfolio WM

Risk-free portfolio

Portfolio WM shows a 15% rate of return for each year, with no risk at all. The reason that stock W and stock M can be combined to form a risk-free portfolio is that their returns move in opposition to each other.

Correlation coefficient

In statistics, the tendency of two variables to move together is called correlation. The **correlation coefficient (r)** is the measure of this tendency. To learn how to calculate the correlation coefficient, consult any beginning statistics text. All of the examples in this workbook will have the correlation coefficient already calculated. You should become familiar with what the correlation coefficient represents and how it is used.

Value of r

The correlation coefficient (r) ranges between +1.0 to -1.0. Let's see what the values represent.

- If **r = +1.0** (perfectly positively correlated), the two variables move up and down in perfect synchronization.
- If **r = -1.0** (perfectly negatively correlated), the two variables always move in exact opposite directions.

- If r falls between $+1.0$ and -1.0 , correlation occurs in varying degrees.
- If $r = 0.0$, the two variables are not related to each other – changes in one variable are independent of changes in the other.

In the example, stock W and stock M have a correlation coefficient of -1.0 (perfectly negatively correlated). Diversification with these two stocks creates a risk-free portfolio.

If two stocks in a portfolio are perfectly positively correlated ($r = +1.0$), and if the standard deviations for the stocks are not equal, diversification does not eliminate any risk. The portfolio of the two stocks will have the same risk as holding either stock on its own.

Impossible to eliminate all risk

Portfolios with two perfectly negatively correlated stocks ($r = -1.0$) or two perfectly positively correlated stocks ($r = +1.0$) are extremely rare. In fact, most pairs of stocks are positively correlated, but not perfectly. On average, the correlation coefficient for the returns of two randomly selected stocks would be about $+0.6$. For most pairs of stocks, the correlation coefficient will be between $+0.5$ and $+0.7$. For this reason, it is impossible to eliminate all risk by combining stocks into a portfolio.

Example

We used portfolio WM to illustrate a point. Let's look at a more realistic example. Consider stock W and stock Y. The rates of return for the two stocks and for a portfolio equally invested in the two stocks are listed in Figure 6.15. The standard deviations for each set of returns, and the portfolio as a whole, have already been calculated. The correlation coefficient is $r = +0.65$.

	Stock W k_w (%)	Stock Y k_y (%)	Portfolio WY k_p (%)
1988	40	28	34
1989	-10	20	5
1990	35	41	38
1991	-5	-17	-11
1992	<u>15</u>	<u>3</u>	<u>9</u>
Average Return	15	15	15
Standard Deviation	22.6	22.6	20.6

Figure 6.15: Return Rates and Standard Deviation for Stocks W and Y and Portfolio WY

You can see that stock W, stock Y, and portfolio WY each have an average return of 15%. The average return on the portfolio is the same as the average return on either stock. The standard deviation is 22.6 for each stock and 20.6 for the portfolio. Even though the two stocks are positively correlated (+0.65), the risk is reduced by combining the stocks into a portfolio. A stock analyst might say that combining the two stocks "diversifies away some of the risk."

Diversification eliminates some risk

This example illustrates that the risk of the portfolio as measured by the standard deviation is *not* the average of the risk of individual stocks in the portfolio. Typically, if the correlations among individual stocks are positive, but less than +1.0, some of the risk may be eliminated.

Positive correlation between same-industry companies

It is important to recognize that companies in the same industry generally have a more positive correlation coefficient for expected returns than companies in different industries. For example, you would expect the correlation between the returns of Ford Motor Company and the returns of General Motors to be more positive than the correlation between the returns of Ford Motor Company and IBM. Thus, a two-stock portfolio of companies within the same industry is probably riskier than a two-stock portfolio of companies operating in different industries. Portfolios should be diversified between industries as well as companies to eliminate the maximum amount of risk.

Systematic and Unsystematic Risk

Expected vs. uncertain return

To evaluate the risk associated with an individual security or a portfolio of securities, the realized return to investors must be broken into two parts:

- 1) Expected return
- 2) Uncertain return

Expected return based on current market factors

For a stock (equity security), the expected return is the part that investors and analysts can predict. These predictions are based on information about current market factors that will influence the stock in the future. We discussed this process in Unit Two in our discussion on interest rates. By studying external and internal factors that affect a company's operations, analysts can assess how each factor may affect the company. The stock price of a company is determined by expectations of a company's future prospects based on the influence of all factors.

Uncertain return influenced by unexpected factors

The other part of the return of a stock is the uncertain or risky part that comes from unexpected factors arising during the period. For example, an unexpected research breakthrough may cause a company's stock to rise; a sudden, unexpected rise in inflation rates may cause a company's stock to fall.

Company specific, diversifiable risk

These two parts of the realized return translate into two types of risk: (1) risk specific to a single company or a small group of companies, and (2) risk that affects all companies in the market. In the stock W and stock Y example, when we created a diversified portfolio to eliminate some of the risk associated with each stock, we eliminated the **company-specific risk**. This type of company-specific, diversifiable risk is called **unsystematic risk**.

Creating a diversified portfolio of stocks enables an investor to eliminate nearly all unsystematic risk. The unexpected events that cause unsystematic risk include things like strikes, lawsuits, winning or losing major contracts, etc. Since these events that make up company-specific risk are essentially random occurrences, diversification allows us to eliminate them as risk factors. In other words, on average, the positive events of all the companies in the portfolio tend to cancel out the negative events.

Risk that affects the entire market

The other type of risk is called **systematic risk (market risk or undiversifiable risk)**. Systematic risk cannot be eliminated by diversification because the unexpected events that create systematic risk affect all companies in the market. Such events may include war, inflation, recession, etc.

The risk of an individual security is not the primary concern of the investor. The relevant riskiness of an individual security is its contribution to the riskiness of a well-diversified portfolio. For example, the riskiness of Ford Motor stock is the contribution that the Ford stock makes to the overall riskiness of the portfolio in which it is held.

Reduction of unsystematic risk

The graph in Figure 6.16 illustrates how the addition of securities to the portfolio eliminates the unsystematic risk, while the systematic risk remains constant. The standard deviation for the expected return of the portfolio is on the vertical axis. The number of securities held in the portfolio is on the horizontal axis.

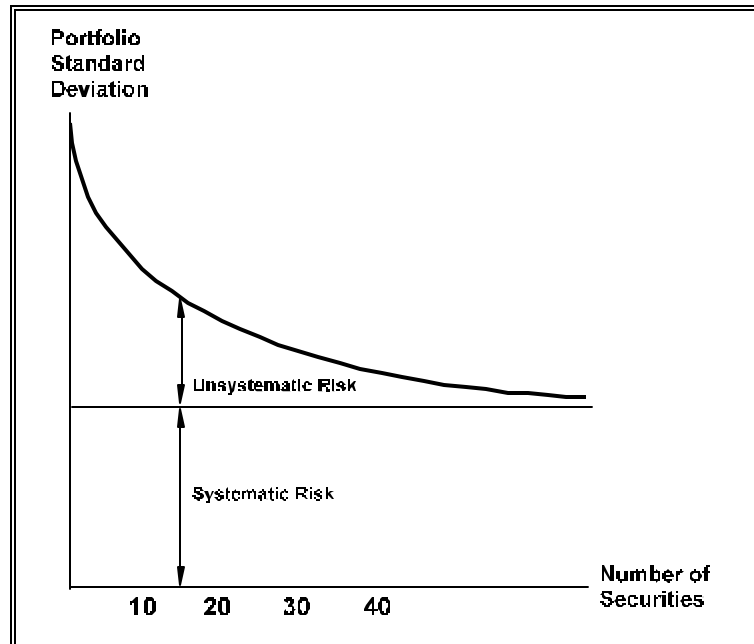


Figure 6.16: Reduction of Unsystematic Risk

You can see that as the number of securities increases, the amount of unsystematic risk declines. Many studies have been completed to determine the optimal size of a portfolio. Analysts say that over 95% of the unsystematic risk is eliminated in a portfolio of about 40 securities.

Market index portfolio

A portfolio that contains all of the stocks in the market, based on weight within the market, is often called the market portfolio. The New York Stock Exchange (NYSE) lists over 1,500 different stocks. The standard deviation for a market index portfolio containing all of the NYSE stocks historically has been about 15.1% ($\sigma_M = 15.1\%$). This represents the amount of risk that can not be eliminated by diversification and corresponds to the risk premium that is added to the risk-free level of interest rates discussed in Unit Two.

Summary

An investor with a diversified portfolio has invested with different companies in several industries to reduce unsystematic (company-specific) risk.

Events that affect the entire market, such as war, inflation, and recession, contribute to the systematic or undiversifiable risk of a portfolio.

The relevant risk of holding a security is the individual security's contribution to the overall risk of the portfolio. Different securities will affect the same portfolio differently. The same security will have different contributions to different portfolios. Investors will not be rewarded for assuming risk that could have been eliminated through diversification. The type of risk that remains after diversification is systematic (market) risk.

Please check your understanding of "systematic" and "unsystematic risk" in Practice Exercise 6.3. In the next section you will see how to measure systematic risk.

PRACTICE EXERCISE 6.3

Directions: Mark the correct answer to each question. Compare your solutions to the correct answers in the Answer Key that follows on the next page.

3. Portfolio diversification is an important strategy for investors because it:

- a) reduces expected returns.
- b) increases expected returns.
- c) reduces systematic risk.
- d) reduces unsystematic risk.

4. The type of risk that investors are compensated for is:

- a) total risk.
- b) unsystematic risk.
- c) systematic risk.
- d) normal or expected return.

Use the following portfolio information for the next question.

Security	Amount Invested	Weight	Expected Return
Stock A	\$5,000	0.25	9%
Stock B	\$5,000	0.25	10%
Stock C	\$6,000	0.30	11%
Stock D	\$4,000	0.20	12%

5. What is the expected return on the portfolio?

- a) 10.00%
- b) 10.25%
- c) 10.45%
- d) 10.50%

ANSWER KEY

3. Portfolio diversification is an important strategy for investors because it:

d) reduces unsystematic risk.

Portfolio diversification reduces unsystematic risk. It does not reduce market risk (systematic risk).

4. The type of risk that investors are compensated for is:

c) systematic risk.

Investors can expect to be rewarded for assuming systematic risk since they can diversify their portfolio to minimize or eliminate unsystematic risk.

5. What is the expected return on the portfolio?

c) 10.45%

Security	Amount Invested	Weight	Expected Return	Weight x Expected Return
Stock A	\$5,000	0.25	9%	2.25%
Stock B	\$5,000	0.25	10%	2.50%
Stock C	\$6,000	0.30	11%	3.30%
Stock D	\$4,000	0.20	12%	2.40%
			Expected Return	10.45%

Beta

Standard deviation is used to measure the risk of an individual stock — the type of risk that can be reduced by diversification. What is needed is a method for measuring an individual stock's contribution to the systematic risk of a portfolio.

Measure of stock's volatility

The **beta** coefficient (δ) is the measure of a stock's volatility relative to that of an average stock. In other words, it reflects the degree to which the price of a stock tends to move with movements in the market.

An average stock is defined as a stock that tends to move up and down in step with the market as a whole. The market is often measured by using an index such as the New York Stock Exchange (NYSE), the Standard and Poor's 500 (S&P 500), or the Dow Jones Industrial average. These indices serve as good representations of the entire United States stock market. For example, the S&P 500 is a weighted-average portfolio of the stocks of 500 of the largest and most stable U.S. companies. It is often selected to represent the movements of the entire U.S. stock market. Analysts use these indices to measure the entire market because information for the companies in an index is easily obtained, making analysis quicker and less expensive.

Covariability of stock with the market

The beta coefficient measures the covariability (relative movement) of a stock as compared to a **benchmark** which, in this case, is the market portfolio. An average-risk stock has a beta coefficient of 1.0. This means that if the market moves up by 10%, the stock will, on average, move up by 10%. Likewise, if the market moves down by 10%, the stock will, on average, fall 10%. Less than average-risk stocks have betas between 0.0 and 1.0, whereas higher than average-risk stocks have betas greater than 1.0.

Example

You can see the covariability of three stocks by looking at the graph in Figure 6.17. The vertical axis represents the return on each individual stock, and the horizontal axis represents the return on the market. Plotted on the graph are the returns for three stocks for the years 1990, 1991, and 1992: H (for high risk), A (for average risk), and L (for low risk).

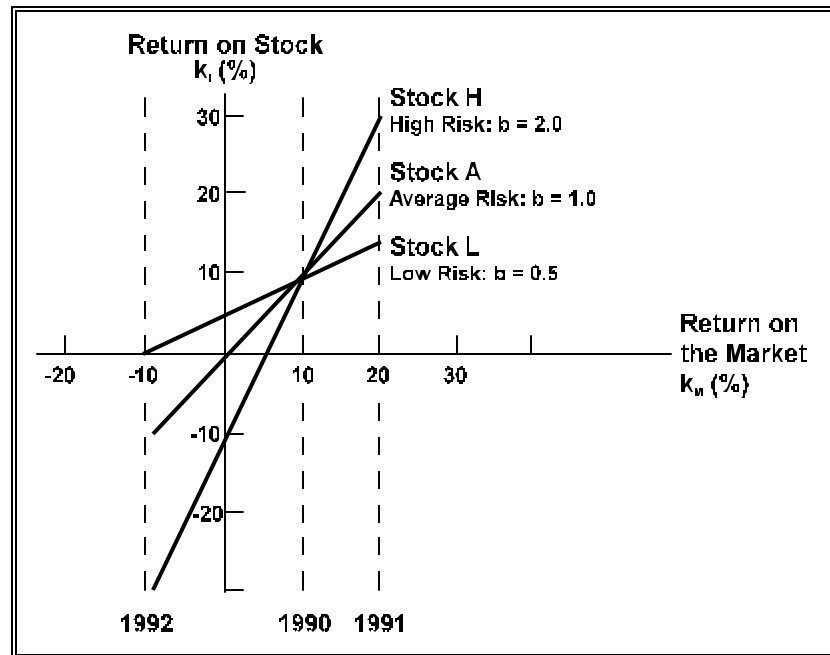


Figure 6.17: Covariability of Stocks H, A, and L with the Market

In 1990, all three stocks had a 10% return — the same as the market's return. In 1991, the market return rose to 20%. Stock A also earned 20%, Stock H soared to a 30% return, and Stock L earned a 15% return. In 1992, the market fell to a -10% return. Stock A also fell to -10%, while Stock H fell to -30% and Stock L earned a 0% return.

Stock H was twice as volatile as the market and, therefore, it carried twice as much risk. Stock H's beta coefficient of 2.0 represents the risk. Stock A moved the same as the market moved and has a beta coefficient of 1.0. The low-risk security, Stock L, moved half as much as the market and, therefore, has a beta of 0.5.

Beta is equal to the slope

The slopes of the lines show how much the return of each stock moves in response to movement in the return on the market as a whole. A security's beta is equal to the slope on this type of graph. There are several different ratings companies that calculate and publish the beta coefficients for thousands of companies.

If a company with a beta coefficient greater than 1.0 is added to an average-risk portfolio, the overall risk of the portfolio will increase. On the other hand, adding a stock with a beta coefficient less than 1.0 will reduce the overall risk of an average-risk portfolio.

Example

Let's analyze a simple example to organize what we have learned. Suppose we have the following information for stock X and stock Y.

	<u>Standard Deviation</u>	<u>Beta</u>
Stock X	40%	0.50
Stock Y	20%	1.50

We know the following about stock X and stock Y:

- 1) Beta represents systematic risk. Stock X has less systematic risk than stock Y.
- 2) Standard deviation represents total risk. Stock X has more total risk than stock Y.
- 3) Total risk equals systematic plus unsystematic risk. Stock X must have more unsystematic risk than stock Y.
- 4) Investors earn a return on systematic risk. Stock Y will have a higher expected return and a higher risk premium even though its total risk is less than the total risk of stock X.

Portfolio beta calculation

The beta of a portfolio can be calculated if the information about the beta of each stock in the portfolio and the amount invested in each stock is known. The formula is a weighted average of the individual stock beta coefficients.

$$\rho_P = \sum_{i=1}^m W_i \rho_i$$

Where:

- ρ_P = Beta coefficient of the portfolio
- m = Number of securities in the portfolio
- W_i = Relative weight invested in each security
- ρ_i = Beta coefficient of security i
- i = Each specific security

Example

To illustrate the use of the formula, suppose that a portfolio contains the following stocks.

Security	Amount Invested	Beta Coefficient
Stock A	\$1,000	0.80
Stock B	\$2,000	0.95
Stock C	\$3,000	1.10
Stock D	\$4,000	1.40

Sum of weighted beta coefficients

To calculate the portfolio beta coefficient, multiply each stock's weight in the portfolio by its beta and add up the products. Two columns have been added to the table: "Weight" — the relative weight in the portfolio of each stock and "Weight x Beta" — the weighted beta coefficients. The beta coefficient of the portfolio is the sum of these products.

Security	Amount Invested	Weight	Beta Coefficient	Weight x Beta
Stock A	\$1,000	0.10	0.80	0.08
Stock B	\$2,000	0.20	0.95	0.19
Stock C	\$3,000	0.30	1.10	0.33
Stock D	\$4,000	0.40	1.40	0.56
Portfolio Beta ρ_P				= 1.16

Remember, the relative weight of a security is its percentage of the total investment. These weights must add to 1.0.

For this example, the beta coefficient of the portfolio is 1.16. Because the beta is larger than 1.0, this portfolio has greater systematic risk than the market portfolio.

As mentioned earlier, investors are rewarded for bearing systematic risk. The next step will be to calculate the amount of the reward.

Summary

Beta is a measure of the systematic risk for a security or a portfolio relative to a market portfolio. Beta does not capture unsystematic, diversifiable risk. The beta coefficient measures the covariability of a security's or portfolio's volatility with the volatility of the market as a whole.

Please check your understanding of "beta" in the Practice Exercise which follows. You will then be ready to continue to the final section of this unit.

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PRACTICE EXERCISE 6.4

Directions: Mark the correct answer to each question. Compare your solutions to the correct answers in the Answer Key that follow on the next page.

6. Beta is a measure of:

- _____ a) the total risk for a security or a portfolio.
- _____ b) the systematic risk for a security or a portfolio relative to a market portfolio.
- _____ c) expected or normal returns for a security or a portfolio.
- _____ d) unique or asset-specific risks for a security or a portfolio.

Use the following portfolio information for the next question.

Security	Amount Invested	Weight	Expected Return	Beta Coefficient
Stock A	\$5,000	0.25	9%	0.80
Stock B	\$5,000	0.25	10%	1.00
Stock C	\$6,000	0.30	11%	1.20
Stock D	\$4,000	0.20	12%	1.40

7. What is the portfolio beta?

- _____ a) 0.90
- _____ b) 0.95
- _____ c) 1.09
- _____ d) 1.10

ANSWER KEY

6. Beta is a measure of:

b) the systematic risk for a security or a portfolio relative to a market portfolio.

7. What is the portfolio beta?

c) 1.09

<u>Security</u>	<u>Amount Invested</u>	<u>Weight</u>	<u>Expected Return</u>	<u>Beta Coefficient</u>	<u>Weight x Beta Coefficient</u>
Stock A	\$5,000	0.25	9%	0.80	0.20
Stock B	\$5,000	0.25	10%	1.00	0.25
Stock C	\$6,000	0.30	11%	1.20	0.36
Stock D	\$4,000	0.20	12%	1.40	0.28
				Portfolio Beta Coefficient =	1.09

SYMBOLS AND DEFINITIONS

Let's take a few moments to review some symbols and definitions and to introduce some new ones. You will have a better understanding of their meaning and use as you complete the final section of this unit, "Risk vs. Return."

SYMBOL	DEFINITION
$E(k_i)$	Expected rate of return on stock i
k_i	Required rate of return on stock i Note: If $E(k_i)$ is less than k_i , you would not purchase this stock; or you would sell it if you owned it. For the marginal (or "representative") investor in the market, $E(k_i)$ must equal k_i , otherwise a "disequilibrium" exists.
k_{RF}	Risk-free rate of return, generally estimated by using the rate of return on U.S. Treasury securities as a proxy
β_i	Beta coefficient of stock i
k_M	Required rate of return on a portfolio consisting of all stocks (market portfolio). It is also the required rate of return on an average ($\delta \mathcal{L} = 1.0$) stock.
$RP_m = (k_M - k_{RF})$	Market risk premium. This is the additional return over the risk-free rate required to compensate investors for assuming an "average" amount of risk. Average risk means $\delta \mathcal{L} = 1.0$. Historically, this premium has averaged 7.4%. (In other words, stocks have averaged a rate of return that is 7.4% higher than long-term U.S. Treasury securities.)
$RP_i = (k_m - k_{RF})\beta_i$	Risk premium on stock i . The stock's risk premium is less than, equal to, or greater than the premium on an average stock depending on whether its beta is less than, equal to, or greater than 1.0. If $\beta_i = 1.0$, then $RP_i = RP_m$.

RISK VS. RETURN

Required Rate of Return

Market risk premium

The market risk premium (RP_m) depends on the degree of aversion that all investors have to risk. In other words, the size of the market risk premium depends on the amount of compensation that investors in the market require for assuming risk. To entice investors to take on additional risk in their portfolio, a high degree of aversion to risk will be matched by a large market risk premium.

Calculation for required rate of return

Refer to the table of symbols and definitions on Page 6-41, as needed, to clarify the following explanation of how to calculate the required rate of return for a stock.

Example

If U.S. Treasury bonds earn 3% and the market portfolio earns 10%, then the market risk premium will be:

$$RP_m = 10\% - 3\% = 7\% \text{ (or } .07)$$

To calculate the risk premium for an individual stock (RP_i), we multiply the market risk premium (RP_m) by the beta coefficient for that stock (ρ_i). In this example, the beta coefficient = 0.8. Therefore, the risk premium for that stock will be:

$$\begin{aligned} (RP_m)\rho_i &= RP_i \\ .07 \times 0.8 &= .056 \end{aligned}$$

The required rate of return investors expect for that stock is:

$$\begin{aligned} k_i &= k_{RF} + (k_m - k_{RF})\rho_i && .03 + (.10 - .03)0.8 \\ k_i &= k_{RF} + (RP_m)\rho_i && .03 + .056 = .086 \text{ (or } 8.6\%) \end{aligned}$$

Investors will expect to earn an 8.6% rate of return on their investment in this stock.

An average risk stock ($\beta = 1.0$) will have a required rate of return of 10%, which is the same as the market's rate of return. A high-risk stock ($\beta > 1.0$) will have a required rate of return higher than 10%. Let's look at a graphic illustration of the relationship between risk and return.

Security market line displays risk vs. return

A **security market line (SML)** is used to graphically display the relationship between risk and return generated by a security. The following figure is an example of a security market line.

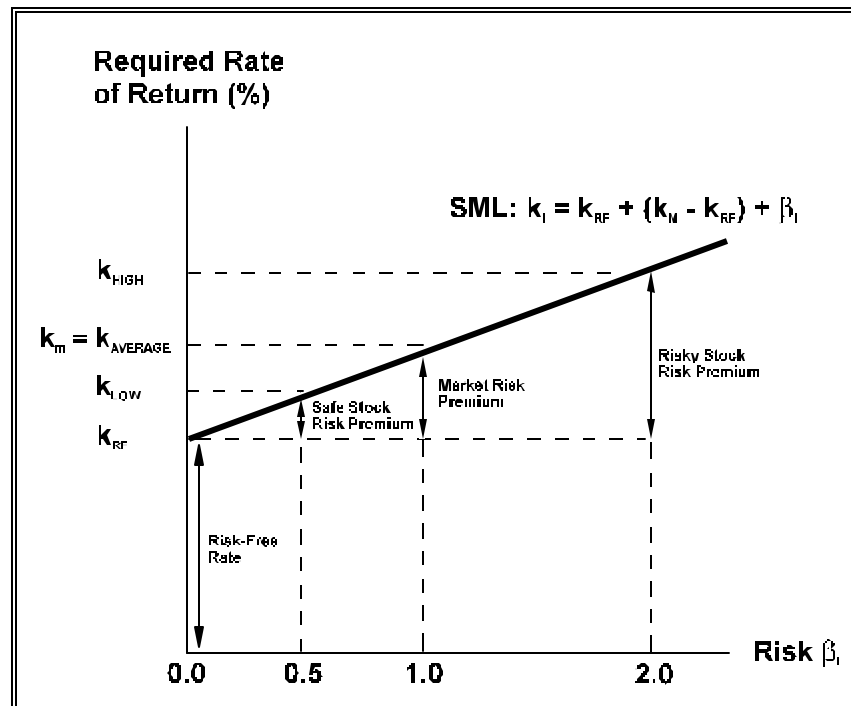


Figure 6.18: Security Market Line

The vertical axis shows the required rate of returns; the horizontal axis shows the risk measured by beta. The market risk is shown at $\beta = 1.0$.

The security market line begins with the risk-free rate of return, where $\beta = 0.0$. This is the rate of return earned with no risk assumed by the investor.

As mentioned previously, an accurate measure of the risk-free rate of return at any single point of time is difficult to obtain. Short-term U.S. Treasury securities are often used as a proxy because they are not subject to default or counterparty risk. Although subject to the risk resulting from interest rate fluctuation, short-term U.S. Treasury Securities are still the best alternatives for estimating the risk-free rate of return.

The line is upwards-sloping to the right. This means that as investments become more risky, investors require higher expected returns to compensate for the additional risk. A relatively safe stock has a much lower risk premium than a relatively risky stock.

Movement of the SML

The security market line can move in two ways. First, a change in expected inflation will cause the SML to shift up or down. For example, if inflation is expected to increase by 3%, investors at every risk level will expect to be compensated for the higher inflation. Every point on the SML will be 3% higher and the line will shift upwards. The second change in the SML represents a change in investors' degree of risk aversion. The **slope** of the line is the risk premium that is required by investors. If, for example, investors become less averse to risk, the slope of the SML will decline (the line becomes flatter). In other words, investors will require a smaller premium on risky securities.

Slope = Required risk premium

We can calculate the slope of the line to determine the risk premium of a security on the SML.

Example

Suppose that the risk-free rate is 3% and the expected rate of return for a relatively safe security ($\delta_i = 0.5$) is 6%.

The formula to determine the slope of the line is:

$$\text{Slope} = [E(k_i) - k_{RF}] / \delta_i$$

Where:

$E(k_i)$ = Expected rate of return of a security

k_{RF} = Risk-free rate of return

δ_i = Beta coefficient of security i

For this example, the slope will be:

$$\text{Slope} = [.06 - .03] / 0.5$$

$$\text{Slope} = .06 \text{ or } 6\%$$

This means that the slope of the security market line for this security indicates a risk premium of 6% per "unit" of systematic risk. The "unit" of risk is the beta coefficient.

Capital Asset Pricing Model (CAPM)

The **capital asset pricing model (CAPM)** is the method most commonly used by analysts and investors to place value on equity securities (stocks), and to determine the **cost of equity** to a company.

Review of capital asset pricing methodology

We have discussed the basics of the CAPM theory throughout our discussion of portfolio risk. A review of these basics will be beneficial to understanding the CAPM theory.

First, we presented the method for calculating an expected return of an individual stock.

Next, the method used to measure the riskiness of a security – which incorporated standard deviation and variance – was introduced, and the relationship between risk and return was recognized. Investors require a higher expected return for assuming more risk.

Then, it was determined that risk is either systematic or unsystematic. Investors are able to diversify investments and combine securities in a portfolio to eliminate unsystematic risk. Therefore, they cannot expect to be compensated for assuming unsystematic risk.

A measure of the systematic risk of a security and its influence on the required return was introduced. The beta coefficient is a measure of the degree to which a security is sensitive to changes in the overall market (systematic risk).

Finally, the rationale for using the rate of return of U.S. Treasury securities to estimate the risk-free rate of return and the method for estimating the required rate of return on the market portfolio was introduced.

CAPM formula

On page 6-42, we introduced a formula for calculating the required rate of return for an individual security. This formula is known as the **capital asset pricing model (CAPM)**. Refer to page 6-41 for the definitions of the symbols.

$$\text{CAPM} = k_i = k_{RF} + (k_m - k_{RF}) \beta_1$$

To compute the CAPM, we start with the risk-free rate of return. Then, a risk premium is added by multiplying the market risk premium by the security's beta coefficient. The result is the required rate of return that investors expect for investing in the stock. The price of the stock is determined by investors buying and selling the shares in the stock market. The equilibrium price that balances the buyers and sellers of the stock becomes the market price.

Abnormal return

An abnormal return is a return that falls above or below the security market line. In other words, an abnormal return is the result of a security that generates a return higher or lower than other securities at the same level of risk (beta). An abnormal return is usually temporary because the market price changes until the security is earning the same return as other securities at that risk level.

Example

For example, suppose a stock selling for \$50 per share is expected to earn \$20 (capital increase plus dividends) in the coming year. The rate of return will be 40% ($\$20 / \$50 = 40\%$).

If other securities with the same beta were expected to earn 20% in the coming year, investors would rush to get in on the good deal (higher return for equal risk). This would cause the price of the stock to rise, because investors would be willing to pay more than \$50 to get in on the higher returns. They would bid the price up to \$100 – the point where the stock's expected return equals 20%. No one would be willing to pay more than \$100 because then their return would be less than 20%.

Weakness of CAPM theory

The capital asset pricing model has some weaknesses.

- Most of the estimates for expected results are based on historical data. Historical data is used to estimate the volatility (beta) of a security. The future volatility of a stock may be very different than its past volatility.
- Companies are rarely completely stable. A company will often make fundamental changes that tend to affect the riskiness of its stock.
- CAPM theory is based on a set of assumptions that is often violated in the "real world." Many analysts feel that the model should also include other economic factors in addition to market sensitivity factor.

CAPM theory preferred by investors

Other models have been developed to try and include a company's sensitivity to inflation, business cycles, etc. However, none of these models have gained the widespread use and acceptance of the CAPM.

The main point to remember is that while there may be inaccuracies in the theory, most investors and analysts act as if the theory is a good one. This means that market prices are being set by using these assumptions.

Since we summarized the process for calculating the rate of return in our discussion of CAPM, we will omit a summary for this section. Please practice what you have learned about "Risk vs. Return" before continuing to a final unit summary of the concepts presented in *Corporate Valuation – Risk*.

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✓ PRACTICE EXERCISE 6.5

Directions: Mark the correct answer to each question. Compare your solutions to the correct answers in the Answer Key that follow on the next page.

8. Onyx Corporation has a beta of 1.50. The risk-free rate of return is 10% and the expected return on the market portfolio is 12%. What rate of return would investors in Onyx Corporation expect to earn?

- _____ a) 10%
- _____ b) 11%
- _____ c) 12%
- _____ d) 13%
- _____ e) 15%

9. Mega Company is a stock that you are considering adding to your portfolio. Mega has a beta of 1.20. The risk-free rate is 13% and the market premium is 2%. What is the expected rate of return you should earn on Mega Company stock?

- _____ a) 13.2%
- _____ b) 15.4%
- _____ c) 16.9%
- _____ d) 18.4%
- _____ e) 19.5%

ANSWER KEY

8. Onyx Corporation has a beta of 1.50. The risk-free rate of return is 10% and the expected return on the market portfolio is 12%. What rate of return would investors in Onyx Corporation expect to earn?

d) 13%

Use CAPM with $\delta_i = 1.50$, $E(k_M) = 12\%$, and $k_{RF} = 10\%$

$$k_i = k_{RF} + (k_M - k_{RF}) \delta_i$$

$$k_i = 10\% + (12\% - 10\%) 1.50$$

$$k_i = 13\%$$

9. Mega Company is a stock that you are considering adding to your portfolio. Mega has a beta of 1.20. The risk-free rate is 13% and the market premium is 2%. What is the expected rate of return you should earn on Mega Company stock?

b) 15.4%

Use CAPM with $\delta_i = 1.20$, $R_{PM} = 2\%$, and $k_{RF} = 13\%$

$$k_i = k_{RF} + (R_{PM}) \delta_i$$

$$k_i = 13\% + (2\%) 1.20$$

$$k_i = 15.4\%$$

PRACTICE EXERCISE*(Continued)*

10. Suppose that U.S. Treasury securities are earning 8% and the expected rate of return on the market is 14%. If a stock has a beta of 0.60, what is the expected rate of return for the stock?
- _____ a) 8.0%
 - _____ b) 8.4%
 - _____ c) 10.6%
 - _____ d) 11.0%
 - _____ e) 11.6%
11. With the market conditions the same as above, another stock has an expected return of 20%. What is the beta for this stock?
- _____ a) 0.9
 - _____ b) 1.0
 - _____ c) 1.5
 - _____ d) 2.0
 - _____ e) 2.5

ANSWER KEY

10. Suppose that U.S. Treasury securities are earning 8% and the expected rate of return on the market is 14%. If a stock has a beta of 0.60, what is the expected rate of return for the stock?

e) 11.6%

Use CAPM with: $\delta_{\mathcal{L}_i} = 0.60$ $E(k_M) = 14\%$ and $k_{RF} = 8\%$

$$\begin{aligned} k_i &= k_{RF} + (k_M - k_{RF}) \delta_{\mathcal{L}_i} \\ k_i &= 8\% + (14\% - 8\%) 0.60 \\ k_i &= 11.6\% \end{aligned}$$

11. With the market conditions the same as above, another stock has an expected return of 20%. What is the beta for this stock?

d) 2.0

Use CAPM with: $k_i = 20\%$ $E(k_M) = 14\%$ and $k_{RF} = 8\%$

Solve for $\delta_{\mathcal{L}_i}$

$$\begin{aligned} k_i &= k_{RF} + (k_M - k_{RF}) \delta_{\mathcal{L}_i} \\ 20\% &= 8\% + (14\% - 8\%) \delta_{\mathcal{L}_i} \\ 12\% &= (6\%) \delta_{\mathcal{L}_i} \\ 2.0 &= \delta_{\mathcal{L}_i} \end{aligned}$$

UNIT SUMMARY

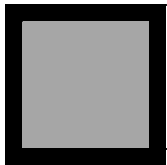
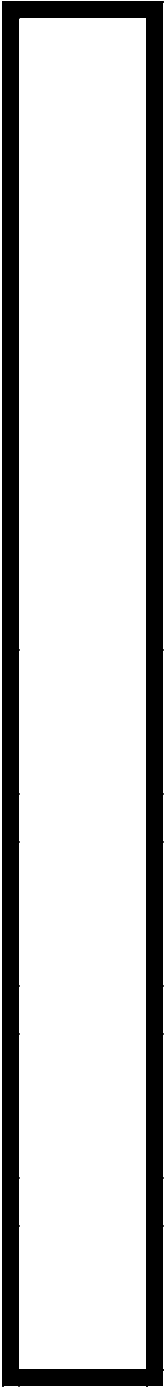
We began this unit with a discussion of the process for calculating the expected return on a security and the expected total cash flow from an investment or project. We then developed the methodology for quantifying the risk associated with expected cash flows. We used the two measurements of variation and standard deviation to determine the amount of volatility among possible cash flows or returns. Standard deviation represents the total risk for a security or portfolio. A lower standard deviation indicates less volatility and, therefore, a lower amount of risk.

Our discussion of portfolio risk indicated that by investing with different companies and different industries, the riskiness of a portfolio may be reduced. This process of diversifying the portfolio only affects the unsystematic or company-specific risk associated with the securities in a portfolio. Systematic (market) risk cannot be eliminated by diversification because the unexpected events that create systematic risk affect all companies in the market.

The relevant risk of holding a security is the individual security's contribution to the systematic risk of the portfolio. The beta coefficient reflects the relative movement of a stock as compared with the market portfolio. An average-risk stock has a beta coefficient of 1.0, which means that if the market moves up by 10%, the stock will, on average, move up by 10%. A lower beta indicates that a stock is less volatile than the market portfolio and, therefore, has less systematic risk. Portfolio beta is the weighted average of the beta coefficients of the individual securities in the portfolio.

The beta coefficient is used in the capital asset pricing model (CAPM) to calculate the required rate of return for an individual security. The rate of return that investors require to invest in a security includes the risk-free rate plus a market (systematic) risk premium. The size of the market risk premium depends on the amount of compensation required by investors for assuming risk. The CAPM is the most widely-accepted method for setting market prices because it is sensitive to systematic risk.

You have demonstrated your understanding of each of these key concepts in the five practice exercises; therefore, this unit has no Progress Check. Please continue your study of corporate value and expected cash flow analysis with Unit Seven: *Corporate Valuation – Cost of Capital*, which follows.



Unit 7

UNIT 7: CORPORATE VALUATION – COST OF CAPITAL

INTRODUCTION

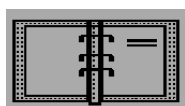
In Unit Five, we discussed the discounting of a company's expected cash flows or the cash flows of a potential project a company is considering. We showed how the discount rate is used to discount the cash flows and arrive at their present value. This discount rate is often called the required rate of return for the project or the opportunity cost for the funds that may be used to invest in the company or project. In Unit Six, you also learned how to adjust rates of return according to the risk involved in the investment. The cost of capital represents the required rate of return. In this unit, you will learn how to calculate the cost of each source of capital and the overall cost of capital for a company.



UNIT OBJECTIVES

After you successfully complete this unit, you will be able to:

- Calculate the cost of debt, preferred stock, and common stock (equity)
- Calculate the weighted average cost of capital



SOURCES OF CAPITAL AND THEIR COSTS

Three major sources

There are three major sources of capital available for financing a project or investment:

- Debt
- Preferred stock
- Common stock

Each of these sources of **capital** has a **cost** associated with it. Some companies can borrow capital at lower rates than other companies because of different risks associated with the companies. Likewise, because of risks involved in their business, some companies have a lower cost associated with preferred stock or common stock than other companies.

Cost of Debt

The **cost of debt** is usually the easiest to calculate. There are two points to consider when calculating the cost of debt.

Appropriate cost is incremental cost

1. The appropriate cost of debt is the incremental cost - not the historical cost.

For example, suppose that a company is considering investing in a project and is planning to issue corporate bonds to pay for part of the investment. The cost of debt is the rate of interest that the company will pay on the new bonds. The rate of interest currently being paid on outstanding bonds is not the appropriate cost of debt.

Interest payments may be tax deductible

2. Interest payments are tax deductible in many countries.

In these cases, the appropriate cost of debt to use is the after-tax cost of debt.

$k_d^* = k_d \times (1 - T)$	
Where:	
k_d^*	= After-tax cost of debt
k_d	= Quoted cost of debt
T	= Tax rate of company

For example, a company can borrow at 8% and its tax rate is 35%, the appropriate after-tax cost of debt would be:

$$k_d^* = 8\% \times (1 - 0.35)$$

$$k_d^* = 5.2\%$$

The cost of debt is adjusted to reflect the tax deductibility of interest payments. Companies do not have to pay taxes on interest payments and, therefore, the cost of debt is lowered. Remember, the tax rate is the expected rate during the period of the investment. It may or may not be the same rate as it has been in the past.

Estimating cost prior to pricing bond issue

There are times when a potential new bond issue has not yet been priced. In this case, the cost of debt can be estimated by substituting the current yield on bonds already issued by the company for the quoted cost of debt (k_d) in the formula.

Example

For example, suppose that a company has 10% annual bonds outstanding. Each bond has a face value of \$1,000, and the bonds are currently selling for \$909 each. The current yield on the bonds is calculated by dividing the annual interest payment by the market price. Thus, the rate of return on these bonds is $\$100 / \$909 = 11.0\%$. The company can estimate that the cost of new debt will be approximately 11%.

Cost of Preferred Stock

To estimate the **cost of** using **preferred stock** as a source of capital, we use the dividend rate that the shares will pay and the price of the shares net of flotation costs. The flotation cost is the amount charged by investment bankers to find investors for the shares ("float the stock"). Once again, these represent *future* figures, not historical ones. The formula is:

$$k_p = D_p / S_N$$

Where:

k_p = Cost of preferred stock

D_p = Annual dividend paid per share

S_N = Price per share, net of flotation costs
(what the company would receive from the issue of new shares)

Example

For example, suppose that Mega Company plans to issue new preferred stock to raise capital for a project. The investment bankers indicate that Mega may issue stock with a par value of \$100 per share that pays a 9% dividend. The investment bankers typically charge 2.5% of the new issue for their services. The cost of the preferred stock can be estimated as follows:

$$\begin{aligned}k_p &= D_p / S_N \\k_p &= \$9 / \$97.50 \\k_p &= 0.0923 \text{ or } 9.23\%\end{aligned}$$

The 9% preferred stock means that the company will pay a \$9 dividend (D_p). The company actually receives \$97.50 per share after paying the flotation costs (2.5%) to the investment bank. The estimated cost of the preferred stock is 9.23%.

Cost of Common Stock (Equity)**Rate of return required by stockholders**

The **cost of common stock (equity)** is the rate of return that is required by stockholders. In other words, it is the minimum rate of return that the corporation must earn for its stockholders in order to maintain the market value of its common stock. As we mentioned earlier, the return an investor receives takes two forms: the payment of dividends by the company and the rise in the value of the shares (capital gain). The investor realizes the gain at the time the stock is sold.

Retained earnings used to enhance value

For a company to grow, it must retain some of its earnings instead of paying all earnings to investors as dividends. Investors expect that the management of the company will use retained earnings to invest in projects that will enhance the value of the firm, and the cost of equity must include this implicit expectation. The cost of equity is the rate of return that investors require in the *future*, not the rate earned by investors in the *past*.

Estimating cost of equity

There are two methods that can be used to estimate the cost of equity.

- 1) Dividend valuation method
- 2) Capital asset pricing model

Dividend Valuation Model**Dividend valuation formula**

The dividend valuation formula was introduced in the discussion of pricing securities (see Unit Four – *Valuing Financial Assets*). The formula is:

$$V = D_0 [(1+G) / (R - G)]$$

Where:

- V = Present value of the common stock (equity)**
D₀ = Most recent dividend paid
G = Estimated growth rate of the dividend
R = The discount rate (or required rate of return)

The alternative formula uses the next dividend (D₁), which is calculated by multiplying the most recent dividend by 1 + the growth rate. The alternative formula is:

$$V = D_1 / (R - G)$$

Present value of the stock

The idea is to find the required rate of return in order to equate the current value of the stock with the present value of the perpetual stream of dividends. The formula for finding the present value of the stock is.

$$P_0 = D_1 / (k_e - G)$$

Where:

- P₀ = Most recent stock price**
D₁ = Next dividend
k_e = Required rate of return
G = Estimated growth rate of dividend

**Cost of equity
(required
return)**

Now, to solve for cost of equity (k_e), we use algebra and rearrange the equation:

$$k_e = (D_1 / P_0) + G$$

Here is an example to illustrate the formula's use.

Example

Alpha Company is expecting a 4% growth rate per year for the foreseeable future. The stock is currently selling for \$25 per share and the last annual dividend to shareholders was \$3. What is the estimated cost of equity for Alpha Company?

Before applying the formula, the next dividend (D_1) is estimated by multiplying the last dividend by 1+ the growth rate ($\$3 \times 1.04 = \3.12). Now the variables can be put into the formula.

$$\begin{aligned} k_e &= (D_1 / P_0) + G \\ k_e &= (\$3.12 / \$25) + 0.04 \\ k_e &= 0.1648 \text{ or } 16.48\% \end{aligned}$$

Alpha's cost of equity is estimated at 16.48%.

**Appropriate
method if
company pays
dividends**

This method is an easy approach to estimating the cost of equity. Stock price and dividend information is relatively easy to obtain for most publicly traded companies. The estimated growth rates may be obtained from several sources, such as the *Value Line Investment Survey* or other published stock analysts' reports. However, if a company does not pay dividends, obviously this is not a good method to use.

Capital Asset Pricing Model (CAPM)

The capital asset pricing model (CAPM) is the second method for estimating a company's cost of equity. The basics of the CAPM and its use were discussed in the last unit (see pages 6-45, 6-46). The CAPM requires an estimate of the:

- Company's beta
- Risk-free rate (U.S. Treasury securities)
- Market premium

The information about U.S. Treasury securities can be obtained easily. The appropriate market rate of return and a company's beta coefficient can also be found in stock analysts' reports.

Let's look at an example of how the CAPM is used to estimate the cost of equity. If you want to review the symbols and the formula, refer to pages 6-41, 6-42.

Example

LDM Manufacturing Company wishes to estimate its cost of equity as part of a project analysis. LDM has estimated its beta to be 0.85. Short-term U.S. Treasury bonds are currently paying 3.6% (k_{RF}), and analysts have projected that a stock market portfolio will average an 11% rate of return (k_M) for the next few years. What is LDM's estimated cost of equity (k_e)?

$$\begin{aligned}k_e &= k_{RF} + (k_M - k_{RF}) \beta_i \\k_e &= 3.6\% + (11\% - 3.6\%) \times 0.85 \\k_e &= 3.6\% + (7.4\%) \times 0.85 \\k_e &= 3.6\% + (6.29\%) \\k_e &= 9.89\%\end{aligned}$$

The estimated cost of equity for LDM Manufacturing Company is 9.89%.

**Incorporates
risk
adjustment
into analysis**

This method is usually preferred over the dividend discount method because it incorporates a risk adjustment into the analysis. The appropriate method for estimating cost of equity may depend on the information that is available to the analyst.

Summary

In this section you learned how to calculate the cost of three types of capital:

Debt

The appropriate cost of *debt* is the after-tax cost of debt.

$$k_d^* = k_d \times (1 - T)$$

Preferred stock

The cost of *preferred stock* is the annual dividend per share divided by the price per share net of flotation costs.

$$k_p = D_p / S_N$$

Common stock (equity)

There are two ways to calculate the cost of *common stock (equity)*: the dividend valuation model and the capital asset pricing model.

The dividend valuation model may be used when the company pays an annual dividend to stockholders.

$$k_e = (D_1 / P_0) + G$$

The capital asset pricing model is based on a company's beta, the risk-free rate, and the market premium. It incorporates a risk adjustment into the analysis.

$$k_e = k_{RF} + (k_M - k_{RF}) \rho_i$$

Before you continue with the discussion of weighted average cost of capital, practice what you have learned about calculating the cost of debt, preferred stock, and equity in the practice exercise that follows. If you have difficulty with any calculations, please review the appropriate text.

PRACTICE EXERCISE 7.1

Directions: Please provide the correct solutions and answers to the questions below, then check them with the Answer Key.

1. Tri-Pro Company wishes to estimate its cost of debt as part of an analysis concerning a possible project. Tri-Pro has outstanding bonds with a par value of \$1,000 and an annual coupon rate of 7%. The bonds are currently selling for \$950. What is Tri-Pro's estimated marginal cost of debt before taxes?

_____ %

2. Tri-Pro Company plans to use preferred stock as a source of capital for its new project. Current Tri-Pro preferred stock has an issue price of \$110 and pays an annual \$10 dividend. What is the expected cost of preferred stock for Tri-Pro?

_____ %

ANSWER KEY

1. Tri-Pro Company wishes to estimate its cost of debt as part of an analysis concerning a possible project. Tri-Pro has outstanding bonds with a par value of \$1,000 and an annual coupon rate of 7%. The bonds are currently selling for \$950. What is Tri-Pro's estimated marginal cost of debt before taxes?

The estimated marginal cost of debt is 7.37%.

This is the recommended way of getting the correct answer:

Use the outstanding value of bonds to make the estimate.

Divide the interest payment by the current price.

$$\$70 / \$950 = 0.0737 \text{ or } 7.37\%$$

Tri-Pro's estimated before-tax cost of debt is 7.37%.

2. Tri-Pro Company plans to use preferred stock as a source of capital for its new project. Current Tri-Pro preferred stock has an issue price of \$110 and pays an annual \$10 dividend. What is the expected cost of preferred stock for Tri-Pro?

The expected cost of preferred stock is 9.1%.

This is the recommended way of getting the correct answer:

$$k_p = D_p / S_N$$

$$k_p = \$10 / \$110$$

$$k_p = 0.0909 \text{ or } 9.1\%$$

Tri-Pro's estimated cost of preferred is 9.1%.

PRACTICE EXERCISE*(Continued)*

3. Tri-Pro's current common stock price is \$30, and the stock recently paid a yearly dividend of \$2.50. Analysts following Tri-Pro's industry feel that Tri-Pro will grow at a 4% annual rate for the next few years. What is the appropriate cost of equity that Tri-Pro should use in its analysis?

_____ %

4. Chicago Incorporated has a beta of 1.25. The market portfolio is expected to earn an average of 8.20% and the risk-free rate is currently estimated to be 3.55%. What is the estimated cost of equity for Chicago Inc.?

_____ %

ANSWER KEY

3. Tri-Pro's current common stock price is \$30, and the stock recently paid a yearly dividend of \$2.50. Analysts following Tri-Pro's industry feel that Tri-Pro will grow at a 4% annual rate for the next few years. What is the appropriate cost of equity that Tri-Pro should use in its analysis?

The cost of equity is 12.7%.

This is the recommended way of getting the correct answer:

Remember to calculate the next dividend.

$$D_1 = \$2.50 (1 + 0.04) = \$2.60$$

Now calculate the cost of equity.

$$k_e = (D_1 / P_0) + G$$

$$k_e = (\$2.60 / \$30.00) + 0.04$$

$$k_e = (0.0867) + 0.04$$

$$k_e = 0.1267 \text{ or } 12.7\%$$

4. Chicago Incorporated has a beta of 1.25. The market portfolio is expected to earn an average of 8.20% and the risk-free rate is currently estimated to be 3.55%. What is the estimated cost of equity for Chicago Inc.?

The estimated cost of equity is 9.36%.

This is the recommended way of getting the correct answer:

Use the CAPM method to estimate the cost of equity.

$$k_e = k_{RF} + (k_M - k_{RF}) \delta_i$$

$$k_e = 3.55\% + (8.20\% - 3.55\%) \times 1.25$$

$$k_e = 3.55\% + 5.8125\%$$

$$k_e = 9.3625\% \text{ or } 9.36\%$$

PRACTICE EXERCISE*(Continued)*

5. The required rate of return is the:
- _____ a) dividend a stockholder expects per share of common stock.
 - _____ b) gain at the time the stock is sold.
 - _____ c) minimum return that is required to maintain the market value of a company's common stock.
 - _____ d) rate earned by a company in the past that has been retained and invested in the company's future growth.
6. The preferred methodology for calculating the cost of equity:
- _____ a) is based on the dividend rate that shares will pay and the price of shares net of flotation costs.
 - _____ b) accounts for the rate of interest and the tax deductibility of interest payments.
 - _____ c) incorporates a risk adjustment into the analysis.
 - _____ d) equates the current value of the stock with the present value of the perpetual stream of dividends.

ANSWER KEY

5. The required rate of return is the:
- c) minimum return that is required to maintain the market value of a company's common stock.**
6. The preferred methodology for calculating the cost of equity:
- c) incorporates a risk adjustment into the analysis.**

If you had any difficulty calculating the correct answer, refer back to the appropriate section of the workbook and do the practice exercise again.

When you feel that you understand this concept completely, move on to the next section.

Weighted Average Cost of Capital

Discount rate for stream of future cash flows

The appropriate methods for estimating the cost of debt, the cost of preferred stock, and the cost of equity were discussed in the previous section. These are the three major sources of capital available to a company for funding a project. The next step is to combine the costs of each of these components to arrive at the appropriate rate for discounting a stream of future cash flows.

Combine costs into one discount rate

The **weighted average cost of capital (WACC)** is obtained by combining each of these costs into one discount rate. The formula to calculate the WACC is:

$$k_a = W_d k_d (1 - T) + W_p k_p + W_e k_e$$

Where:

k_a = Weighted average cost of capital

W_d = Percentage of capital using debt

k_d = Cost of debt

T = Marginal tax rate of company

W_p = Percentage of capital using preferred stock

k_p = Cost of preferred stock

W_e = Percentage of capital using equity

k_e = Cost of equity

Weighted averages have been used several times throughout this course, so you should be very familiar with their calculation. You are simply multiplying the cost of each component by its relative weight in the capital structure of the company. As we mentioned in the discussion on the cost of debt, interest payments are often tax-deductible, so an adjustment is made to the formula to account for this $(1 - T)$.

Example

We will illustrate the WACC calculation in this example. XYZ Corporation has a:

- Cost of equity of 12.6%
- Cost of preferred stock of 9.8%
- Cost of debt of 6.5%
- Marginal tax rate of 35%

XYZ is considering a \$1,000,000 project and plans to fund it with \$500,000 in debt, \$400,000 in equity, and \$100,000 in preferred stock. What is the appropriate discount rate that XYZ should use in discounting the expected cash flows from the project? The answer is obtained by using the known values in the WACC formula.

$$\begin{aligned}
 k_a &= W_d k_d (1 - T) + W_p k_p + W_e k_e \\
 k_a &= 0.50 (6.5\%)(1 - 0.35) + 0.10 (9.8\%) + 0.40 (12.6\%) \\
 k_a &= 2.1125\% + 0.980\% + 5.04\% \\
 k_a &= 8.13\%
 \end{aligned}$$

Goal: to create value for shareholders

The discount rate that XYZ should use to discount the expected cash flows is 8.13%. The idea of the weighted average cost of capital is to account for the costs of all of the capital components. By doing so, the company will only accept projects that create value for the shareholders of the company. This should be the main goal for managers of the firm.

Use future discount rate

There are some important points to keep in mind when using the weighted average cost of capital. The cost of each component is estimated at the margin or future cost - not the historical cost. We often use historical data to forecast future conditions, but it is important to discount future cash flows with a future discount rate.

Appropriate rate for type of financing

Many projects may use only one or two of the funding sources to finance a project. The appropriate discount rate for a project financed entirely by equity is the cost of equity. A simple rule to remember is that cash flows should be discounted at the rate of the cost of capital that purchased those cash flows.

In the real world, the cost of debt is the least expensive, and the cost of equity the most expensive, of the capital components. Many analysts and managers are tempted to use only debt to finance projects because it is the cheapest. That is not realistic because, to limit their risk, most bankers (debt holders) will eventually require that the owners of the company contribute some equity capital to the company.

Capital Structure**Mix of funding sources**

The term capital structure refers to the mix of debt, preferred stock, and equity used as funding sources by a company. Many companies have a target capital structure that dictates the ideal percentage of each source of capital to be used in order to maximize the value of the firm.

Example: target capital structure

For example, a company may have a target capital structure of 50% debt, 40% equity, and 10% preferred stock. In order to maintain that target structure, new growth comes 50% from debt, 40% from equity, and 10% from preferred. Usually, projects that are part of the core business of a company are financed according to the company's target capital structure. An acquisition of another company, or some other project that is outside the core business, may have a different capital structure.

Another factor to consider is that a new project may have a different risk than the core business of the firm. It may be necessary to adjust the cost of capital used to discount the cash flows of the new project to represent that risk. This can be accomplished by analyzing the beta of companies involved in projects of comparable risk and, if appropriate, changing the cost of equity used for the new project. Once again, the key is to discount the expected cash flows at the discount rate that represents the cost of capital used to obtain those cash flows.

Flotation Costs

Investment bank's charges

As we mentioned earlier, flotation costs are the fees that investment bankers charge for providing their services.

Example

For example, ABC Company wants to fund a project with equity and sells 100,000 shares of common stock at \$20 per share. The company has gross proceeds of \$2,000,000 from the sale of the stock and the investment bank charges 6% of the proceeds for its services. After paying the 6% (\$120,000) flotation cost to the investment bank for finding buyers for the stock, ABC Company nets \$1,880,000.

Present value net of costs

In calculating the net present value of a project, we have to deduct these flotation costs to arrive at the appropriate net present value (NPV).

NPV review

Let's review the procedure for calculating the net present value of a project:

1. Establish a set of expected cash flows for a project or a company. (We will discuss some methods for forecasting cash flows in the next unit.)
2. Find an appropriate cost of capital to discount the cash flows.

We may have to estimate the cost of several different funding sources to obtain the appropriate weighted average cost of capital. The discounting process leads to the present value of the cash flows.

3. After discounting, subtract the funds invested in order to receive the cash flows. This gives the net present value of the project.
4. Finally, subtract the flotation costs necessary to raise the investment capital needed to receive the cash flows. This result provides the necessary information to make a decision on whether to accept or reject the project. If the NPV of the project less flotation costs is greater than zero, the project creates value for the firm and should be accepted.

Summary

The required rate of return (cost of capital) takes into consideration the different sources of capital and their individual costs.

The appropriate discount rate for a set of expected cash flows is the weighted average cost of capital, based on the proportions of each funding source.

Please practice what you have learned about the weighted average cost of capital by completing Practice Exercise 7.2. When you have successfully completed the exercises, continue to Unit Eight: *Corporate Valuation – Estimating Corporate Value*.

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PRACTICE EXERCISE 7.2

Directions: Calculate the correct solutions, then check them with the Answer Key.

7. Tri-Pro Company uses a target capital structure of 40% debt, 25% of preferred stock, and 35% of common stock. Tri-Pro's estimated cost of debt is 7.37%, its estimated cost of preferred stock is 9.1%, and its estimated cost of equity is 12.7%. If Tri-Pro's marginal tax rate is expected to be 35%, what is the appropriate cost of capital to discount Tri-Pro's possible projects?

_____ %

8. Chicago Incorporated has a target structure of 45% debt and 55% equity. If Chicago's estimated cost of debt is 4.25%, its estimated cost of equity is 9.36%, and its marginal tax rate is 36%, what is its weighted average cost of capital?

_____ %

ANSWER KEY

7. Tri-Pro Company uses a target capital structure of 40% debt, 25% of preferred stock, and 35% of common stock. Tri-Pro's estimated cost of debt is 7.37%, its estimated cost of preferred stock is 9.1%, and its estimated cost of equity is 12.7%. If Tri-Pro's marginal tax rate is expected to be 35%, what is the appropriate cost of capital to discount Tri-Pro's possible projects?

The cost of capital is 8.64%.

This is the recommended way of getting the correct answer:

$$k_a = W_d k_d (1 - T) + W_p k_p + W_e k_e$$

$$k_a = (0.40)(7.37\%)(1 - 0.35) + (0.25)(9.1\%) + (0.35)(12.7\%)$$

$$k_a = 1.9162\% + 2.2750\% + 4.4450\%$$

$$k_a = 8.6362\% \text{ or } 8.64\%$$

8. Chicago Incorporated has a target structure of 45% debt and 55% equity. If Chicago's estimated cost of debt is 4.25%, its estimated cost of equity is 9.36%, and its marginal tax rate is 36%, what is its weighted average cost of capital?

The weighted average cost of capital is 6.37%.

This is the recommended way of getting the correct answer:

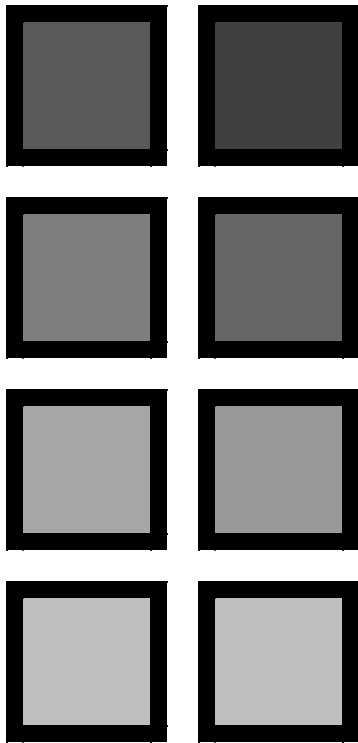
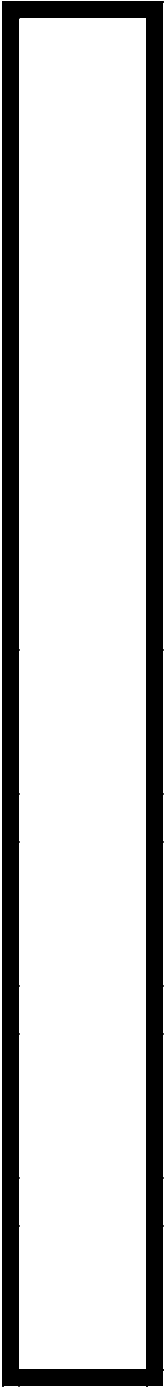
$$k_a = W_d k_d (1 - T) + W_p k_p + W_e k_e$$

$$k_a = (0.45)(4.25\%)(1 - 0.36) + (0.00)(0.00\%) + (0.55)(9.36\%)$$

$$k_a = 1.224\% + 0.00\% + 5.148\%$$

$$k_a = 6.3720\% \text{ or } 6.37\%$$

If you had any difficulty calculating the correct answers, refer to the appropriate section of the workbook and do the Practice Exercise again.



Unit 8

UNIT 8: CORPORATE VALUATION – ESTIMATING CORPORATE VALUE

INTRODUCTION

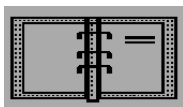
You learned about the discounting process earlier in the course; now you will learn how to estimate cash flows. These estimations are an important part of placing a value on a corporation or making a decision concerning a potential project. An important point to remember when estimating cash flows is that they are *just estimates!* In the real world, risk and volatility affect potential cash flows. In this unit, we focus on methods that help minimize potential errors when making forecasts.



UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize the basic methodology for predicting cash flows based on certain assumptions
- Estimate the residual value of a project or firm
- Estimate the value of a company using the discounted cash flow method
- Recognize other methodologies for estimating value



FORECASTING CASH FLOWS

In order to understand the concept of cash flow, it will be helpful to look at two equations that we can derive from Unit One, *Financial Statement Analysis* and Unit Three, *Time Value of Money*, in the "Present Value" section on discrete and continuous discounting.

Corporate value = Market value of the debt + Market value of equity

Corporate value = Present value of future net cash flow

Basic accounting identity

The first equation is another form of the basic accounting identity: Assets = Liabilities + Equity. We can say that the value of a company is equal to the value of its assets. Remember, capital for purchasing assets is raised from three sources: debt, preferred stock, and common stock.

Economic identity

The second equation is often referred to as an economic identity and means that the value of the company is the present value of the cash flow generated by its assets. These two identities will be used to develop the concepts concerning estimating and discounting cash flows.

Free Cash Flow Calculation**Cash available for distribution to investors**

Free cash flow is the cash available for distribution to the capital providers of the firm. The following is a good formula to use in calculating free cash flow:

(1)		Sales	
(2)	-	Operating expenses	
	=	Earnings before interest and taxes (EBIT)	
(3)	-	Taxes (on EBIT)	
(4)	+	Depreciation and other non-cash items in the income statement	
(5)	-	Increase in working capital investment	
(6)	-	Capital expenditures	
	=	Free cash flow	

Elements of the calculation

Let's review each element of the calculation.

- (1) Obtain the sales figure from the income statement.
- (2) Deduct the operating expenses from sales to arrive at earnings before interest and taxes (EBIT).

(3) Deduct the taxes on EBIT.

This is generally not the tax figure from the balance sheet, because that tax figure has allowed for the tax-deductibility of interest payments made during the period. Generally, this tax can be estimated by multiplying EBIT by the marginal tax rate (tax rate before deferments, etc.)

(4) Add back the depreciation that was deducted as a part of operating expenses.

It is added because it does not represent a cash expenditure made by the company — it is merely an accounting convention.

(5) Deduct the investment made in working capital.

Working capital constitutes the funds necessary for the operations of a business. It is calculated by subtracting current liabilities from current assets on the balance sheet.

An increase in working capital means that the company has invested cash in its day-to-day operations. You may remember in our discussion of the cash flow statement that increases in accounts such as INVENTORY and ACCOUNTS RECEIVABLE represent uses of cash, whereas increases in accounts such as NOTES PAYABLE and ACCOUNTS PAYABLE represent sources of cash.

(6) Deduct the capital expenditures.

These expenditures include purchases of property, plant, equipment, etc. This represents the amount of cash the company has spent to maintain its operations. An analyst may not have access to detailed information about planned capital expenditures or working capital investment, especially when studying a company other than the analyst's own. In this case, the analyst will use an incremental rate of investment to make forecasts.

The result of the calculation is the cash flow that is available for the company to distribute in the form of interest payments on debt or divided payments on preferred stock and equity.

Assumption-based Cash Flow Forecasting

Assumptions about future business conditions

Most analysts make assumptions about future business conditions based on research of the company and the industry in which it operates. The analyst then applies the assumptions to the cash flow calculation in order to forecast future cash flows. A set of assumptions, and the future cash flows that are derived from the assumptions, are listed in Figure 8.1.

XYZ Corporation Projected Cash Flows						
Assumptions:						
	Sales Increase:	10% per Year				
	Profit Margin:	8%				
	Tax Rate:	35%				
	Incr. in Working Capital Investment:	10% of Incremental Sales				
	Incr. Net Fixed Capital:	12% of Incremental Sales				
(In Millions \$)						
		Year 0	Year 1	Year 2	Year 3	Year 4
(1) Sales		287.6	316.4	348.0	382.8	421.1
(2) Operating Profit		22.5	25.3	27.8	30.6	33.7
(3) Taxes		8.1	8.9	9.7	10.7	11.8
(4) Incr. in Working Capital			2.9	3.2	3.5	3.8
(5) Capital Expenditures			3.5	3.8	4.2	4.6
(6) Free Cash Flow			10.0	11.1	12.2	13.5

Figure 8.1: Predicted Cash Flows Based on Assumptions

Starting with the assumptions that are given, let's analyze the data.

(1) Sales

The analysts have made an assumption concerning the future sales growth of XYZ Corporation. They feel that XYZ's sales will grow 10% per year for the next four years. That assumption is based on their study and knowledge of XYZ's business and competition.

In Year 0, XYZ had \$287.6 million in sales. To project the sales in Year 1, we multiply last year's sales by 1.10 (1 + growth rate). In this case, $\$287.6 \times 1.10 = \316.4 . Years 2, 3, and 4 sales figures also are calculated by multiplying the previous year's sales by 1 + the growth rate.

(2) Operating Profit

Next, we calculate the operating profit for each year. The assumption is that XYZ can expect an 8% profit margin for each year of the analysis. That assumption is based on XYZ's past profit margins and the analysts' future expectations for the firm. In the projected cash flow calculation, we multiply each year's sales figure by the expected profit margin. For example, for Year 1, the calculation is $\$316.4 \times 0.08 = \25.3 .

(3) Taxes

The analysis projects that the cash tax rate (tax rate that will actually be paid) for the next four years will be approximately 35% p.a. Because tax laws are complex, vary from government to government, and are often subject to change, we will not spend much time discussing them in this course. In this analysis, it is important to remember that the cash tax rate is the appropriate rate to use. (This rate may be different than the marginal tax rate used on the income statement and may or may not be the rate that the firm has paid in the past.) Generally, the analyst knows the appropriate rate that should be applied. Remember, the analyst essentially is making reasonable estimates.

(4) Increase in Working Capital

In our discussion of the free cash flow calculation, we defined the term "working capital." It refers to the cash that the firm has committed to inventories, accounts receivable, etc. less the sources of that cash, which may include trade receivables, short-term bank notes, and accrued wages.

Analysts estimate the increase in working capital using a percentage of incremental sales, which means that a certain percentage of each sales increase is committed to working capital. The theory is that the firm must grow larger to generate increased sales.

The calculation is made by subtracting the previous years' sales from the projected sales figure for the year that is being analyzed, then multiplying the result by the appropriate percentage of incremental sales. For example, to project Year 1's increase in working capital, subtract Year 0 sales from Year 1's projected sales and multiply by 10%. The increase in working capital is: $(\$316.4 - \$287.6) \times 0.10 = \$2.9$.

(5) Capital Expenditures

To project fixed capital asset expenditures, we can also use an incremental sales percentage. For a company to generate additional sales, it usually must spend cash on new equipment or new plants. These capital expenditures are cash outflows from the company, just as the increase in working capital is a cash outflow. Remember, this is the capital expenditure net of depreciation. For example, for Year 1, the capital expenditure is $(\$316.4 - \$287.6) \times 0.10 = 3.5$.

(6) Free Cash Flow

To calculate the projected free cash flow for each year of the analysis, subtract the cash outflows of taxes, working capital investment, and net fixed capital asset expenditures from the operating profit. For example, for Year 1, the free cash flow is $25.3 - 8.9 - 2.9 - 3.5 = 10$. We will discuss how to use these projected cash flows a little later in the course.

"What-if" type analysis

Many analysts use this type of framework to make estimates of future cash flows. Each analysis is based on the amount of information that is available. Analysts use a spreadsheet program (Lotus 1-2-3, Excel, etc.) to make the calculations. In that way, they can easily perform a "what-if" type analysis to see the effect of changes in each of their assumptions on the firm's projected cash flows.

More accurate analysis with inside information

Analysts working inside a company have access to much more complete information than those studying a competitor from outside the firm. Analysts inside a company can conduct a more in-depth study that actually projects the entire income statement and balance sheet. From that, they can build an estimated cash flow statement, much as we did in Unit One. For example, inside analysts have access to the exact figures that the company is planning to spend on working capital and fixed assets; therefore, they do not have to make estimates using incremental increases in sales.

We suggest that you work through the cash flow calculations in Figure 8.1 to make sure that you are comfortable with all of the figures. You may want to try and replicate the analysis on a spreadsheet program to begin building a framework that you can use in the future.

To review, free cash flow is the amount of cash generated by a company's operations for distribution to its debt and equity investors. We can estimate these cash flows for several years in the future in order to estimate the value of the company to its shareholders.

RESIDUAL VALUE

Value at the end of the forecast period

In a perfect world, we would be able to project cash flows for the expected life of the company. However, it is impractical, and often inaccurate, to forecast cash flows far into the future. Most forecasts project cash flows for the next five to ten years. However, most companies or projects still have value at the end of the forecast period. This value is often referred to as the **residual value** of the firm.

The residual value is an important part of an analysis and must be based on valid assumptions. A short-term, one-time project that is planned to be in operation for a specific amount of time may have no value at the end of the forecast period. In those cases, it is appropriate to have the residual value equal zero. Otherwise, most companies and projects will have some value at the end of the forecast period.

Methodology for estimating residual value

There are two ways that the analyst can estimate residual value and apply it to a company valuation analysis:

- Perpetuity method
- Growing perpetuity method

Perpetuity Method

Assume perpetual cash flows

The first way to estimate the residual value is to treat it as a perpetuity. In other words, the analyst assumes that after the forecast period, the company will generate a perpetual cash flow. To estimate the perpetual cash flow, we take the sales figure from the last year of the forecast and subtract the operating profit and taxes. Since it is assumed that the company is no longer growing, it isn't necessary to allow for increases in working capital and capital expenditures. You may recall that the value of a perpetuity is the perpetual cash flow divided by the appropriate discount rate.

Example

For example, if we assume that XYZ Corporation will generate \$13 million in cash flow each year starting in Year 5 and the discount rate for XYZ is 10%, then the residual value for XYZ will be:

$$\begin{aligned}\text{Residual Value} &= (\text{Perpetual cash flow}) / (\text{Discount rate}) \\ &= (\$13 \text{ million}) / (0.10) \\ &= \$130 \text{ million}\end{aligned}$$

As you can see, the residual value can be a substantially large part of the company's value. Therefore, to avoid unacceptable error, it is important that the assumptions made in computing the residual value are valid.

Company is no longer creating value

You may recall that companies that earn more than the required rate of return create value for shareholders; those that earn less destroy value. The theory of residual value is that after the forecast period, the company (or project) is no longer able to produce real growth in value; it is earning exactly the required rate of return for the shareholders.

Growing Perpetuity Method

Assumes continued growth after forecast period

This method assumes that the perpetuity continues to grow after the forecast period. The residual value is equal to the present value of the growing perpetuity.

$$\text{Residual value} = [\text{CF} \times (1 + g)] / (k_a - g)$$

Where:

CF = First cash flow

g = Growth rate

k_a = Discount rate

Example

For example, suppose that XYZ Corporation is expected to have a growing perpetuity of cash flows starting in Year 5. In that year, the projected cash flow is \$13 million, with a growth rate of 3% and a discount rate of 8.13%.

$$\begin{aligned} &= [\$13 \times (1.03)] / (0.0813 - 0.03) \\ &= [\$13.39] / (0.0513) \\ &= \$261.0 \text{ million} \end{aligned}$$

One key point to remember when using the growing perpetuity method: *the growth rate must be less than the discount rate* or the formula is not valid.

Other Methods

Based on specific values

The estimation techniques we have discussed may not be necessary if the analyst has a specific value to use for the residual value. For example, the project may have no value to the company at the end of the forecast, so it is appropriate to use \$0 as the residual value for the valuation. Perhaps the company can be sold for an already agreed upon price (or at a very accurate estimated price) at the end of the forecast; then the analyst would use this figure as the residual value of the company for the valuation computation. The estimation techniques are used when this kind of information is not available.

Summary

In our discussion of cash flow forecasting, we defined free cash flow as the cash that is available to distribute for the use of capital. The calculation of free cash flow is based on certain assumptions about future incremental cash flows.

The residual value of a company is the value at the end of the forecast period. The value may equal the perpetual cash flow divided by the appropriate discount rate. This means that the company earns exactly the required rate of return for shareholders.

The residual value also can represent the present value of a growing perpetuity. This method assumes that the company's earnings will continue to grow after the forecast period.

Whether it is a constant perpetuity or a growing perpetuity, the residual value refers to the value of the company or project at a specific time in the *future*. In the next section, we will discuss how to find the value of the firm at the *present* time.

Practice what you have learned about forecasting cash flows and calculating residual value by completing the exercise that follows. Then continue to the next section, "Discounted Cash Flow Method."

PRACTICE EXERCISE 8.1

Directions: Calculate the correct values to complete the cash flow projection. Check your solution with the Answer Key on the next page.

- Analysts have made the following assumptions concerning the next four years for LDM Incorporated.

Assumptions:

Sales Increase:	15% per Year
Profit Margin:	20%
Tax Rate:	34%
Incr. in Working Capital Investment:	15% of Incremental Sales
Incr. Net Fixed Capital:	18% of Incremental Sales
Residual Value:	\$16,000 Perpetuity Starting in Year 5
Residual Value Discount Rate:	15%

Calculate the projections for Years 1, 2, 3, and 4 and the projected residual value.

	Year 0	Year 1	Year 2	Year 3	Year 4
Sales	10,000.0	_____	_____	_____	_____
Operating Profit	2,000.0	_____	_____	_____	_____
Taxes	680.0	_____	_____	_____	_____
Incr. in Working Capital		_____	_____	_____	_____
Capital Expenditures		_____	_____	_____	_____
Free Cash Flow		_____	_____	_____	_____
Residual Value					_____

ANSWER KEY

1. Analysts have made the following assumptions concerning the next four years for LDM Incorporated.

Your cash flow calculation should look like this:

Assumptions:

Sales Increase:	15% per Year
Profit Margin:	20%
Tax Rate:	34%
Incr. in Working Capital Investment:	15% of Incremental Sales
Incr. Net Fixed Capital:	18% of Incremental Sales
Residual Value:	\$16,000 Perpetuity Starting in Year 5
Residual Value Discount Rate:	15%

Calculate the projections for Years 1, 2, 3, and 4 and the projected residual value.

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>
Sales	10,000.0	11,500.0	13,225.0	15,208.8	17,490.1
Operating Profit	2,000.0	2,300.0	2,645.0	3,041.8	3,498.0
Taxes	680.0	782.0	899.3	1,034.2	1,189.3
Incr. in Working Capital		225.0	258.8	297.6	342.2
Capital Expenditures		270.0	310.5	357.1	410.6
Free Cash Flow		1,023.0	1,176.4	1,352.9	1,555.9
Residual Value					106,666.7

If you have any questions about these numbers, refer to the discussion on how to calculate each value in the computation.

DISCOUNTED CASH FLOW METHOD

Appropriate discount rate

In Unit Seven, we discussed the calculation for the required rate of return for each of the sources of capital. We also discussed the weighted-average cost of capital (WACC), which combines the required rates of the different sources into one discount rate. A company that uses several different sources of capital to finance its operations should apply a weighted-average cost of capital to discount cash flows. The weights are based on the amount of capital derived from each source.

However, if a project is to be funded entirely by one source of capital (e.g. equity capital), then the cost of that source is the appropriate discount rate.

Placing Value on a Company

Example

To illustrate the discounting process, we will discount the projected cash flows of XYZ Corporation using its WACC. In Unit Seven, we described XYZ's target capital structure of 50% debt, 10% preferred stock, and 40% common stock with required rates of return of 6.5%, 9.8%, and 12.6%, respectively. XYZ has a marginal tax rate of 35%. Let's review the calculation from page 7-15:

$$\begin{aligned}
 k_a &= W_d k_d (1 - T) + W_p k_p + W_e k_e \\
 k_a &= 0.50 (6.5\%)(1 - 0.35) + 0.10 (9.8\%) + 0.40 (12.6\%) \\
 k_a &= 2.1125\% + 0.980\% + 5.04\% \\
 k_a &= 8.13\%
 \end{aligned}$$

The WACC for XYZ Corporation is 8.13%. This is the discount rate the company uses for discounting the projected cash flows of XYZ Corporation in order to place a value on the company. We will use the residual value calculated with the growing perpetuity method. Let's see how the present value computation is made.

(In Million \$)			
Year	Cash Flow	Discount Factor	PV of CF
1	10.0	$1/(1.0813)^1$	9.4281
2	11.1	$1/(1.0813)^2$	9.4936
3	12.2	$1/(1.0813)^3$	9.6499
4	13.5	$1/(1.0813)^4$	9.8753
Residual	261.0	$1/(1.0813)^4$	190.9319
		PV =	229.3788

Figure 8.2: Present Value of XYZ Corporation's Cash Flows

Sum of discounted projected cash flows

We have discounted the projected cash flows for XYZ (Page 8-4) using the WACC. The sum of the discounted cash flows equals the present value of the corporation. Notice that we have discounted the residual value as a cash flow to be received in Year 4. When we calculated the residual value (Page 8-9), we discounted the growing perpetuity at the time it was to be received (Year 4). To calculate the present value of the firm at Year 0, we discount this value to the present time. The \$229.38 million is the total corporate value of the firm.

Remember the two equations that we gave at the beginning of this section:

$$\text{Corporate value} = \text{Market value of the debt} + \text{Market value of equity}$$

$$\text{Corporate value} = \text{Present value of future net cash flow}$$

Finding market value of equity

We have calculated the present value of the future net cash flows, so we know that the corporate value of XYZ is \$229.38 million. To find the *market value of the equity*, we subtract the market value of the debt and the market value of the preferred stock from the total corporate value. In XYZ Corporation, the value of the debt is \$168.8. We arrived at that figure by adding the current liabilities and the long-term debt from the accounts on the current balance sheet (see Page 1-2).

$$\text{Total debt} = \text{Current liabilities} + \text{Long-term debt}$$

$$\text{Total debt} = \$61.4 \text{ million} + \$107.4 \text{ million}$$

$$\text{Total debt} = \$168.8 \text{ million}$$

Subtract market value of debt

Remember, it is important to subtract the *market value* of the debt rather than the book value. Since current liabilities are relatively short-term, usually we can assume that the book value is close to the market value. For long-term debt, that assumption may not be valid, especially in an environment where long-term interest rates have changed radically since the debt was issued. In this case, it may be more appropriate to value the bonds using the methods described in Unit Four, in the section on valuing financial instruments. For the purpose of this example, we will assume that book value is close to market value for the long-term debt.

Subtract value of preferred stock

We also subtract the value of the preferred stock from the corporate value to arrive at the value of the equity. The same assumptions and calculations made in valuing long-term debt are used in finding the value of the preferred stock. Our assumption is that the book value and market value of the preferred stock are almost identical; we will use the book value of XYZ's preferred stock of \$12.3 million.

Calculating market value of equity

If we return to our XYZ example, we can now calculate the market value of the common equity held by the shareholders.

<u>Year</u>	<u>Cash Flow</u>	<u>Discount Factor</u>	<u>PV of CF</u>
1	10.0	$1/(1.0813)^1$	9.4281
2	11.1	$1/(1.0813)^2$	9.4936
3	12.2	$1/(1.0813)^3$	9.6499
4	13.5	$1/(1.0813)^4$	9.8753
Residual	261.0	$1/(1.0813)^4$	<u>190.9319</u>
		PV =	229.3788
	Less Market Value of Debt		168.80
	Less Market Value of Preferred		<u>12.30</u>
	Market Value of Common Equity		48.28
	Shares Outstanding		8.00
	Price per Share		6.04

Figure 8.3: Market Value of Common Equity

Market value per share

The price per share is the market value of the common equity divided by the number of shares outstanding. Based on the assumptions concerning XYZ's growth and capital expenditures, the analyst believes that the common shares of XYZ Corporation are worth \$6.04 each.

Test value sensitivity of assumptions

The next step is for the analyst to go back and review the assumptions to make sure they are reasonable. The analyst will also try to develop "what-if" scenarios by changing the assumptions (usually one at a time) to isolate the value sensitivity of the assumption. For example, in the XYZ analysis, the analyst may wish to calculate the value of the firm using a 7% growth rate rather than a 10% growth rate (Figure 8.4).

XYZ Corporation Projected Cash Flows					
Assumptions:					
Sales Increase:	7% per Year (Changed from 10%)				
Profit Margin:	8%				
Tax Rate:	35%				
Incr. in Working Capital Investment:	10% of Incremental Sales				
Incr. Net Fixed Capital:	12% of Incremental Sales				
(In Millions \$)					
	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>
Sales	287.6	307.7	329.3	352.3	377.0
Operating Profit	22.5	24.6	26.3	28.2	30.2
Taxes	8.1	8.6	9.2	9.9	10.6
Incr. in Working Capital		2.0	2.2	2.3	2.5
Capital Expenditures		2.4	2.6	2.8	3.0
Free Cash Flow		11.6	12.4	13.2	14.2

Figure 8.4: Projected Cash Flows Based on a 7% Growth Rate

We now can discount these cash flows as we did on page 8-15.

<u>Year</u>	<u>Cash Flow</u>	<u>Discount Factor</u>	<u>PV of CF</u>
1	11.6	$1/(1.0813)^1$	10.7278
2	12.4	$1/(1.0813)^2$	10.6055
3	13.2	$1/(1.0813)^3$	10.4408
4	14.2	$1/(1.0813)^4$	10.3873
Residual	261.0	$1/(1.0813)^4$	<u>190.9319</u>
		PV =	233.0933
	Less Market Value of Debt		168.80
	Less Market Value of Preferred		<u>12.30</u>
	Market Value of Common Equity		51.99
	Shares Outstanding		8.00
	Price per Share		6.50

Figure 8.5: Market Value of Common Equity Based on 7% Growth Rate

You can see that by changing the assumption of a 10% growth rate to a 7% rate, the market value of the company becomes 6.50 per share.

The analyst may continue to create different scenarios to get an idea of the range of prices for the stock (or a range of values for a project). The analyst may even try to estimate the probabilities of each scenario occurring in order to find the most likely value. This information is useful as part of the decision-making process concerning the investment in a company or project.

Because the residual value is such a high proportion of the total value of the discounted cash flows in the XYZ analysis, the analyst may double check the assumptions concerning residual value to make sure they are reasonable. As you can see, creating a spreadsheet to perform this analysis saves a lot of time when recalculating the cash flows based on different assumptions.

Summary

We can summarize the discounted cash flow method of estimating corporate value using the following steps:

- Make reasonable assumptions concerning the future operations of the company (or project).
- Forecast the cash flows of the company (or project) for a reasonable time period.
- Estimate the residual value of the company or project at the end of the forecast period.
- Calculate the appropriate discount rate for the company or project, based on the future capital structure of the company (or project).
- Discount the forecasted cash flows using the appropriate discount rate.
- Add the discounted cash flows to find the estimated corporate value.
- Subtract the market value of the debt and preferred stock to find the estimated market value of the equity.
- Divide the value of the equity by the number of common shares outstanding to find the estimated market price of the shares.

Before continuing to "Other Valuation Methodologies," please practice what you have learned about estimating corporate value by completing the following Practice Exercise.

PRACTICE EXERCISE 8.2

Directions: In Practice Exercise 8.1, you calculated the forecasted cash flows for LDM Incorporated based on a given set of assumptions. Use the results listed below to answer Question 2. Check your solutions with the Answer Key on the next page.

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>
Sales	10,000.0	11,500.0	13,225.0	15,208.8	17,490.1
Operating Profit	2,000.0	2,300.0	2,645.0	3,041.8	3,498.0
Taxes	680.0	782.0	899.3	1,034.2	1,189.3
Incr. in Working Capital		225.0	258.8	297.6	342.2
Capital Expenditures		270.0	310.5	357.1	410.6
Free Cash Flow		1,023.0	1,176.4	1,352.9	1,555.9
Residual Value					106,666.7

2. Analysts have estimated the weighted average cost of capital for LDM to be 8.65%. The current market value of LDM's debt is \$42,300 and LDM has no preferred stock. If there are 8,200 shares of LDM outstanding, what is the estimated market price of LDM's common stock?

ANSWER KEY

2. Analysts have estimated the weighted average cost of capital for LDM to be 8.65%. The current market value of LDM's debt is \$42,300 and LDM has no preferred stock. If there are 8,200 shares of LDM outstanding, what is the estimated market price of LDM's common stock?

<u>Year</u>	<u>Cash Flow</u>	<u>Discount Factor</u>	<u>PV of CF</u>
1	1,023.0	1/(1.0865) ¹	941.6
2	1,176.4	1/(1.0865) ²	996.5
3	1,352.9	1/(1.0865) ³	1,054.8
4	1,555.9	1/(1.0865) ⁴	1,116.5
Residual	106,666.7	1/(1.0865) ⁴	76,543.8
		PV =	80,653.2
Less Market Value of Debt			42,300.0
Less Market Value of Preferred			0.0
Market Value of Common Equity			38,353.2
Shares Outstanding			8,200.0
Price per Share			4.68

We have discounted the projected cash flows by the weighted average cost of capital (WACC) and subtracted the market value of the debt to arrive at the market value of the equity. We then divided the value of the equity by the number of shares to estimate the price per share.

OTHER VALUATION METHODOLOGIES

The discounted cash flow method is generally considered the most accurate method for estimating the value of a company or for estimating the value created by a potential project. By focusing on cash flows rather than net income or other values, the analyst eliminates the bias incurred by various accounting conventions. There are several outstanding books that highlight the advantages of cash flow over net income as the basis for estimating a company's value. You may want to reference one of them during your study, especially if you have more questions concerning discounting cash flows.

Good "first cut" types of analyses

You should recognize some other methods used to estimate corporate value. These methods are not as accurate as discounting cash flows, but they are still being used by many managers — especially those without a strong analytical background. Because they are not nearly as rigorous as the discounted cash flow method, you might consider them good "first-cut" types of analyses that give the analyst a beginning point for comparison.

These methods include:

- Price / earnings ratio
- Market / book ratio
- Liquidation value
- Dividend value
- Other price / characteristic ratios

Price / Earnings Ratio

We introduced this ratio in the financial ratio section of Unit One. To refresh your memory, the formula for calculating the Price / Earnings (P/E) ratio was given as:

$$\text{P/E} = (\text{Market price per share}) / (\text{Earnings per share})$$

High P/E means investor optimism

The P/E ratio tells the analyst how much investors are paying for each dollar of income generated by the firm. High P/E ratios indicate that investors are very optimistic about the future prospects of the company. They are willing to pay more for the current earnings of the firm because they believe that future earnings will be even higher. Likewise, lower P/E ratios indicate less investor optimism.

P/E ratios are often used to compare companies that are potential investment opportunities. It is appropriate to compare companies within the same industry because of their similar structure and operations. For example, analysts would compare the P/E ratios of Ford Motor Company and General Motors as part of an analysis. They would not compare the P/E ratio of Ford with the P/E ratio of IBM.

The problem with using P/E ratios in making investment decisions is that the analysis is short-sighted. Some investors feel that by choosing companies with lower P/E ratios, they are getting a bargain.

Let's look at an illustration.

Example

An investor is considering two similar companies. Company A has a P/E ratio of 8 and Company B has a P/E ratio of 12. The investor chooses Company A because s/he believes that the price is lower per dollar of income generated by the company. However, Company B may still be a better investment because of its future earnings potential. The P/E ratio is based on the most recent earnings figure – not the future earnings potential; therefore, it is often considered too short-sighted to be the only investment criterion.

Many analysts will use the P/E ratio to estimate the price of a company's stock. They multiply the estimated earnings for the coming year by the P/E ratio. For example, Company B has a P/E ratio of 12 and expected earnings per share of \$3.80. The estimated price of the stock is $\$3.80 \times 12 = \45.60 .

Valuable as part of analysis

The use of the P/E ratio is appropriate as part of thorough investment analysis. In fact, all of these methods have some merit, not as stand-alone decision-making tools, but as part of a more thorough analysis.

Market / Book Ratio

We also discussed the market / book ratio in Unit One. The formula was given as:

$$\text{Market / Book} = (\text{Market price per share}) / (\text{Book value per share})$$

Relationship between market price and value

This ratio gives the relationship between the market price that investors are paying for the company and the value of the company according to accounting methods. In other words, the market / book ratio shows how many times greater the market price of the company is than the book value of the company.

Some investors consider the book value as the floor for the market price of the company. They believe that if the market value ever falls below the book value, the company could sell off its assets at book value to maintain its price. Those investors may also argue that low market / book ratios indicate that the company is a safer investment. However, the book value may not be the price the company could get by selling off its assets and, therefore, the ratio may be misleading.

The market book ratio is even less useful as a stand-alone tool than the P/E ratio. Problems occur even when trying to compare companies in the same industry. Most accounting conventions require that assets be carried at their historical cost on the books. It then becomes more difficult to find a company with similar operations and similar assets to serve as an appropriate point of comparison. Companies with older, less costly assets will have lower market / book ratios than similar companies with newer, more modern assets — all other factors being equal.

However, some less sophisticated investors continue to use the market / book ratio as a major investment decision-making tool. The sophisticated analyst will use the information derived from this ratio as part of a more thorough analysis.

Liquidation Value

Amount left for shareholders

If all of the assets of a company are sold and the proceeds are used to pay the creditors of the company, the amount left to distribute to shareholders is the **liquidation value**. The liquidation value is often considered the floor price of the stock, because if the stock price falls below this value, investors will be better off if the company is liquidated.

Liquidation value is often used as a criterion for potential takeover strategies. A corporate raider may notice that the price of the stock is less than the liquidation value and make an offer for all of the company's stock. S/he then gains control, sells off the assets, and pays the creditors. The remaining value, less the original investment, represents the investor's gain.

However, it may be extremely difficult to estimate the liquidation value of a company. This is especially true if there is little or no market for the assets of the company. The analyst will need a thorough understanding of the company and its industry to make an accurate estimate of the liquidation value of a company.

Dividend Value

Perpetuity vs. growing perpetuity

In our discussion on valuing financial instruments, we introduced the dividend valuation method for estimating the value of the equity. We discussed two methods for calculating the value — depending on whether the dividend payment on the common stock is a perpetuity or a growing perpetuity.

To review, the formula for estimating the value of common stock as a perpetuity was given as:

$$V = D \times (1 / R)$$

The growing perpetuity formula is:

$$V = D_1 / (R - G)$$

Remember that both methods require an estimated discount rate (R) and the most recent dividend payment (D). The growing perpetuity also needs an estimated growth rate (G) to complete the calculation.

However, if a company does not pay dividends, or if our assumptions concerning future dividends do not fit a perpetuity, then this method is of little value. For example, if a company is erratic in its dividend payments, then the assumption of a dividend perpetuity may not be valid.

Other Ratios

Help to understand a company or industry

There are other ratios that may assist the analyst in understanding a company or industry:

- The price / revenues ratio may provide insight into how much investors are paying for the revenues being generated by the company.
- The price / EBIT (earnings before interest, taxes, and depreciation) ratio shows how much value market investors are placing on a company's earnings (without interest payments, taxes, and depreciation).
- Many analysts develop ratios that correspond to some type of operating statistic of the company or industry.

Example

In the airline industry, for example, an analyst may use a ratio relating revenues with passenger seat-miles to measure the distance one seat on one airplane is available on a flight. An airplane with 150 seats flying on a 500-mile trip has 75,000 passenger seat-miles available for the trip. This statistic will give the analyst an idea of how much revenue is being generated by each seat on each plane. Every industry has similar measures that are used by analysts to gain an understanding of how well companies are performing.

All of these methods have some merit and validity. The thorough analyst will estimate corporate value using several different methods and will compare these results with the estimates obtained using the discounted cash flow method under different scenarios. The analyst will then have a good idea of the assumptions that are being made concerning the future of the company and if these assumptions are valid and reasonable.

FACTORS AFFECTING VALUE

Factors outside the company's control

When making assumptions concerning the future of a company or project, the analyst must consider many factors. Unfortunately, many analysts only consider factors the company can control, which is often a narrow approach. There are many factors outside the company that may affect the future of the company. We want to discuss some of these factors and try to understand their effects on valuation estimation.

Market Liquidity

Ability to buy and sell in the market

Market liquidity refers to the ease with which securities are bought and sold in a market. For example, the United States securities markets are very large — many buyers and sellers participate every trading day. The prices asked by sellers and the prices offered by buyers are usually very close, and transactions are completed easily.

Other markets are considerably less liquid for investors, perhaps due to the small number of investors participating in the market, government regulations concerning ownership of securities, tax regulations, or other factors. These conditions lead to large spreads between sellers' asking prices and buyers' offered prices.

Example

For example, suppose that an investor has completed a discounted cash flow analysis and estimates that a share of TTM Manufacturing Company has a value of \$6.50. The shares are currently being sold for \$5.95 and the investor would like to purchase some shares for his portfolio. However, the market in which TTM trades requires a surcharge of \$1.00 per share for foreign ownership of shares. Since the investor is not a citizen of the country in which TTM trades, the investment price is \$6.95 per share. At this price, the investment is no longer attractive.

Even if the investor is a citizen of the country, he will not be able to buy the security unless there are current holders of TTM stock who are willing to sell their shares. In order to entice a current investor to sell, the buyer may have to substantially increase the offer above the \$5.95 listed price. However, if the price the buyer must offer is too high to justify the investment, the buyer loses out on the opportunity to invest in TTM. Likewise, an investor who currently holds securities in this market and wants to close out a position, may have to sell the securities for less than the estimated value.

Market liquidity is a very important factor in many small, emerging securities markets where investors may have to pay premiums for an opportunity to invest in the market. This is the kind of factor the investor must consider when making investment decisions.

Country Conditions**Political and economic structure**

Another factor to consider when completing an analysis is the conditions of the country in which the company conducts its operations. The political structure of a country may possibly affect the value of the company. Some governments are more friendly toward large firms than other governments. One country may treat a certain industry more favorably than another country.

The future economic factors of the country are also important to consider in an analysis. Inflation, investment regulations, and taxation rules are factors that may affect the value of a company and should be considered when making assumptions concerning the future prospects of the company.

Industry Conditions

It is difficult to analyze a company without studying the industry in which it competes. Most analysts have an understanding of industry conditions and a company's competitors — issues that must be thoroughly considered when making assumptions.

Example

For example, a company has had growth rates of about 25% per year for the past five years. If the analyst knows that there will be several new competitors in the industry and the potential number of buyers for the product is limited, can the analyst reasonably expect similar growth rates in the future? These are the types of questions that are asked and answered by analysts when making assumptions concerning the future.

Synergies

Whole is worth more than sum of parts

In conceptual terms, synergy refers to the phenomenon of the whole being worth more than the sum of the parts. For example, two companies combined into one may be worth more than both companies as stand-alone entities. The combined company may be able to eliminate some duplicate costs and operations or combine functions to increase profitability.

These synergies are often discovered when undertaking a thorough analysis of the operations of the companies. In a buyout scenario, one company may be willing to pay more for a potential takeover target than another bidding competitor because of these potential synergies. However, synergies often are more perceived than realized, so care should be taken when adding a premium to an investment opportunity just because of potential synergies.

UNIT SUMMARY

We began this unit with the concept of forecasting cash flows and discounting them by using the cost of capital. You also saw how to estimate the residual value of the firm after the forecast period.

Finally, you learned some other methods for estimating the value of a company. These methods were compared to the discounted cash flow method and their relative strengths and weaknesses were discussed. We also discussed factors that may affect the value of the company. These factors are considered when making assumptions about a company's future prospects. They often are used to help assess whether assumptions are reasonable and valid.

You have completed Unit Eight: *Corporate Valuation – Estimating Corporate Value* and you have demonstrated your ability to complete the calculations presented in this unit. Please complete Progress Check 8 to check your understanding of the concepts, then continue to Unit Nine: *Fixed Income Securities*. If you answer any questions incorrectly, please review the appropriate text.

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✓ PROGRESS CHECK 8

Directions: Select the correct answer for each question. Check your solution with the Answer Key on the next page.

1. An estimate of future free cash flows:

- _____ a) indicates the amount of cash generated by a company's operations that is available to debt and equity investors.
- _____ b) provides a summary of a company's positive and negative cash flows.
- _____ c) assumes a 0% growth in a company's operations.
- _____ d) provides a much more accurate estimate of future cash flows when calculated by analysts outside the company.

2. A project that is completed at the end of the forecast period:

- _____ a) has residual value that is equal to the value of the project.
- _____ b) has a residual value of \$0.00.
- _____ c) has a residual value equal to the sale price.
- _____ d) requires that the perpetuity method be used to calculate residual value.

3. The appropriate discount rate to use in the discounted cash flow method for estimating corporate value is the:

- _____ a) risk-free market interest rate.
- _____ b) investors' required rate of return.
- _____ c) weighted average of the possible returns.
- _____ d) weighted average cost of capital.

ANSWER KEY

1. An estimate of future free cash flows :
 - a) **indicates the amount of cash generated by a company's operations that is available to debt and equity investors.**

2. A project that is completed at the end of the forecast period:
 - b) **has a residual value of \$0.00.**

3. The appropriate discount rate to use in the discounted cash flow method for estimating corporate value is the:
 - d) **weighted average cost of capital.**

PROGRESS CHECK*(Continued)*

4. The sum of the discounted projected cash flows equals the:
- _____ a) market value of the equity.
 - _____ b) market value of current liabilities plus long-term debt.
 - _____ c) corporate value.
 - _____ d) value of preferred stock plus the value of equity.
5. An analyst has projected cash flows for ABC Corporation based on assumptions about growth in sales and profit margin and increases in capital investments. The analyst decides to change the assumption about sales growth and redo the projection. The analyst is:
- _____ a) changing the company's strategy for future sales growth.
 - _____ b) isolating the sensitivity of the corporation's value to the estimate of the future growth rate.
 - _____ c) estimating the probabilities of each scenario.
 - _____ d) effecting an increase in the market price of the stock.
6. Select **four** factors a company cannot control that may affect the company's estimated future value.
- _____ a) Market liquidity, synergies, industry conditions, management profile
 - _____ b) Country conditions, market liquidity, pooling of resources, tax increases
 - _____ c) Market liquidity, synergies, country conditions, industry conditions
 - _____ d) Management profile, tax increases, market liquidity, synergies

ANSWER KEY

4. The sum of the discounted projected cash flows equals the:
- c) corporate value.**
5. An analyst has projected cash flows for ABC Corporation based on assumptions about growth in sales and profit margin and increases in capital investments. The analyst decides to change the assumption about sales growth and redo the projection. The analyst is:
- b) isolating the sensitivity of the corporation's value to the estimate of the future growth rate.**
6. Select **four** factors a company cannot control that may affect the company's estimated future value.
- c) Market liquidity, synergies, country conditions, industry conditions**

PROGRESS CHECK*(Continued)*

7. Match each of the valuation methodologies in Column A with its definition in Column B.

Column A

___ Dividend value

___ Market / book ratio

___ Price / earnings ratio

___ Liquidation value

Column B

a) Amount investors are paying for each dollar of income generated by the firm

b) Amount left for shareholders after assets are sold and creditors are paid

c) Amount investors are paying relative to value of the company according to accounting methods

d) Value of equity based on value of perpetuity

ANSWER KEY

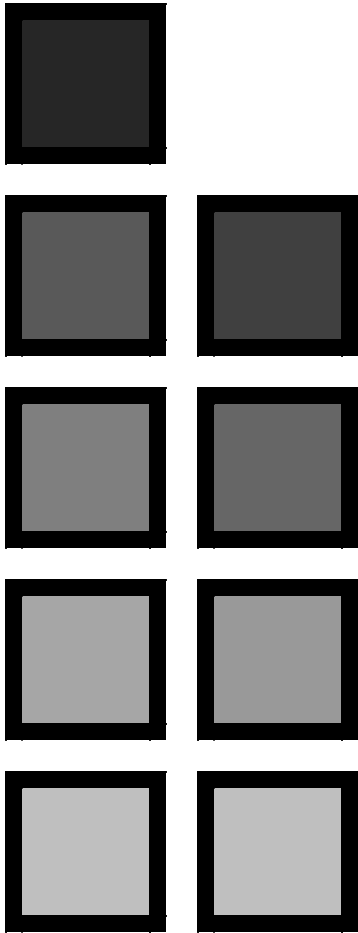
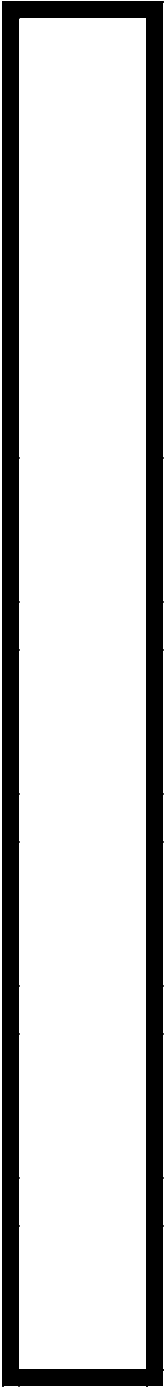
7. Match each of the valuation methodologies in Column A with its definition in Column B.

Column A

- d** Dividend value
c Market / book ratio
a Price / earnings ratio
b Liquidation value

Column B

- a) Amount investors are paying for each dollar of income generated by the firm
b) Amount left for shareholders after assets are sold and creditors are paid
c) Amount investors are paying relative to value of the company according to accounting methods
d) Value of equity based on value of perpetuity



Unit 9

UNIT 9: FIXED INCOME SECURITIES

INTRODUCTION

In Unit Four, we introduced the pricing of fixed income securities (bonds) and explained how to compute the present value of coupon payments and principal repayment. The important point to remember is that a bond in the open market trades for a price equal to its present value.

The focus of this unit is on the calculations that are used to price bonds and determine their yields. Of all debt instruments available worldwide, U.S. Treasury securities (debt instruments) represent the largest market, are the most liquid, and are considered to be of the highest credit quality (i.e., free from default risk). Furthermore, many companies throughout the world use U.S. Treasuries as part of their investment portfolios.

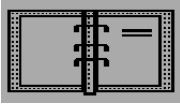
Throughout this unit, U.S. Treasury securities are used in pricing and yield examples designed to illustrate the various principles and calculations. An understanding of U.S. Treasuries will provide a foundation for your continuing study of more complex and risky debt securities. It is important to remember that given matching characteristics, the calculations used to price U.S. Treasury instruments are applicable to any other type of debt security.



UNIT OBJECTIVES

When you have successfully completed this unit, you will be able to:

- Select the correct formula for computing the price of a bond
- Recognize the relevant data needed to calculate the price, discount rate, and yield of Treasury securities and other debt instruments
- Convert yields of various debt instruments to bond equivalent yields (BEY)
- Calculate the duration of a bond and a portfolio of bonds



INTRODUCTION TO BOND PRICING AND YIELD MATHEMATICS

Nonmarketable and marketable securities

Through the Treasury Department, the United States Government issues many types of debt securities for purchase by investors. Instruments that do not have a secondary market (e.g. U.S. savings bonds and retirement bonds) are considered to be nonmarketable. They are purchased directly from the government and held until maturity.

In this unit, we will discuss the pricing of debt instruments that are classified as **marketable** — that is, they are bought and sold in a secondary market. Specifically, we will discuss:

- Treasury bills
- Treasury notes
- Treasury bonds
- Eurobonds

U.S. Treasury Bills

Zero-coupon instruments with maturity under a year

The shortest term debt instruments are **U.S. Treasury Bills**. These bills are issued in maturities of 13, 26, and 52 weeks. The government holds an auction every week for the purchase of 13- and 26-week bills, which are **zero-coupon** instruments. Coupon payments are not made during the life of these bills, and they are bought and sold on a discount basis — often referred to as the **discount rate** or yield of the bill.

The 52-week bills are auctioned every four weeks. Semi-annual interest payments are made on these bills.

Pricing U.S. Treasury Bills

Non-interest bearing security

To calculate the price of zero-coupon Treasury bills or any non-interest bearing security, we use the following formula:

$$P = F [1 - ((R \times D_M) / 360)]$$

Where:

P	=	Current price of the security
F	=	Face value of the security
D_M	=	Number of days until the security matures (U.S. Treasury bills assume a 360-day year)
R	=	Discount yield (yield to maturity)

Example

To illustrate the use of this formula, consider the following question. At what price would we expect a \$1,000 Treasury bill to be trading in the market with 64 days to maturity and a 6.5% discount **yield to maturity**? Apply the formula and perform the computations. In this example, $D_M = 64$ and $R = 0.065$.

$$\begin{aligned} P &= F [1 - ((R \times D_M) / 360)] \\ P &= \$1,000 [1 - ((0.065 \times 64) / 360)] \\ P &= \$1,000 [1 - 0.01166] \\ P &= \$1,000 [0.98844] \\ P &= \$988.44 \end{aligned}$$

We would expect the bill to sell for \$988.44 in the bond market. This price would appear in the financial section of *The Wall Street Journal* as 98.84. The quoted price is a percentage of the selling price.

Calculating the Discount Rate

There may be times when the price of the security is known but the discount rate used in pricing the instrument is unknown. To calculate the rate, we use the pricing formula and solve for R. The variables have the same value as defined for the pricing formula. The formula for calculating the discount rate is:

$$R = [(F - P) / F] \times (360 / D_M)$$

Example

For example, suppose that a 45-day, \$1,000 Treasury bill is priced at \$988. Find the discount rate that was used to price the security. To solve, let $F = \$1,000$, $P = \$988$, and $D_M = 45$ days.

$$\begin{aligned} R &= [(F - P) / F] \times (360 / D_M) \\ R &= [(\$1,000 - \$988) / \$1,000] \times (360 / 45) \\ R &= [0.0120] \times (8.00) \\ R &= 0.0960 \text{ or } 9.60\% \end{aligned}$$

Yield to maturity

A discount rate of 9.60% was used to calculate the price of the Treasury bill (\$988). This rate is often referred to as the yield to maturity of the security. Some investors might say, "The security is priced to yield 9.60%."

Rate of Return

True (actual) yield

Because an investor pays less than face value for a zero-coupon (discounted) security, the investor earns a rate of return on the investment. This rate of return, which is based on the difference between the purchase price and sale price, is always greater than the discount rate, which is based on the face value of the security.

One convention is to state this rate of return on a 365-day year basis. The rate of return is the simple interest earned on the investment, which is often referred to as the **true (actual) yield** of the bill. The formula is:

$$i = [(P_s - P_b) / P_b] \times [365 / D_H]$$

Where:

i = Simple annual interest (rate of return) earned on a 365-day year basis

P_s = Price at which security was sold

P_b = Price at which security was bought

D_H = Number of days security was held

Example

Suppose an investor purchases a \$10,000, 90-day Treasury bill for \$9,600 and then sells it after 30 days for \$9,700. What is the annual rate of return (simple interest) earned by the investor? Apply the formula with these values: $P_s = \$9,700$, $P_b = \$9,600$, and $D_H = 30$.

$$\begin{aligned} i &= [(P_s - P_b) / P_b] \times [365 / D_H] \\ i &= [(\$9,700 - \$9,600) / \$9,600] \times [365 / 30] \\ i &= [0.0104] \times [12.1667] \\ i &= 0.1267 \text{ or } 12.67\% \end{aligned}$$

The investor has earned an annual rate of return of 12.67% for a 30-day investment in the Treasury bill. If the investor holds the bill to maturity, then the face value of the security is used as the selling price when calculating the simple interest earned.

Bond equivalent yield

The true or actual yield is sometimes called the **bond equivalent yield (BEY)** of the Treasury bill. This yield allows for direct comparison to a Treasury note with bond yields that are priced on a 365-day year basis and make semi-annual coupon payments. Comparisons should be made between instruments with the same amount of time to maturity. For example, the yield on a Treasury bill with 30 days to maturity can be compared to the yield on a Treasury note or bond also with 30 days until maturity.

The calculation of a bond equivalent yield for Treasury bills with maturities between six months and one year is complicated by the fact that Treasury notes and bonds make coupon payments during the period. That formula will not be given here, but be aware that an adjustment is necessary for the comparison to be valid.

Money market yield

The term money market yield (MMY) refers to the rate of return earned during a 360-day year, rather than a 365-day year. The calculation is identical to the true yield calculation, except that we substitute 360 for 365 in the formula. Certificates of deposit (CD) and other short-term instruments are often quoted on a money market yield basis.

Summary

U.S. Treasury Bills are short-term (up to one year) debt securities issued through the U.S. Treasury Department. They are marketable securities because they are bought and sold in a secondary market.

Zero-coupon securities are those for which no coupon payments are made during the life of the securities. They are bought and sold on a discounted basis.

The price of a non-interest bearing security is calculated with this formula:

$$P = F [1 - ((R \times D_M) / 360)]$$

Yield to maturity is the discount rate of a security. The formula for calculating the discount rate is:

$$R = [(F - P) / F] \times (360 / D_M)$$

The rate of return on a zero-coupon security is greater than the discount rate. The true (actual) yield is based on a 365-day year and is calculated as:

$$i = [(P_S - P_b) / P_b] \times (365 / D_H)$$

Money market yield (MMY) refers to a rate of return based on a 360-day year.

Before we continue to the next section regarding U.S. Treasury notes and bonds, please complete the Practice Exercise which follows.

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PRACTICE EXERCISE 9.1

Directions: Use the list of formulas to solve Exercises 1- 4. Perform the calculations to find the correct solutions. Check your solutions with those of the Answer Key.

List of Formulas

$$(1) \quad P = F [1 - ((R \times D_M) / 360)]$$

$$(2) \quad i = [(P_s - P_b) / P_b] \times [365 / D_H]$$

$$(3) \quad R = [(F - P) / F] \times (360/D_M)$$

Data	Solutions
<p>1) Compute the price of a 26-week Treasury bill that has:</p> <ul style="list-style-type: none"> ▪ A face value of \$10,000 ▪ A yield to maturity of 4.75% ▪ 60 days until maturity <p>1a) Now, consider that this bill is held until maturity. Compute its actual yield.</p>	
<p>2) Compute the price on a 13-week Treasury bill that has:</p> <ul style="list-style-type: none"> ▪ 24 days until maturity ▪ A 3.25% discount rate ▪ A \$1,000 face value 	
<p>3) Compute the discount rate used to price a 26-week Treasury bill that has:</p> <ul style="list-style-type: none"> ▪ A \$10,000 face value ▪ A price at time of issue of \$9,400 	
<p>4) Compute the yield to maturity of a 13-week Treasury bill that has:</p> <ul style="list-style-type: none"> ▪ A \$1,000 face value ▪ 45 days until maturity ▪ A current price of \$987.50 	

ANSWER KEY1) **\$9,920.83***Use the zero-coupon pricing formula with $D_M = 60$, $R = 0.0475$, and $F = \$10,000$*

$$\begin{aligned}
 P &= F [1 - ((R \times D_M) / 360)] \\
 P &= \$10,000 [1 - ((0.0475 \times 60) / 360)] \\
 P &= \$10,000 [1 - 0.0079] \\
 P &= \$9,920.83
 \end{aligned}$$

1a) **4.85%***Use the rate of return (bond equivalent) formula with $P_s = \$10,000$, $P_b = \$9,920.83$, and $D_H = 60$.*

$$\begin{aligned}
 i &= [(P_s - P_b) / P_b] \times [365 / D_H] \\
 i &= [(\$10,000 - \$9,920.83) / \$9,920.83] \times [365 / 60] \\
 i &= [0.0080] \times [6.0833] \\
 i &= 0.0485 \text{ or } 4.85\%
 \end{aligned}$$

2) **\$997.83***Use the zero-coupon pricing formula with $D_M = 24$, $R = 0.0325$, and $F = \$1,000$*

$$\begin{aligned}
 P &= F [1 - ((R \times D_M) / 360)] \\
 P &= \$1,000 [1 - ((0.0325 \times 24) / 360)] \\
 P &= \$1,000 [1 - 0.0022] \\
 P &= \$997.83
 \end{aligned}$$

3) **11.87%***Use the discount rate formula with $F = \$10,000$, $P = \$9,400$, and $D_M = 182$ (26 weeks \times 7 days/week)*

$$\begin{aligned}
 R &= [(F - P) / P] \times (360 / D_M) \\
 R &= [(\$10,000 - \$9,400) / \$10,000] \times (360 / 182) \\
 R &= [0.0600] \times [1.9780] \\
 R &= 0.1187 \text{ or } 11.87\%
 \end{aligned}$$

4) **10.00%***Use the discount rate formula with $F = \$1,000$, $P = \$987.5$, and $D_M = 45$*

$$\begin{aligned}
 R &= [(F - P) / P] \times (360 / D_M) \\
 R &= [(\$1,000 - \$987.5) / \$1,000] \times (360 / 45) \\
 R &= [0.01250] \times [8.0000] \\
 R &= 0.1000 \text{ or } 10.00\%
 \end{aligned}$$

Semi-annual payments and maturities from 1 to 10 years

U. S. Treasury Notes

Treasury notes are securities (bonds) with original maturities ranging from 1 to 10 years. These debt instruments make semi-annual interest payments at a coupon rate that is quoted on a per annum basis. The notes are auctioned at the time of issue. Two-year notes are issued monthly. Three-, four-, five-, and seven-year notes are issued quarterly, but on different cycles.

Pricing U.S. Treasury Notes

To refresh your memory, you learned in Unit Four that the formula for pricing a semi-annual, coupon-paying instrument is:

$$V = C[1 / (1 + R)]^1 + C[1 / (1 + R)]^2 + \dots + C[1 / (1 + R)]^T + F[1 / (1 + R)]^T$$

Where:

- V = Present value of the bond
- C = Coupon payment (coupon rate multiplied by face value)
- R = Discount rate (current prevailing rate)
- F = Face value of the bond
- T = Number of compounding periods until maturity

Now we will customize this formula so that it can be applied to any semi-annual, interest-paying bond, such as the U.S. Treasury note. To do so, we combine all coupon payments into one summation. The formula can now be expressed as:

$$P = \sum_{i=1}^{2T} \frac{(C / 2)}{[1 + (Y / 2)]^i} + \frac{F}{[1 + (Y / 2)]^{2T}}$$

Where:

- P = Price of the bond
- C = Annual coupon payment
- Y = Annual yield to maturity rate (discount rate)
- F = Face value of the bond
- T = Number of years until maturity
- Sigma = Sum of the series
- i = 1 = Series begins at first possible occurrence

Since all U.S. Treasuries are quoted in terms of annual discount rates and annual coupon payments, we must adjust for the semi-annual interest payments. What we essentially are doing is discounting the interest payments (C) at the quoted yield to maturity rate (Y). In a five-year note, there are 10 interest payments made before maturity. The last term in the formula discounts the principal over the life of the note.

Example

An example may help you understand these adjustments. Suppose we want to find the price of a \$1,000 face value note that has:

- A 5-year maturity
- An annual coupon payment of \$80
- An annual yield to maturity of 9.06%
- Ten interest payments that will be made during the life of the note

Two methods for finding the price of a note

There are two methods that can be used to compute the price of the note. Both require the use of a financial calculator. One involves the use of the formula, the other uses the calculator's bond function.

Formula with calculator

- 1) Use the formula with your financial calculator, as we did when computing the present value of a set of cash flows.

Enter:

- Number of payments as 10
- Size of the payments as \$40
- Interest rate as 4.53%
- Future value as \$1,000

Press the *present value* key.

Your answer should be \$958.12, the present value of the note.

**Calculator
bond function**

- 2) Use the bond functions of your calculator. Make sure that your calculator is set to the proper type of bond. In this case, it is a semi-annual bond on a 365-day year basis.

Enter:

- Maturity date of the bond (the date five years from now)
- Annual coupon rate of 8.0%
- Annual yield to maturity of 9.06%

Press the *price* key.

The note price displayed should be 95.8124. This is a percentage price quote. To find the dollar price, divide 95.8124 by 100 and multiply that quotient by the face value $[(95.8124/100) \times \$1,000 = \$958.12]$.

Check the owner's manual that came with your calculator for instructions about entering the appropriate data.

Calculating Yield to Maturity

At times, you may know the price of the security, but wish to calculate its yield to maturity. Use your calculator to save time. Enter the bond information, then press the *yield* key to compute the answer.

Example

Suppose that the price of a seven-year \$10,000 security is \$9,560. The annual coupon is \$900, and the coupon payments are made semi-annually. What is the yield to maturity of this security?

Make sure that you set the type of the bond to accurately reflect the bond in our example — a 365-day year, semi-annual bond. Using your calculator, enter 14 for the number of payments, enter 9% as the annual coupon rate, and 95.6 as the percentage price of the security. Press the *yield* key. Your answer should be 4.9429%, which is the semi-annual yield to maturity. To find the bond equivalent rate, multiply your answer by 2. The result should be 9.8858% — the yield to maturity of the bond, often referred to as the bond equivalent yield (BEY).

U.S. Treasury Bonds

Maturities greater than 10 years – semi-annual payments

Treasury bonds are securities with maturities greater than ten years. The most common maturities are 20 and 30 years. Like Treasury notes, Treasury bonds are coupon instruments that make semi-annual interest payments to investors. These bonds are also issued at a quarterly auction. The price and yield relationships and calculations for Treasury bonds are the same as they are for Treasury notes, with one exception – some original issue 30-year bonds are callable at par plus **accrued interest** during the last five years of maturity.

Call Provision

Issuer option to pay off bond

The U.S. Government and many companies issue bonds (usually long-term) with a call provision, which gives the issuer the option to pay off the bond at some time before the maturity date. The call provision specifies when, and at what price, the bonds may be called. In the case of Treasury bonds, the call price is usually par (face value), and the call date may be any time during the last five years of the bond. The rate of return on callable bonds is often referred to as "yield to first call." Let's see how a call provision might work.

Example

Suppose that in January of 1969, the U.S. Government issued 30-year bonds to investors that were callable at par at any time during the last five years of the life of the bonds. The bonds carried an annual coupon of 5.5%.

Now it's January of 1994 and current 30-year securities have a 4.5% annual coupon. The old bonds are selling for \$1,044.33 per bond; they are selling at a premium (above par) because interest rates have fallen from 5.5% to 4.5%.

In this case, the Government is making higher interest payments on the old bonds than on new issues. The government exercises the call provision and pays investors par value (\$1,000 per bond) plus any interest accrued since the last interest payment. The investors lose the opportunity to sell the bonds at the premium price. They also lose the stream of interest payments at 5.5%, which they would have received until the bonds matured.

**Callable
bonds are
less valuable**

Therefore, bonds with call provisions are less valuable to investors than bonds that are not callable. If the bonds are selling at a premium, the convention is to price the bond using the date of first call as the maturity date. If the bond is at a discount, then the original maturity date is the one used in price and yield calculations. Many financial calculators now allow the user to input the date of the first call of a callable bond and will price the bond accordingly.

Summary

Treasury notes are securities with maturities ranging from 1 year to 10 years and are semi-annual coupon paying securities. Pricing is done using the bond function of a financial calculator.

Treasury bonds also pay coupons on a semi-annual basis, but have maturities greater than 10 years, usually 20 or 30 year maturities.

A call provision gives the issuer of a bond the option to pay off the bond before the maturity date.

Please complete Practice Exercise 9.2 before continuing to the sections on "Eurobonds," "Other Bonds," and "Accrued Interest."

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PRACTICE EXERCISE 9.2

Directions: Use the bond function of your financial calculator to compute the correct answer to each problem. Check your solutions with the Answer Key on the next page.

5. The U.S. government plans to issue new 20-year \$10,000 face value bonds with a coupon rate of 8.5%. The bonds will be priced to yield 8.75%. The expected price of the bonds when they are issued will be \$_____.

6. On April 5, 1971, a \$10,000 face value Treasury bond was available to investors for \$9,250. The bond had a coupon rate of 4.25%, and its maturity date was January 23, 1998. The yield to maturity of the bond for investors was _____%.

7. A new issue of 3-year, \$1,000 face value Treasury notes is available to investors. The 3-year notes have a coupon rate of 6.75% and are priced to yield 6.75%. The price investors will pay for the notes is \$_____.

ANSWER KEY

5. The U.S. government plans to issue new 20-year \$10,000 face value bonds with a coupon rate of 8.5%. The bonds will be priced to yield 8.75%. The expected price of the bonds when they are issued will be **\$9,765.82.**

Use the bond features on your calculator to solve this problem.

Enter:

- 8.5% as the coupon rate*
- 8.75% as the yield to maturity*
- 20 as the number of years until maturity*

Press the price key and you should get 97.6582 as the price. Remember that this is the percentage price.

To get the dollar price, divide 97.6582 by 100 and then multiply this figure by the face value of \$10,000 for a \$9,765.82 dollar price.

6. On April 5, 1971, a \$10,000 face value Treasury bond was available to investors for \$9,250. The bond had a coupon rate of 4.25%, and its maturity date was January 23, 1998. The yield to maturity of the bond for investors was **4.75%.**

Use the bond features on your calculator to solve this problem. Enter 4.25% as the coupon rate. Enter April 5, 1971 as the purchase date and January 23, 1998 as the maturity date. You will probably need to convert the dollar price of the security into a percentage price. To do so, divide the dollar price of \$9,250 by the face value of \$10,000 and multiply the quotient by 100. Enter 92.50 as the percentage price and press the yield to maturity key. You should get 4.7471 or 4.75% as the yield to maturity.

7. A new issue of 3-year, \$1,000 face value Treasury notes is available to investors. The 3-year notes have a coupon rate of 6.75% and are priced to yield 6.75%. The price investors will pay for the notes is **\$1,000.00.**

Enter the appropriate dates for three years maturity; enter 6.75% for both the coupon rate and the yield to maturity. When you press the price key, you will get 100.00 as the percentage price which means that the security will sell at face value.

Hint: *This problem can be solved without a calculator. Just remember that when the discount rate is equal to the coupon rate, the bond will sell at par (face value).*

Eurobonds

Issued in an international market

Eurobonds represent another large group of fixed income securities. A Eurobond is a generic term used to describe a debt instrument issued by a company in an international market that is outside any domestic regulations.

Annual coupon payments

Eurobonds typically make annual coupon payments to the investor – generally made on a 365-day year basis. We can calculate the price of a Eurobond, or any annual coupon paying instrument, with this formula:

$$P = \sum_{i=1}^T \frac{C}{(1+Y)^i} + \frac{F}{(1+Y)^T}$$

Where:

- P** = Current price of the bond
- C** = Annual coupon payment
- Y** = Annual yield to maturity rate (discount rate)
- F** = Face value of the bond
- T** = Number of years until maturity
- \sum = Sum of the series
- i = 1** = Series begins at first possible occurrence

Remember, the price of bonds may be found using your financial calculator — it is not necessary to memorize all of these pricing formulas. The important point of this section is to identify the proper variables to enter into your calculator so that you can compute the answer. The formulas are used as reference points to enhance your understanding of the concepts.

Use the bond functions of your calculator to solve for the price of the Eurobond described in the next example.

Example

What is the price of a 7-year Eurobond with \$10,000 par value, coupon rate of 7.8%, and priced to yield 6.4%?

To solve this problem, enter the appropriate dates for a seven year maturity, enter 7.8% as the coupon rate and 6.4% as the yield to maturity. You should get \$107.7054 when you push the *price* key. Since this is the percentage price, divide it by 100 and then multiply this by the face value of \$10,000 to get the actual sale price of \$10,770.54.

Comparing U.S. bonds to Eurobonds

U.S. Treasury securities make semi-annual interest payments and Eurobonds make annual interest payments. Now the question becomes: Which bond offers the investor the higher return on investment? Clearly, we need a way to compare the two investments.

Suppose that we are considering a U.S. Treasury bond with a 6.5% yield to maturity and a Eurobond with a 6.75% yield to maturity. There is a simple formula that relates the two quoted yields. It is:

$(1 + Y_E) = [1 + (Y_T / 2)]^2$	
Where:	
Y_E	= Annual Eurobond yield to maturity
Y_T	= Annual Treasury bond yield to maturity

Example

Using this formula, the Eurobond annual yield that is equivalent to the Treasury yield of 6.5% is:

$$\begin{aligned} (1 + Y_E) &= [1 + (Y_T / 2)]^2 \\ (1 + Y_E) &= [1 + (0.065 / 2)]^2 \\ (1 + Y_E) &= [1 + 0.0325]^2 \\ (1 + Y_E) &= 1.0661 \\ Y_E &= 0.0661 \text{ or } 6.61\% \end{aligned}$$

The annual 6.5% yield to maturity on the Treasury security is equivalent to a 6.61% annual yield to maturity for a Eurobond. Therefore, the 6.75% Eurobond appears to be the better investment, all other things being constant.

Other Bonds

Make certain comparisons are valid

Like the U.S. Treasury, many other governments issue bonds. Corporations also issue bonds for investors to purchase. The calculations for the price and yield of these bonds are based on the same principles as for Treasury securities. Make certain that comparisons between securities are valid. For example, an adjustment to computations is needed to allow direct and valid comparison of a corporate bond that pays semi-annual coupons, on a 360-day year, to a Treasury bond which is priced on a 365-day year.

Accrued Interest

Additional adjustment for true selling price

To find the true selling price of a security that makes periodic interest payments, one other adjustment is required. Because it is common for a bond or note to sell on a date other than the date interest is paid, the buyer will pay the seller the interest accrued from the date the last coupon payment was made until the security is sold. Otherwise, the buyer will receive the entire interest payment even though s/he owned the bond for only part of the coupon period.

Semi-annual coupon payments

The convention is to adjust the coupon payment by the number of days in the coupon period that the security is actually held by the seller. We use this formula:

$$AI = (C / 2) \times (D_H / D_C)$$

Where:

AI = Accumulated interest on the security

C = Annual coupon payment

D_H = Number of days that the security was held

D_C = Number of days in the coupon period

Number of days in coupon period The number of days in the coupon period can vary from 181 to 184, depending on the dates the coupon is paid. For example, if the coupon payments are made on May 15 and November 15 of each year, there are 184 days from May 15 to November 15 and 181 days from November 15 to May 15.

Annual coupon payments However, if the security pays one annual coupon (such as a Eurobond), then two adjustments to this formula are needed. These adjustments are: (1) the annual coupon is not divided by 2 and (2) the number of days in the coupon period will be 360 or 365, depending on the type of bond and its conventions.

Examples Two examples will help demonstrate the use of these formulas.

1. What is the accumulated interest on a \$10,000 face value 9.8% coupon Treasury bond that has 130 days until the next coupon payment, and the coupon payment period has 182 days? Use the formula with \$980 as the annual coupon payment, and 52 (182 day coupon period minus 130 days remaining in the period) as the number of days that the security has been held.

$$AI = (C / 2) \times (D_H / 182)$$

$$AI = (\$980 / 2) \times (52 / 182)$$

$$AI = \$140.00$$

The buyer of the security would have to add \$140.00 to the quoted price of the security to compensate the seller for the 52 days in the coupon period that the seller owned the security.

2. What is the accumulated interest for a \$1,000 face value 6.5% Eurobond that was held for 270 days since the last coupon payment? In this case, the annual coupon payment is \$65 dollars and $D_H = 270$.

$$AI = C \times (D_H / 365)$$

$$AI = (\$65) \times (270 / 365)$$

$$AI = \$48.08$$

The seller would receive the transaction price plus \$48.08 compensation for interest earned from the last coupon payment until the security was sold.

BOND PRICE QUOTES

Newspaper source

Most newspapers contain a financial section that quotes recent stock and bond prices. *The Wall Street Journal* is generally considered a very good source for financial information.

Treasury Bill Quotes

Figure 9.1 contains price information for U.S. Treasury bills as it may appear in the financial section.

TREASURY BILLS					
<u>Maturity</u>	<u>Days to Mat.</u>	<u>Bid</u>	<u>Asked</u>	<u>Chg.</u>	<u>Ask Yld.</u>
Jan 13 '94	1	3.06	2.06	+0.07	3.00
Jan 20 '94	8	2.97	2.87	2.91
Jan 27 '94	15	2.90	2.80	+0.01	2.84
Feb 03 '94	22	2.90	2.80	+0.01	2.84
Feb 10 '94	29	2.90	2.80	-0.01	2.85
Feb 17 '94	36	2.89	2.85	-0.01	2.90
Feb 24 '94	43	2.89	2.85	-0.01	2.90
Mar 03 '94	50	2.86	2.82	-0.02	2.87
Mar 10 '94	57	2.92	2.88	-0.03	2.93
Mar 17 '94	64	2.93	2.91	-0.03	2.97
Mar 24 '94	71	2.94	2.92	-0.03	2.98
Mar 31 '94	78	2.93	2.91	-0.02	2.97
Apr 07 '94	85	2.99	2.97	-0.02	3.03

Figure 9.1: Price Quotes for U.S. Treasury Bills

An explanation of each column of information beginning with the top left column is as follows:

<i>Maturity:</i>	The maturity date of the bill
<i>Days to Mat.:</i>	Number of days until the security matures
<i>Bid:</i>	Discount rate that buyers are asking for when purchasing the bill
<i>Asked:</i>	The discount rate that sellers are requesting when they sell the bill

The difference between these two rates is often referred to as the bid/ask spread. The transaction price will occur somewhere between these two rates.

<i>Chg.:</i>	The net change in the asked discount rate from the previous trading day
<i>Ask Yld.:</i>	The yield to maturity of the bill, based on the asked discount rate

Example

An example will illustrate the relationships between the columns of information in Figure 9.1. Follow the instructions.

Maturity: Beginning at the top of Column 1, read down the column to find the bill that matures on Feb 10, '94.

Now read across the row.

<i>Days to Mat.:</i>	This bill has 29 days until maturity.
<i>Bid:</i>	The bid discount rate is 2.90%.
<i>Asked:</i>	The asked rate is 2.80%.
<i>Chg.:</i>	This indicates that the asked rate is down 1/100 of 1% from the last price quote (the day before). The term basis point refers to 1/100 of 1%. An analyst might say, "The asked rate is one basis point less than the previous day."
<i>Ask Yld.:</i>	The yield to maturity of the bill, based on the asked rate, is 2.85%.

Verify the yield to maturity

We can verify the yield to maturity for this bill using the simple interest formula. To make that calculation, we first need to find the dollar price of the bill.

$$P = F [1 - ((R \times D_M) / 360)]$$

$$P = \$10,000 [1 - ((0.028 \times 29) / 360)]$$

$$P = \$9,977.44$$

Now, use a face value of \$10,000 as the selling price of the bill and use the simple interest formula.

$$i = [(P_s - P_b) / P_b] \times [365 / D_H]$$

$$i = [(\$10,000 - \$9,977.44) / \$9,977.44] \times [365 / 29]$$

$$i = 0.0285 \text{ or } 2.85\%$$

This is the yield to maturity of the Feb 10, '94 bill, which corresponds to the Ask Yld. column in the figure.

Treasury Note and Bond Quotes

Newspaper source

Price quotes for U.S. Treasury notes and bonds are shown in Figure 9.2. Let's see how they may appear in the financial section of the newspaper.

<u>Rate</u>	<u>Maturity Mo/Yr</u>	<u>Bid</u>	<u>Asked</u>	<u>Chg.</u>	<u>Ask Yld.</u>
6	Nov 94n	102:01	102:03	3.45
8 1/4	Nov 94n	103:28	103:30	3.46
10 1/8	Nov 94	105:13	105:15	3.47
11 5/8	Nov 94n	106:23	106:25	+ 1	3.38
4 5/8	Nov 94n	100:30	101:00	3.46
4 5/8	Dec 94n	101:00	101:02	+ 1	3.50
7 5/8	Dec 94n	103:27	103:29	+ 1	3.48
8 5/8	Jan 95n	104:29	104:31	3.56
4 1/4	Jan 95n	100:21	100:23	+ 1	3.55
3	Feb 95	100:10	101:10	1.78
5 1/2	Feb 95n	101:31	102:01	+ 1	3.59
7 3/4	Feb 95n	104:12	104:14	+ 1	3.57
10 1/2	Feb 95	107:11	107:13	+ 3	3.53
11 1/4	Feb 95n	108:06	108:08	+ 3	3.49

Figure 9.2: Price Quotes for U.S. Treasury Notes and Bonds

The columns provide the following information:

<i>Rate:</i>	Coupon rate being paid by the security
<i>Maturity Mo/Yr:</i>	Maturity month and year. [An (n) next to the year indicates that the security is a note.]
<i>Bid:</i>	Bid price quoted in percentage terms
<i>Asked:</i>	Asked price quoted in percentage terms (The numbers following a colon mean "32nds of a dollar." Therefore, a price quote of 101:22 means that the price of the security is 101 22/32.)
<i>Chg:</i>	Net price change in the asked price from the previous day, reported in 32nds
<i>Ask Yld.:</i>	Yield to maturity of the security, based on the asked price

Some long-term bonds have call features. If the bond is selling at a premium (above par value), then the yield is calculated using the earliest call date. If the bond is selling at a discount (below par value), then the yield is calculated until maturity.

Verifying yield to maturity

We can calculate the yield to maturity if we know the date (day, month, year) when the security matures. Treasury notes come due on the second and fourth Friday of each month. Let's look at an example to illustrate the relationship.

Example

Use your calculator to verify the yield of a bond in Figure 9.2.

- Locate the Jan 95n security that shows an 8 5/8% coupon rate (Column 2, Row 8).
- Enter the maturity date, Jan. 13, 1995.
- Enter 8.625% (8 5/8%) for the coupon rate.
- Enter 104.96875 (104:31 or 104 31/32) for the current percentage price.
- Press the *yield to maturity* key.

The calculator should display 3.5638% as the yield, which corresponds to the 3.56% quote.

Summary

Eurobonds make up another large group of fixed income debt securities issued in a currency that is different from the currency of the country in which it is issued. The price is calculated using this formula:

$$P = \sum_{i=1}^T \frac{C}{(1+Y)^i} + \frac{F}{(1+Y)^T}$$

Quotes for Treasury bills, notes, and bonds may be found in the financial section of some newspapers.

Interest earned by the seller, during the coupon period in which a bond is sold, is accrued and added to the price paid by the buyer.

You will learn about the significance of bond duration in the final section of this unit. First, however, please practice what you have learned about Eurobonds and accrued interest in the exercise which follows.

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PRACTICE EXERCISE 9.3

Directions: Use the bond features on your calculator to calculate the correct answer to each question. Check your solutions with the Answer Key on the next page.

8. A new Eurobond issue is available to investors. The bonds carry a 7.375% coupon rate, have a maturity of 10 years, and are priced to yield 8.0%.
What price percentage would investors expect to pay for the new bonds? Give your answer in percentage terms.

_____ %

9. A Eurobond with five years to maturity has a percentage price of 97.85%. If the coupon rate of the security is 6.325%, what is the yield to maturity of the bond?

_____ %

ANSWER KEY

8. A new Eurobond issue is available to investors. The bonds carry a 7.375% coupon rate, have a maturity of 10 years, and are priced to yield 8.0%. What price percentage would investors expect to pay for the new bonds? Give your answer in percentage terms.

95.81

Use the bond features on your calculator to solve this problem. Enter 7.375% as the coupon rate and 8.0% as the yield to maturity. Enter the appropriate dates for a 10 year maturity. Press the price key to arrive at the answer 95.8062.

9. A Eurobond with five years to maturity has a percentage price of 97.85%. If the coupon rate of the security is 6.325%, what is the yield to maturity of the bond?

6.85%

Use the bond features on your calculator to solve this problem. Enter 6.325% as the coupon rate, 97.85 as the percentage price of the security, and enter the appropriate dates for a five year maturity. Press the yield to maturity key and you should get 6.8472.

PRACTICE EXERCISE*(Continued)*

10. What is the Treasury security equivalent yield for the Eurobond in Problem 9?

_____ %

11. An investor is considering the purchase of a Treasury note from another investor. The annual coupon of the note is \$90, it has been 36 days since the last coupon payment, and the coupon payment period is 183 days. How much would the new investor have to add to the purchase price of the security to compensate the old investor for accrued interest?

\$ _____

ANSWER KEY

10. What is the Treasury security equivalent yield for the Eurobond in Problem 9?

4.79%

Use the yield equivalence formula with $Y_E = 0.0485$ and solve for Y_T .

$$\begin{aligned} (1 + Y_E) &= [1 + (Y_T/2)]^2 \\ (1 + 0.0485) &= [1 + (Y_T/2)]^2 \\ (1.0485)^{1/2} &= [1 + (Y_T/2)] \\ (1.0240) - 1 &= Y_T/2 \\ 0.0240 \times 2 &= Y_T \\ 0.0479 &= Y_T \text{ or } Y_T = 4.79\% \end{aligned}$$

An annual Eurobond yield of 4.85% is equivalent to an annual Treasury yield of 4.79%.

11. An investor is considering the purchase of a Treasury note from another investor. The annual coupon of the note is \$90, it has been 36 days since the last coupon payment, and the coupon payment period is 183 days. How much would the new investor have to add to the purchase price of the security to compensate the old investor for accrued interest?

\$8.85

Use the accrued interest formula with $C = \$90$ and $D_H = 36$.

$$\begin{aligned} AI &= (C/2) \times (D_H/183) \\ AI &= (\$90/2) \times (36/183) \\ AI &= 45 \times 0.1967 \\ AI &= \$8.85 \end{aligned}$$

The new investor would have to pay the old investor \$8.85 to compensate for the accrued interest.

DURATION

Factors That Affect Bond Prices

In a competitive market, a company's bonds must offer fair expected rates of return to entice investors. The return to the investor will be in the form of price gains during the life of the bond plus the coupon payments made by the issuer of the bond.

Sensitivity to interest rates

In Unit Four, we discussed the relationship between market interest rates and bond prices. We said that as interest rates fall, bond prices rise and vice versa. For example, a bond with an 8% coupon will sell at par (face value) when other competitive yields are also 8%. If the market rate rises to 9%, the bond price will fall until its expected return matches the 9% market rate.

Effect of maturity on sensitivity to interest rates

So, we know that bond prices are sensitive to changes in market interest rates. In addition, the prices of long-term bonds are more sensitive to changes in interest rates than the prices of short-term bonds. Given this relationship between the sensitivity of bond prices to changes in market interest rates and the life of a bond, it is particularly important to have an appropriate definition of the life or term of a bond.

Effective life of the bond

Just as the quoted coupon rate doesn't convey the true yield of a bond, the effective life of a bond is more complicated than just looking at the maturity date. Bond price sensitivity is more appropriately related to the weighted average life of a bond than to the maturity date.

If you were to receive \$1,000 after 20 years and no coupon payments during that time, the effective life of the bond would be 20 years. However, if, in addition to the \$1,000, you receive \$100 per year for each of the 20 years, the weighted average term of the payout is less than 20 years. The higher the coupon payments relative to the par value, the shorter the average life of the payout.

Duration of a Bond

Average maturity of bond's cash flows

Duration is the measure we use to estimate the average maturity of a bond's cash flows. It represents the weighted average life of the bond, where the weights are based on the present value of the individual cash flows relative to the present value of the total cash flows (current price of the bond). The formula for calculating the duration of a bond is:

$$D = \sum_{t=1}^T \frac{[CF_t / (1 + y)^t]}{P}$$

Where:

- D** = Duration of the security
- t** = Time in years until each payment is made
- T** = Time in years until the security matures
- CF_t** = Cash flow (coupon or principal) payments made by the security
- y** = Yield to maturity of the security. If the coupons are semi-annual, then this rate needs to be a semi-annual rate ($y / 2$).
- P** = Price of the security

Example

To illustrate the duration calculation, we will calculate the duration of a three-year note that pays semi-annual coupons and has the following characteristics:

- Percentage selling price of 95.65
- Annual coupon of 7%
- Annual yield to maturity (bond equivalent yield) of 8.678%

Adjust annual coupon rate for semi-annual payments

Because the coupons are paid semi-annually, it is necessary to convert the discount rate of 8.678% to a fully-compounded annual discount rate. To do this, we divide the annual discount rate by two, add one, square the result, and subtract one.

$$\begin{aligned} 0.08678 / 2 &= 0.04339 \\ (1 + 0.04339)^2 - 1 &= 0.0887 = 8.87\% \end{aligned}$$

The fully-compounded annual discount rate is 8.87%. We will use this rate in our calculations. You can see the results arranged in the table in Figure 9.3.

(1) Time	(2) Cash Flow	(3) Present Value at Semi-annual Rate	(4) Present Value / Bond Price	(5) (1) x (4) Time- weighted Value
0.5	3.50	3.3545	0.0351	0.0175
1.0	3.50	3.2150	0.0336	0.0336
1.5	3.50	3.0813	0.0322	0.0483
2.0	3.50	2.9531	0.0309	0.0617
2.5	3.50	2.8303	0.0296	0.0740
3.0	103.50	<u>80.2159</u>	<u>0.8386</u>	<u>2.5159</u>
Total		95.6501	1.0000	2.7511

Figure 9.3: Duration for a Three-year Coupon Bond

**Explanation
of the table**

An explanation of each column will help you understand the calculations.

Time: The time in years from the present time until each coupon payment. The first coupon is paid in 0.5 years, and so on.

Cash Flow: The cash flows in percentage terms that are received as coupon payments.

Present Value at Semi-annual Rate: Each cash flow discounted at the appropriate rate (in this case, it is the fully-compounded annual rate of 0.0887). The present value of the first semi-annual coupon is $[3.5 / (1.0887)^{0.5}] = 3.3545$. The sum of the values in Column 3 should be equal to the market price of the bond. (There may be a slight rounding error.)

Present Value / Bond Price: Weights for each cash flow are determined by dividing the present value of the cash flow by the market price of the bond (95.65). The calculation for the first cash flow is $3.3545 / 95.65 = 0.0351$. The weights in Column 4 should add up to 1.0.

Time-weighted Value: The time until the cash flow is received (Column 1) is multiplied by the weight for the cash flow (Column 4). The first calculation is $0.0351 \times .05 = 0.0175$. The sum of the time-weighted values in Column 5 is the weighted average life (duration) of the bond. In this example, the duration of the 3-year coupon bond is 2.7511 years.

Two points

There are two important points to consider when calculating the duration of a bond:

Appropriate discount rate

1. The discount rate used when discounting cash flows must be appropriate for the type of bond. For example, the bond equivalent yield is the correct rate to use for an annual bond. For a semi-annual bond, the fully-compounded annual discount rate is appropriate.

Consistent use of percentages or dollars

2. It is important to be consistent in the use of percentages or dollar values for cash flows and prices. Either one is appropriate, but do not mix them.

Example

Another example of a duration calculation follows.

What is the duration of a 3-year Eurobond, with a coupon of 7.5% and a percentage price of 96.50? First we calculate the BEY (yield to maturity) of the security. Using your calculator, enter the appropriate dates for maturity, the coupon rate of 7.5%, and the percentage price of 96.50. The yield to maturity should be 8.8797%. This is the appropriate rate to use to discount annual cash flows of the Eurobond. We can now build the worksheet and fill in the information as we make the calculations.

(1) Time	(2) Cash Flow	(3) Present Value at Annual Rate	(4) Present Value / Bond Price	(5) Time- weighted Value
1.0	7.50	6.8883	0.0714	0.0714
2.0	7.50	6.3266	0.0656	0.1311
3.0	107.50	<u>83.2851</u>	<u>0.8631</u>	<u>2.5892</u>
Total		96.5000	1.0001	2.7917

Figure 9.4: Duration for a Three-year Eurobond

The duration of this Eurobond is 2.7917 years. Use the totals to ensure that the calculation is correct. The sum of the values in Column 3 equals the quoted percentage price of the Eurobond. The sum of the weights equals 1.0 in Column 4.

Duration of a perpetuity

The duration of a perpetuity with equal payments is calculated as $(1 + r) / r$ where r equals the yield of the security. For example, consider a perpetuity that is priced to yield 10% annually. The duration would be:

$$(1 + r) / r$$

$$(1 + 0.10) / 0.10 = 11 \text{ years}$$

Even though the instrument makes an annual payment forever, the duration is only 11 years.

Duration of a Portfolio

Weighted average

The duration of the portfolio is a weighted-average of the durations of each of the bonds held in the portfolio. For example, suppose an investor has a portfolio with 86.6% of the funds invested in 4-year notes with a duration of 3.6554 years and 13.4% of the funds invested in 10-year bonds with a duration of 8.9652 years. The duration of the portfolio is:

$$D_p = (3.6554)(0.866) + (8.9652)(0.134) = 4.3669 \text{ years}$$

The weights are the percentage of the total portfolio invested in each security.

Duration Relationships

Influences on duration calculation

As you can see from the calculations, duration is influenced by three important relationships:

- Maturity date
- Coupon rate
- Yield to maturity

Duration is positively correlated with maturity, but it moves in the opposite direction of coupon rates and yield to maturity.

Maturity

- A bond's duration generally increases with the time to maturity, holding the coupon rate constant.

Duration is a measure of the average maturity of the bond's payments, and the largest payment is generally made at maturity. Therefore, an increase in the time to maturity will increase the duration of the instrument.

The duration of a zero-coupon bond equals the time to maturity. One cash flow is received when the bond matures in the amount of the face value. Compared to coupon bonds of the same maturity, zero-coupon bonds have the greatest duration and, therefore, have the greatest sensitivity to changes in market interest rates.

Coupon rate

- A bond's duration is lower when the coupon rate is higher.

High coupon rate bonds produce higher cash flows during the life of the bond and, therefore, duration is weighted toward the early or middle years. For low coupon bonds, duration is weighted more heavily toward the final payment at maturity.

Yield to maturity

- The duration of a coupon bond is higher when the bond's yield to maturity is lower, holding all other factors constant.

Recall that our calculation of duration requires that the cash flows be discounted by the yield to maturity rate (bond equivalent rate). A smaller discount rate places less value on earlier cash flows in the discounting process.

Analysts use duration to track the sensitivity of bond prices to changes in interest rates.

Example

Suppose that you have to decide whether to invest in an 8% coupon rate with a 20-year maturity or a 12% coupon rate bond with a 25-year maturity. Which bond will have the larger increase in price if interest rates decline?

The bond with the shorter maturity (8% coupon rate for 20 years) has a longer duration than the bond with the greater maturity (12% for 25 years) and, thus, is the most price sensitive.

Summary

Duration represents the weighted average life of the bond. The weights are based on the present value of the individual cash flows relative to the present value of the total cash flows (current price of the bond).

The longer the duration, the greater the impact of a change in interest rates on price.

The duration of a coupon bond is higher when the bond's yield to maturity is lower.

The higher the coupon rate, the lower the duration.

Duration is primarily used as a measure to judge bond price sensitivity to interest rate changes.

Please complete Practice Exercise 9.4, which will give your experience with calculating the duration of a bond. When you have checked your answers, continue on to the Unit Summary that follows.

PRACTICE EXERCISE 9.4

Directions: Calculate the correct answer(s) for each question. Check your solutions with the Answer Key on the next page.

12. A 2-year Treasury note with a coupon rate of 6.5% has a percentage issue price of 98.50. What is the duration of the instrument?

Duration = _____ years

Use this table to help with your calculation.

Price: _____ BEY: _____
 Coupon: _____ Discount Rate: _____
 Maturity: _____

Time	Cash Flow	Present Value at Semi-annual Rate	Present Value / Bond Price	Time-weighted Value
		_____	_____	_____
TOTAL				

13. What is the duration of the following portfolio?

<u>Percentage of Portfolio</u>	<u>Security</u>	<u>Duration</u>
42.5%	2-year notes	1.8654
37.8%	5-year notes	4.7333
19.7%	10-year bonds	9.6655

ANSWER KEY

12. A 2-year Treasury note with a coupon rate of 6.5% has a percentage issue price of 98.50. What is the duration of the instrument?

Duration = 1.9067 years

Use this table to help with your calculation.

Price: 98.50 BEY: 7.3199%
 Coupon: 6.50% Discount Rate: 7.4538%
 Maturity: 2 years

Make sure that you have calculated the correct discount rate.

Find the yield to maturity of the note using the bond features on your calculator. That rate should be 7.3199%.

Convert this to an equivalent semi-annual discount rate.

Divide 0.073199 by 2, add 1, square this result, and subtract 1.

You should get 0.074538.

Use this rate to discount the cash flows and your table should look like this.

Time	Cash Flow	Present Value at Semi-annual Rate	Present Value / Bond Price	Time-weighted Value
0.5	3.25	3.1353	0.0318	0.0159
1.0	3.25	3.0246	0.0307	0.0307
1.5	3.25	2.9178	0.0296	0.0444
2.0	103.25	<u>89.4224</u>	<u>0.9078</u>	<u>1.8157</u>
TOTAL		98.5001	0.9999	1.9067

13. What is the duration of the following portfolio?

$$(0.425) (1.8654) + (0.378) (4.7333) + (0.197) (9.6655) = 4.4861 \text{ years}$$

UNIT SUMMARY

In this unit, we introduced the basic methodologies for pricing and calculating yield on debt securities that trade in the secondary markets. Specifically, we discussed U.S. Treasury bills, notes, bonds, and Eurobonds.

Treasury bills are short-term, zero-coupon (non-interest bearing) securities. The quoted price of a Treasury bill is discounted from the face value at a discount rate that is often referred to as the yield to maturity of the security. The discount rate is based on the face value of the security; the rate of return to the investor or true yield of the bill is based on the difference between the purchase price and the sale price.

Treasury notes are bonds with maturities ranging from one to ten years that make semi-annual interest payments. Since all Treasuries are quoted in terms of annual discount rates and annual coupon payments, the price of Treasury notes is adjusted for the semi-annual interest payments. You learned how to calculate the price of a note and the yield to maturity.

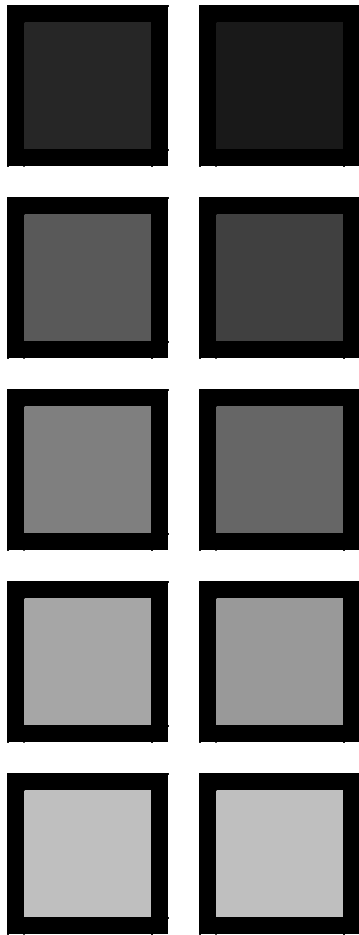
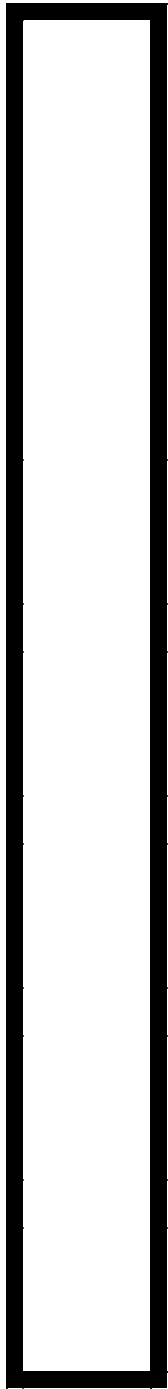
Treasury bonds are securities with maturities greater than ten years. The most common maturities are 20 to 30 years. Since bonds make semi-annual coupon payments to investors, the price and yield calculations are the same as they are for Treasury notes.

A Eurobond is a debt instrument issued by a company in an international market that is outside any domestic regulations. They typically make annual coupon payments to the investor.

The true selling price of a security that makes periodic interest payments includes an adjustment for accrued interest which is owed to the seller by the buyer. The convention is to adjust the payment by the number of days in the coupon period that the security is actually held by the seller.

Bond prices are affected by their sensitivity to market interest rates, the maturity of the bond, and the effective life of the bond. Duration is the measure we use to estimate the average maturity of a bond's cash flows.

You have demonstrated your understanding of each of these key concepts in the four practice exercises; therefore, this unit has no Progress Check. Please continue your study of securities with Unit Ten: *Derivative Securities*, which follows.



Unit 10

UNIT 10: DERIVATIVE SECURITIES

INTRODUCTION

In Unit Two, we introduced the three general subsets of capital markets: bond (debt) markets, equity markets, and derivative markets. In this unit we will focus on the derivative markets.

There are three basic forms of derivative products: forwards, options, and swaps. These instruments are often referred to as derivative securities because their values are derived from the value of the underlying security. In this unit you will be introduced to some of the derivative instruments that investors use to manage exposure to risk.

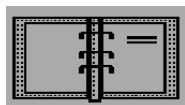
We will focus on the characteristics of options and swaps and how the payoffs are calculated for both types of instruments.



UNIT OBJECTIVES

After you successfully complete this unit, you will be able to:

- Recognize terminology that is associated with options trading
- Recognize the payoff profiles for call options and put options
- Identify the characteristics of interest rate swaps and currency swaps



OPTIONS

An option contract between a buyer (holder) and a seller (writer) describes the rights of the option holder and the obligations of the option writer.

The purchase price of an option is called the premium. This is the compensation that the holder of the option pays to the writer for the rights described in the option contract.

Calls and puts

There are two types of options: **call options** and **put options**.

Call options - give the holder the *right to buy* an asset for a specified price on or before a given expiration (maturity) date.

Put options - give the holder the *right to sell* an asset for a specified price on or before a given expiration (maturity) date.

The specific asset named in the option contract is called the underlying asset. The price at which the underlying asset may be bought is called the exercise or strike or contract price. Purchasing (or selling) the underlying asset of an option contract is referred to as exercising the option.

The key thing to remember is that the holder of the option has the *right* (but not the obligation) to exercise the option.

**In the money /
Out of the
money**

An option is "in the money" when its exercise would produce profits for its holder. An option is "out of the money" during the time when its exercise would not be profitable for its holder. This means that a call option on a stock is in the money when the exercise price is below the market price of the stock; a put option is profitable when the exercise price is above the market price of the stock.

**American /
European**

Options may be either American or European. American options give the holder the right to exercise the option at any time up to the expiration date. European options give the holder the right to exercise the option only on the expiration date.

Background and Markets

When options were first traded, contracts were arranged between pairs of investors and customized to meet specific needs. These customized contracts could vary according to exercise price, expiration date, and underlying asset. Because of these infinite possibilities, it was nearly impossible for a secondary options trading market to exist. Investors would enter into customized agreements according to their investment needs; if their needs changed, it was difficult to find another investor to buy the option contract.

Secondary markets

This lack of liquidity and standardization was one of the driving forces behind the creation of the Chicago Board Options Exchange (CBOE) in 1973. The CBOE enacted rules to standardize option contracts in order to restrict the number of contract types that would trade on the exchange. Soon, other option exchanges were created. This led to a broader standardization of option contracts and was the key to creating a secondary market for options and increasing the liquidity of option contracts.

Customized option contracts are still available to investors — they are traded in over-the-counter (OTC) markets. These customized options are usually considerably more expensive than the standard option contracts because of the lack of liquidity.

Standardized contracts

The first standardized contracts were for the purchase or sale of common stock. However, investors soon created demand for options on a variety of financial and physical assets, including options on:

- Common stock
- Stock indices
- Debt instruments
- Interest rate futures
- Foreign currency futures
- Agricultural commodities
- Precious metals

These options are now written and traded by investors on several different exchanges.

Option Clearing Corporation (OCC)

Another important event occurring with the establishment of standardized option contracts was the creation of the Option Clearing Corporation (OCC). The OCC is the clearinghouse for options trading. It is the single guarantor of all the options listed on the options exchange. With the OCC in place, an option buyer need not be concerned with counterparty risk (lack of performance, for example).

Because options on stock are among the most common, most of our discussion and examples in this unit focus on stock options. However, the basic principles of options are the same, regardless of the underlying asset.

Payoff Profile for Calls and Puts

The following abbreviations will be used in this section for the diagrams and formulas concerning options and their uses.

- S** = Current stock price
- S_T** = Stock price at time T
- X** = Exercise price
- C** = Price of call option
- P** = Price of put option

Call Options

A call option gives the holder the right to buy an asset for a specified price on or before a given expiration (maturity) date. The payoff profile (gain or loss) for the holder and writer of a call option is shown in Figure 10.1.

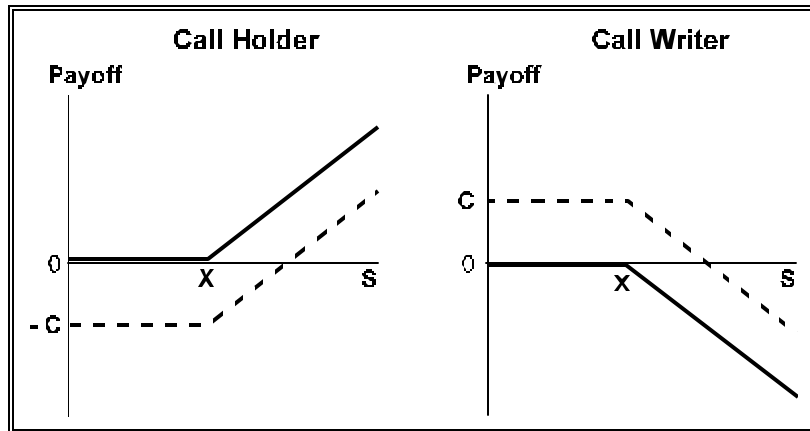


Figure 10.1: Payoff Profile for Call Holder and Call Writer

Payoff for call option holder

The diagram on the left side of Figure 10.1 is the payoff for the holder of a call option. The solid line represents the payoff for the call option. The dashed line is the payoff of the option, net the cost of the option. If the price of the stock never rises above the exercise price, the holder of the option is only liable for the premium paid to buy the call option.

This payoff of the option is also expressed using the following mathematical relationship:

$$\text{Call payoff} = \text{MAX}(0, S - X)$$

When written out, this means that the payoff to the holder of a call option is the maximum of 0 (zero) and the difference between the price of the underlying stock (S) and the exercise price (X) of the option.

Example

Here is an example of how this works. LDM Company has a call option with an exercise price of \$30. If the stock price (X) of LDM Company is less than \$30, the option is out of the money, and the payoff of the option is \$0. If the stock price rises above \$30, the option is in the money, and the payoff for the call option is the price of the stock minus the exercise price ($S - X$). For instance, if the stock rises to \$42, the option payoff is $42 - 30 = \$12$ per share.

An option buyer is able to participate in any price change in the underlying asset without having to buy the asset itself, which would require a substantially larger investment. The option's cost (premium) is usually only a small fraction of the underlying asset's market price. Most options are written for a block of shares, such as 100.

With an American option, the payoff profile is valid anytime during the life of the option. In other words, the holder of the option can exercise the option and claim the payout. In a European option, the only stock price we can consider is the price at the time the option expires.

**Payoff for a
call option
writer**

The diagram on the right side of Figure 10.1 is the payoff profile for the call option writer. In this case, the writer receives the price of the call option up front. Essentially, the writer is making a bet that the price of the underlying stock will not rise above the exercise price. If the bet is correct, the holder of the option will never exercise the option and the writer has the premium as profit. Options trading is a zero-sum transaction — any profits gained by one counterparty are exactly matched by losses incurred by the other counterparty.

If the price of the stock is greater than the exercise price on the option plus the premium, the holder of the option will most likely exercise the option. In this case, the writer is obligated to sell the shares of stock to the holder at the exercise price. If the writer did not own the underlying stock when the option was written (called writing a naked option), then the writer has to buy the stock at the current market price and sell it to the holder of the option at the exercise price. The writer's loss will be the difference in the two prices (net of the price received for the call option).

Put Options

A put option gives the holder the right to sell an asset for a specified price on or before a given expiration (or maturity) date. The payoff profiles for the holder and writer of a put option are given in Figure 10.2.

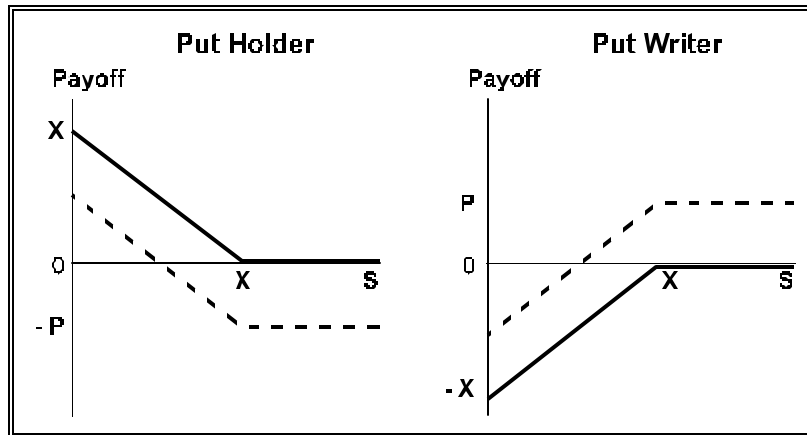


Figure 10.2: Payoff Profile for Put Holder and Put Writer

**Maximum
payoff for put
holder**

The figure on the left is the payoff profile for the holder of a put option. The maximum profit is the exercise price of the put option less the price paid for the option. This would occur if the stock price fell to \$0.

The mathematical relationship is:

$$\text{Put payoff} = \text{MAX}(0, X - S)$$

With a put option, the option is in the money when the underlying stock price is less than the exercise price. The payoff to the holder is the difference between the exercise price and the underlying stock price less the amount paid for the put option. Once again, the solid lines represent the payoff of the put option; the dashed lines are payoff, net the price paid for the option.

Example

As an example, LDM Company put options have an exercise price of \$45. As long as the stock price of LDM remains below \$45, these put options are in the money. If the stock price is at \$32, the payoff for the holder of a put option would be $\$45 - \$32 = \$13$ per share. If the investor paid \$90 for the put option contract, and it was for 100 shares, the net payoff would be $(\$13 \times 100) - \$90 = \$1,210$.

Payoff for put writer

The profits for the writer of a put option are exactly the opposite of the holder of the put. Essentially, the writer of the put is betting that the price of the stock will rise above the exercise price. If that occurs, the writer's payoff is the put option premium. If the stock price is not above the exercise price, the writer of the put is obligated to purchase the underlying stock for the exercise price. This position (called writing a naked put) is somewhat risky, and most investors combine writing puts with other strategies to limit their potential losses.

The transactions for put options work in the same way as the transaction for call options. Investors' accounts with brokers are credited and debited for the net amount of each option transaction.

In the world of finance, very few option contracts are completed. Usually, investors will close out their option positions by taking the opposite side of the transaction before the exercise date; that is, the holder of a call option will become the writer of an identical option shortly before the exercise date.

OCC processing

Option transactions take place electronically through the OCC and its member brokers. The clearinghouse processes all transactions and acts as the counterparty on both sides of an option contract to ensure performance. If an option holder exercises an option, the OCC randomly assigns an exercise notice to a broker's account that reflects the writing of the same option. The broker then assigns the notice to one of its clients (option investors) on either a random or a "first in-first out" basis.

Margin money and margin calls

Investors are required to post margin money with their brokers to assure performance of their obligations. The broker can then deposit or withdraw the funds from the investor's account to correspond with the profit or loss on the option transactions. If an investor's account balance becomes too low (a point where the broker no longer feels that the investor can meet the possible obligations), the investor will receive a margin call. A margin call requires the investor to deposit more funds into the margin account.

SWAPS

Reduces exposure to rate changes

A swap is an exchange of cash flows between two counterparties. The purpose of a typical swap is to reduce the exposure a company may have to a change in interest rates or a change in the exchange rate between two currencies. The two most commonly used swaps are interest rate swaps and foreign currency swaps.

Interest Rate Swaps

Fixed / Floating rate securities

In the previous unit, we discussed several aspects of fixed-income securities. Most of these instruments make fixed coupon payments at some regular interval. However, many other debt instruments do not make fixed interest payments; the rates fluctuate based on a pre-determined floating benchmark. These instruments are called floating rate securities. A floating benchmark is an interest rate that is established by a third party.

London Interbank Offered Rate (LIBOR)

The most common benchmark used in floating rate instruments is the London Interbank Offered Rate (LIBOR). This is the rate that banks use to borrow from each other in the Eurodollar market.

Example of floating rate

Here is an explanation of how it works. Suppose that XYZ Corporation issues floating rate securities with an interest rate of LIBOR plus 50 b.p. (basis points – one basis point is 1/100 of 1% or 0.01%; 50 basis points equals 0.5%). The convention is that on the date the interest payment is due, the amount of the payment will be based on a rate equal to the current LIBOR plus 1/2%. In this case, if LIBOR is 5.25%, XYZ will pay interest based on a 5.75% rate. As LIBOR changes, so will the company's interest payments.

**Example of
fixed / floating
rate swap**

Most interest rate swaps take place between two parties exposed to opposite types of interest rate exposure (risk). To illustrate the process, suppose that a savings and loan institution (S&L) is one of the counterparties. The assets of the S&L are predominantly long-term with fixed rates, such as conventional home mortgages, which pay roughly 8%. Its liabilities are mostly short-term, floating rate deposit accounts. If interest rates rise, the payments they are required to make on the deposits will increase, but the interest the S&L receives from the mortgages remains the same. The S&L is exposed to interest rate risk.

On the other side of the swap is DEF Corporation, which has long-term non-callable bonds as liabilities. The interest rate on the bonds is 8%. DEF has invested in short-term, floating rate assets at roughly LIBOR + 0.5% as part of its portfolio. A fall in interest rates will cause losses for DEF and, therefore, it is also exposed to interest rate changes – but its exposure is opposite that of the S&L.

A swap could work as follows. The S&L agrees to make fixed-rate payments to DEF Corporation based on some agreed upon amount (known as the notional principal) at a fixed interest rate. In return, DEF makes floating rate payments to the S&L based on the notional principal at LIBOR plus a premium of 50 b.p. The swap transaction is illustrated in Figure 10.3.

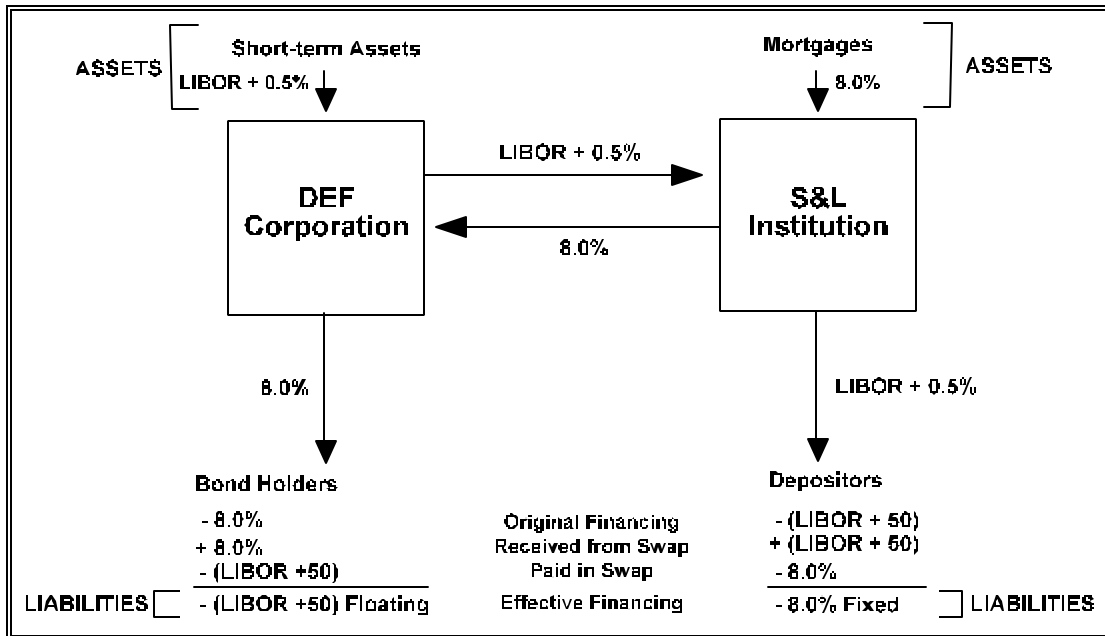


Figure 10.3: Fixed / Floating Interest Rate Swap

Both parties reduce exposure

The swap has allowed both parties to minimize interest rate exposure. DEF Corporation now receives 8% fixed-rate payments from the S&L and pays its bond holders a fixed rate of 8%. The S&L receives a floating rate payment of LIBOR + 50 b.p. from DEF, which matches the floating rate that the S&L is paying its depositors on their accounts.

The swap has helped both parties reduce their interest rate exposure by matching the assets and liabilities of both parties.

There are two other points about interest rate swaps that you should remember.

No exchange of principal

First, even though the payments are based on a notional principal, that principal is never exchanged between the counterparties. In our example, if the notional principal was US \$10,000,000, the fixed rate payment made by S&L to DEF Corporation would be \$800,000 each year for the period of the swap. The payment made by DEF to the S&L would be based on LIBOR. If LIBOR was 7.25%, then DEF's payment would be $\text{LIBOR} + 0.50\% = \$775,000$.

Decrease basis points

Second, it is possible for a company to cut basis points off the interest payments of its debt by using swaps. This occurs when the differential spreads of borrowing costs between companies with differing credit ratings are significantly different.

Example

Consider this example of how the rates may be reduced. These are the rates available in the market to XYZ Corporation and LDM Corporation:

	<u>Fixed Rate</u>	<u>Floating Rate</u>
XYZ Corp.	8.00%	LIBOR + 1%
LDM Corp.	10.50%	LIBOR + 2%

Because XYZ has a better credit rating, its borrowing costs are lower than those for LDM. However, note that LDM pays 2.5% more than XYZ for fixed-rate debt, but only 1% more for floating-rate debt. A swap can be designed to take advantage of the difference and lower the borrowing costs for both companies.

Suppose that XYZ is currently paying 8% to bondholders but needs 10-year floating-rate financing; LDM is currently paying LIBOR plus 200 b.p. but needs 10-year fixed-rate financing. XYZ Corporation needs floating rate financing and would have to pay $\text{LIBOR} + 1\%$ in the market; LDM Corporation needs fixed rate financing and would have to pay 10.5% in the market.

The two companies negotiate through an intermediary to design a swap that will provide each company with the type of financing it needs at a lower cost than it can get in the market.

LDM Corporation agrees to pay XYZ Corporation fixed rate payments of 8.0%. XYZ agrees to pay LDM a floating rate of LIBOR + 0.5%. You can see the cash flows in Figure 10.4.

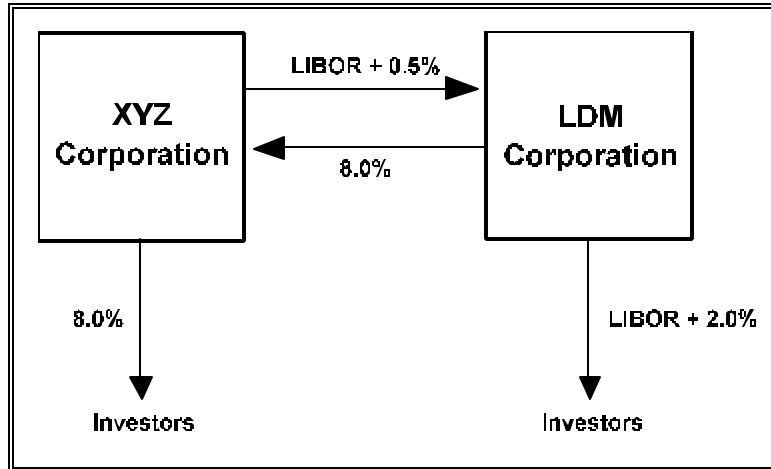


Figure 10.4: Cash Flows for Interest Rate Swap

This table helps organize the cash flows:

<u>Cash Flow</u>	<u>XYZ</u>	<u>LDM</u>
Payment to investors	- 8.00%	- LIBOR + 2.00%
Pays in swap	- LIBOR + 0.50%	- 8.00%
Receives in swap	+8.00%	+LIBOR + 0.50%
Net payment	- LIBOR + 0.50%	- 9.50%

Benefits of the swap

Let's see how each company benefits from the swap.

	<u>XYZ</u>	<u>LDM</u>
Rate available in market	LIBOR + 1.0% Floating	10.50% Fixed
Net payment in swap	LIBOR + 0.5%	9.50%
Savings	0.5% Floating	1.0% Fixed

Both companies now have the type of financing they need — and at a lower cost than if the swap agreement had not been arranged!

Banks as intermediaries

In the example, we said that LDM and XYZ arranged the swap through an intermediary. Most companies lack the resources and expertise to find suitable counterparties and evaluate their credit-worthiness. A large, money-center bank will act as an intermediary in most swap transactions. The bank serves as a clearinghouse for the interest payments and guarantees performance by each counterparty.

From the company's perspective, the swap is essentially made with the bank — it may not know or even care who the counterparty is. Of course, the bank charges a fee for providing these services, usually in the form of a few basis points added to the overall rate that is paid in the swap. The bank usually receives a fee from both counterparties, so the all-in-cost of the interest payments for each company will include the fees paid to the bank.

Currency Swaps**Exchange of cash flows in different currencies**

A currency swap is an exchange of cash flows denominated in one currency for the cash flows in a different currency. The driving force behind the swap might be that a company may be able to borrow in one currency at a lower rate than in another currency. Let's look at an illustration that shows how a currency swap might work.

Example

Consider a German company and a U.S. corporation that are trying to raise financing for their respective businesses in their home currencies. The German company has borrowed in the Deutsche mark (DM) bond market extensively in the past few months. This means that there could be a relative overabundance of DM-denominated debt issued by German companies and investors' demand is low. However, there is relatively little DM-denominated debt issued by U.S. corporations in that market. This situation may mean that a U.S. company could issue bonds at a lower rate than the German firm in the DM market.

In the US\$ market, investors perceive that the German company is a better credit risk than the U.S. corporation, and the German company could issue bonds at a lower rate in the US\$ market. This is an opportunity for a swap to occur between the two companies. The German company could issue in the US\$ market and the U.S. corporation could issue in the DM market. The two companies would exchange initial receipts from the bond issues (they would most likely be equal) and the U.S. company would then pay the interest and repay the principal of the US\$-denominated bonds issued by the German company. The German company would do likewise on the DM-denominated bonds.

Remember that the swap only makes sense if both companies can lower their all-in-cost of borrowing funds. Also remember that a bank or some other intermediary will usually be part of the arrangement and will charge fees for its service.

UNIT SUMMARY

An option is a contract between a buyer and a seller. A call option gives the buyer the right to buy an asset. A put option gives the buyer the right to sell an asset. The underlying asset is the specific asset named in the option contract. Exercising the option is either the buying or the selling of the underlying asset.

An option is in the money when it produces a profit at the time of exercise. An option is out of the money when it does not produce a profit at the time of exercise.

An American option gives the right to exercise the option at any time up to the expiration date. European option gives the right to exercise the option only on the expiration date.

The Option Clearing Corporation (OCC) is the clearinghouse for options trading.

Call holders and put writers post margin money with brokers to assure performance of investors' obligations. A margin call requires the deposit of additional funds into the margin account.

A swap is an exchange of cash flows between two counterparties.

An interest rate swap is used by a company to meet the need for fixed or floating interest rates at lower rates than may be available in the market.

A foreign currency swap is a method for companies to borrow at a lower rate in one currency than in another currency.

Swaps are arranged through an intermediary and are intended to reduce the borrowing costs for both counterparties.

Congratulations! You have completed the Basics of Corporate Finance course. Please complete the final Progress Check which follows to check your understanding of securities.

You have studied the basic concepts and calculations that are the foundation for future courses related to corporate finance and the capital and derivative markets. You should now have a strong working vocabulary for the financial world and an understanding of the basic types of analyses used in the corporate finance field. The index and glossary are provided to help you use this workbook as an easy reference in the future. Use it as you pursue more advanced courses in the corporate finance series.

✓ PROGRESS CHECK 10

Directions: Select the correct answer for each question. Check your solution with the Answer Key on the next page. If you are not able to complete the Progress Check correctly, read the appropriate text again.

1. Match the terms in Column A with the correct definition in Column B.

Column A	Column B
A. Underlying asset	_____ An exchange of cash flows between two counterparties
B. Exercising the option	_____ The specific asset named in the option contract
C. Margin money	_____ The right to buy an asset
D. Interest rate swap	_____ The right to sell an asset
E. Put	_____ The buying or selling of the underlying asset
F. Call	_____ When the option produces a profit at the time of exercise
G. In the money	_____ Price at which underlying asset may be bought or sold
H. Strike price	_____ Account maintained by investors with brokers to assure performance of investors' obligations

2. The holder of an American call option:

- _____ a) must exercise the option on the expiration date.
- _____ b) is obligated to exercise the option either on or before the expiration date.
- _____ c) may exercise the option only on the exercise date.
- _____ d) has the right to exercise on or before the exercise date.

ANSWER KEY

1. Match the terms in Column A with the correct definition in Column B.

Column A	Column B
A. Underlying asset	<u> D </u> An exchange of cash flows between two counterparties
B. Exercising the option	<u> A </u> The specific asset named in the option contract
C. Margin money	<u> F </u> The right to buy an asset
D. Interest rate swap	<u> E </u> The right to sell an asset
E. Put	<u> B </u> The buying or selling of the underlying asset
F. Call	<u> G </u> When the option produces a profit at the time of exercise
G. In the money	<u> H </u> Price at which underlying asset may be bought or sold
H. Strike price	<u> C </u> Account maintained by investors with brokers to assure performance of investor's obligations

2. The holder of an American call option:

d) has the right to exercise on or before the exercise date.

*The holder has the **right** to exercise the option on or before the exercise date, **not** the **obligation**.*

PROGRESS CHECK
(Continued)

3. The only loss sustained by the holder of a call that expires "out of the money" is the:
- _____ a) difference between the strike price and the price of the underlying asset.
 - _____ b) value of the stock if the price drops to \$0.
 - _____ c) premium paid for the call option.
 - _____ d) amount of the strike price.
4. Select a condition that may allow an interest rate swap between two companies to be profitable.
- _____ a) Floating rates are expected to rise because of inflation.
 - _____ b) The credit rate differential spread between fixed and floating rates for the two companies is significant.
 - _____ c) There is a significant difference between U.S. Treasury bill and U.S. Treasury bond rates.
 - _____ d) Fixed rates are expected to fall.

ANSWER KEY

3. The only loss sustained by the holder of a call that expires "out of the money" is the:

c) premium paid for the call option.

The holder of an option is only liable for the price paid for the option. If the option expires "out of the money," he is not required to exercise the option and incur additional losses.

4. Select a condition that may allow an interest rate swap between two companies to be profitable:

b) The credit rate differential spread between fixed and floating rates for the two companies is significant.

This condition may allow both companies to lower their cost of debt, provided that one needs floating-rate and the other needs fixed-rate. Also the calculations must include the intermediary and its fees.

PROGRESS CHECK
(Continued)

5. In an interest rate swap transaction, the notional principal is:
- _____ a) exchanged between the two parties at the beginning of the agreement and again on the maturity date.
 - _____ b) loaned by one counterparty to the other in exchange for either fixed or floating rate interest.
 - _____ c) never exchanged between the counterparties.
 - _____ d) different for both counterparties.
6. Consider the rates available in the market to Mega Corporation and Super, Inc.

	<u>Fixed Rate</u>	<u>Floating Rate</u>
Mega Corporation	9.00%	LIBOR + 1.5%
Super, Inc.	11.25%	LIBOR + 2.25%

Currently, Mega Corporation is paying 9.00% to bondholders, but it wants to replace this fixed rate financing with floating rate financing. Super, Inc. pays LIBOR + 2.25% to investors in need of fixed rate financing. Super agrees to pay Mega fixed rate payments of 9%; Mega agrees to pay Super floating rate payments of LIBOR + 0.75%. How much would each company save on the financing it needs by entering into a fixed / floating rate swap transaction?

Mega _____%

Super _____%

ANSWER KEY

5. In an interest rate swap transaction, the notional principal is:

c) **never exchanged between the counterparties.**

6. Consider the rates available in the market to Mega Corporation and Super, Inc.

	<u>Fixed Rate</u>	<u>Floating Rate</u>
Mega Corporation	9.00%	LIBOR + 1.5%
Super, Inc.	11.25%	LIBOR + 2.25%

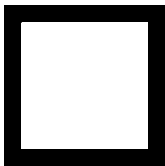
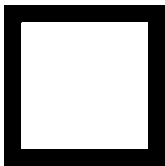
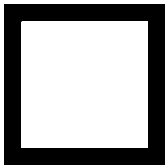
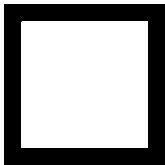
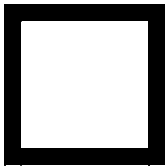
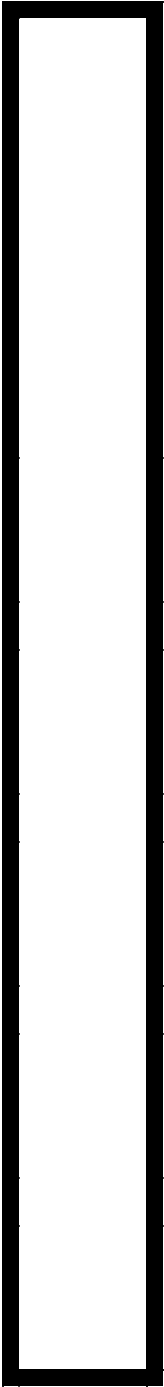
Currently, Mega Corporation is paying 9.00% to bondholders, but it wants to replace this fixed rate financing with floating rate financing. Super, Inc. pays LIBOR + 2.25% to investors in need of fixed rate financing. Super agrees to pay Mega fixed rate payments of 9%; Mega agrees to pay Super floating rate payments of LIBOR + 0.75%. How much would each company save on the financing it needs by entering into a fixed / floating rate swap transaction?

Mega 0.75%

Super 0.75%

<u>Cash Flow</u>	<u>Mega Corporation</u>	<u>Super, Inc.</u>
Payment to investors	- 9.00%	- LIBOR + 2.25%
Pays in swap	- LIBOR + 0.75%	- 9.00%
Receives in swap	<u>+9.00%</u>	<u>+LIBOR + 0.75%</u>
Net payment	- LIBOR + 0.75%	- 10.50%
Rate available in market	LIBOR + 1.50% Floating	11.25% Fixed
Net payment in swap	<u>LIBOR + 0.75%</u>	<u>10.50%</u>
Savings	0.75% Floating	0.75% Fixed

Both companies save 0.75% by entering into an interest rate swap to get the type of funding they need.



Appendices

APPENDIX

GLOSSARY

Accrued Interest	The amount of interest accumulated on a debt security between interest paying dates
Acid-test Ratio	See quick ratio
Annuity	A series of payments or deposits of equal size spaced evenly over a specified period of time
Annuity Due	Annuity where the payments are to be made at the beginning of each period
Asset-specific Risk	The amount of total risk that can be eliminated by diversification by creating a portfolio. Also known as company-specific risk or unsystematic risk .
Average Collection Period	Average number of days necessary to receive cash for the sale of a company's products. It is calculated by dividing the value of the accounts receivable by the average daily sales for the period.
Basic Earnings Power Ratio	Percentage of earnings relative to total assets; indication of how effectively assets are used to generate earnings. It is calculated by dividing earnings before interest and taxes by the book value of all assets.
Basis Point	One one-hundredth of one percent
Benchmark	A standard by which something may be compared and measured
Beta	A measure of the riskiness of a specific security compared to the riskiness of the market as a whole; measure of the systematic risk of a security or a portfolio of securities

Bond	A long-term debt instrument in which the issuer (borrower) is obligated to pay the investor (lender) a specified amount of money, usually at specific intervals, and to repay the principal amount of the loan at maturity. The periodic payments are based on the rate of interest agreed upon at the time the instrument is sold.
Bond Equivalent Yield	Bond yield calculated on an annual percentage rate method
Book Value	The value of an asset as carried on the balance sheet of a company. In reference to the value of a company, it is the net worth (equity) of the company.
Book Value per Share	The book value of a company divided by the number of shares outstanding
Call Option	A contract that gives the holder the right to buy an asset for a specified price on or before a given expiration (maturity) date
Capital Asset Pricing Model (CAPM)	A model for estimating equilibrium rates of return and values of assets in financial markets; uses beta as a measure of asset risk relative to market risk
Capital Budgeting	The process of ranking and selecting investment alternatives and capital expenditures
Capital Market	A market that specializes in trading long-term, relatively high risk securities
Capital Structure	The combination of debt, preferred stock, and common stock used by a company to provide capital for the purchase of its fixed assets
Common Stock	A financial security that represents an ownership claim on the assets and earnings of a company. This claim is valid after the claims of the debt providers and preferred stockholders have been satisfied.
Company-specific Risk	See asset-specific risk

Compound Interest	Interest paid on principal and on interest earned in previous periods
Continuous Compounding	The process of continuously adding interest to a principal plus interest amount and calculating the resulting compound amount
Continuous Discounting	The process of calculating the present value of a stream of future cash flows by discounting over a continuous period of time
Correlation Coefficient	A measure of the tendency of two variables to change values together
Cost of Capital	The minimum rate of return a company must earn in order to meet the rate of return required by the investors (providers of capital) of the company
Cost of Common Stock	The rate of return required by the investors in the common stock of the company. A component of the cost of capital.
Cost of Debt	The cost of debt (bonds, loans, etc.) that a company is charged for borrowing funds. A component of the cost of capital.
Cost of Equity	Same as the cost of common stock. Sometimes viewed as the rate of return stockholders require to maintain the market value of the company's common stock.
Cost of Preferred Stock	The rate of return required by the investors in the preferred stock of a company. A component of the cost of capital.
Coupon(s)	The periodic interest payment(s) made by the issuer of a bond (debt security). Calculated by multiplying the face value of the security by the coupon rate.
Coupon Rate	The rate of interest paid on a debt security. Generally stated on an annual basis, even if the payments are made at some other interval.

Current Ratio	A measure of the ability of a company to use its current assets to pay its current liabilities. It is calculated by dividing the total current assets by the total current liabilities.
Debt Ratio	The percentage of debt that is used in the total capitalization of a company. It is calculated by dividing the total book value of the debt by the book value of all assets.
Derivative	An instrument with a value that is derived from the value of the underlying security
Discount Rate	The rate of interest used to calculate the present value of a stream of future cash flows
Discounting	The process of calculating the present value of a stream of future cash flows
Discrete Compounding	The process of adding interest to a principal plus interest amount and calculating the resulting compound amount at specific intervals, such as monthly or annually
Diversification	The process of spreading a portfolio over many investments to avoid excessive exposure to any one source of risk
Duration	The weighted average of the time until maturity of each of the expected cash flows of a debt security
Earnings per Share	A measure of the earnings generated by a company on a per share basis. It is calculated by dividing income available for distribution to shareholders by the number of common shares outstanding.
Effective Annual Yield	Annualized rate of return on a security computed using compound interest techniques
Effective Interest Rate	The rate of interest actually earned on an investment. It is calculated as the ratio of the total amount of interest actually earned for one year divided by the amount of the principal.

Eurobond	A debt security issued in a market other than the home market of the company issuing the security
Expected Value	The value of the possible outcomes of a variable weighted by the probabilities of each outcome
Face Value	The nominal value of a security. Also called the par value .
Fixed Assets Turnover Ratio	A measure of the utilization of a company's fixed assets to generate sales. It is calculated by dividing the sales for the period by the book value of the net fixed assets.
Fixed Charge Coverage Ratio	A measure of how well a company is able to meet its fixed charges (interest and lease payments) based on the cash generated by its operations. It is calculated by dividing the earnings before interest and taxes by the total interest charges and lease payments incurred by the firm.
Free Cash Flow	The funds available for distribution to the capital providers of the company after investments inside the company have been made
Future Value	The amount a given payment, or series of payments, will be worth at the end of a specified time period, if invested at a given rate
Independent Projects	A situation where an increase (or decrease) in the benefits of one project has no effect on the benefits of another project. Also, a situation where the acceptance of one project does not preclude the acceptance of another project.
Internal Rate of Return (IRR)	The discount rate that equates the present value of the net cash inflows with the present value of the net cash outflows (investments). The IRR measures the profitability (rate of return) of an investment in a project or security.
Inventory Turnover Ratio	Provides a measure of how often a company's inventory is sold or "turned over" during a period. It is calculated by dividing the sales figure for the period by the book value of the inventory at the end of the period.

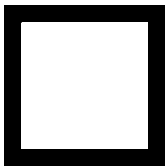
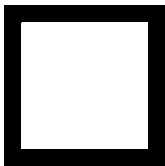
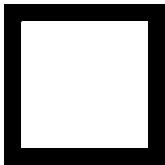
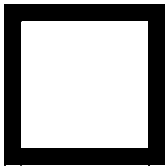
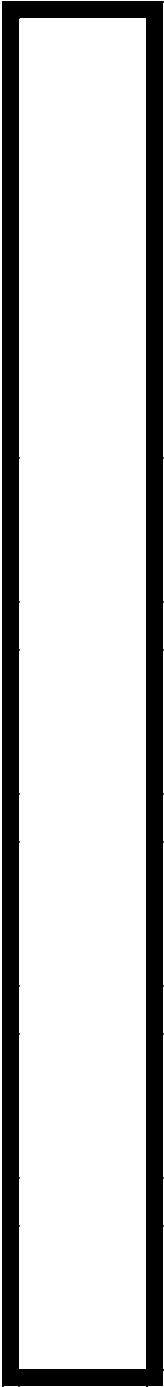
Investment	The commitment of funds (capital) in anticipation of an increased return of funds at some point in the future
Liquidation Value	The net proceeds (after taxes and expenses) of selling the assets of a company at fair market prices
Liquidity	The ease with which assets or securities can be sold for cash on short notice at a fair price
Market Risk	The amount of total risk that cannot be eliminated by portfolio diversification. The risk inherent in the general economy as a whole. Also known as systemic risk .
Market to Book Ratio	Measure of the book value of a company on a per share basis. It is calculated by dividing the book value of the company by the number of common shares outstanding.
Maturity	The date or the number of days until a security is due to be paid or a loan is to be repaid
Money Market	A market that specializes in trading short-term, low-risk, very liquid debt securities
Net Present Value (NPV)	The present value of all future cash inflows minus the present value of all cash outflows
Nominal Interest Rate	The rate of interest quoted, or stated, to be paid on a security
Operating Cash Flow	Income available after the payment of taxes, plus the value of the non-cash expenses
Option	See call option and put option
Ordinary Annuity	An annuity where the payments are made at the end of each period
Par Value	Nominal value of a security. Same as face value .

Payback Period	The number of years necessary for the net cash flows of an investment to equal the initial cash outlay
Perpetuity	A special case of an annuity with no set maturity. Payments are made forever.
Portfolio	A collection of securities and investments held by an investor
Portfolio Diversification	See diversification
Portfolio Weight	The percentage of a total portfolio represented by a single specific security. It is calculated by dividing the value of the investment in a specific security by the value of the investment in the total portfolio.
Preferred Stock	A type of equity security where holders have a claim on the assets and earnings of a company after the debt providers but before the holders of common stock. Preferred stock generally pays a fixed or floating rate dividend each year.
Present Value (PV)	The dollar value at the present time (year zero) of a single cash flow or a stream of future cash flows. The present value is calculated by discounting the future cash flows.
Price to Earnings (P/E) Ratio	A measure of how much investors are willing to pay for each dollar of a company's reported profits. It is calculated by dividing the market price per share by the earnings per share.
Primary Market	Market where debt and equity securities are sold by an issuing company to investors to raise capital for its operations
Probability Distribution	A list of all possible outcomes and the chance of each outcome occurring
Profit Margin Ratio	A measure of how much profit is earned on each dollar of sales. It is calculated by dividing the net income available for distribution to shareholders by the total sales generated during the period.

Profitability Index	A method for determining the profitability of an investment. It is calculated by dividing the present value of the future net cash flows by the initial cash investment.
Project	An investment opportunity for a company
Put Option	A contract that gives the holder the right to sell an asset for a specified price on or before a given expiration (maturity) date
Quick Ratio	A measure of how easily a company can use its most liquid current assets to meet its current liabilities. It is calculated by subtracting the book value of the inventories from the total book value of current assets and dividing the result by the total book value of current liabilities. Also known as acid-test ratio .
Ratio Analysis	The process of using financial ratios, calculated from key accounts found in a company's financial statements, to make judgements concerning the finances and operations of the firm
Real Interest Rate	The rate of interest paid on an investment adjusted for inflation
Replacement Value	The amount necessary to duplicate a company's assets at current market prices
Residual Value	The value attributed to a company to represent all future cash flows after the end of the forecast period
Return on Common Equity Ratio	A measure of the percentage return earned on the value of the common equity invested in the company. It is calculated by dividing the net income available for distribution to shareholders by the book value of the common equity.
Return on Total Assets Ratio	A measure of the percentage return earned on the value of the assets in the company. It is calculated by dividing the net income available for distribution to shareholders by the book value of all assets.
Risk Premium	The additional rate of return required on a risky project (investment) when compared to a risk-free project (investment)

Risk-free Rate	The rate of return on an investment with known future benefits; a riskless rate of return, often estimated using the return earned on short-term U.S. Treasury securities
Secondary Market	The market where securities are exchanged between investors. Secondary market transactions have no effect on the issuing company.
Security Market Line	A graph illustrating the equilibrium relationship between the expected rate of return on securities and their risk as measured by the beta coefficient
Simple Interest	Interest paid only on the principal; calculated by multiplying the interest rate by the principal
Standard Deviation	A statistical term that measures the dispersion of a variable around its expected value. The standard deviation is often used as a measure of risk when applied to a return on an investment.
Swap	An exchange of cash flows between two counterparties. The counterparties may exchange flows in different currencies (currency swap) or exchange floating interest rate payments for fixed rate payments (interest rate swap).
Systematic Risk	The amount of total risk that cannot be eliminated by portfolio diversification. The risk inherent in the general economy as a whole. Also known as market risk .
Times Interest Earned Ratio	A measure of how well a company is able to meet its interest payments based on the cash generated by its operations. It is calculated by dividing the earnings before interest and taxes by the total interest charges incurred by the firm.
Total Asset Turnover Ratio	A measure of the utilization of all of a company's assets to generate sales. It is calculated by dividing the sales figure for the period by the book value of the net fixed assets.
Total Debt to Total Assets Ratio	See debt ratio

Unsystematic Risk	The amount of total risk that can be eliminated by diversification by creating a portfolio. Also known as asset-specific risk or company-specific risk .
Valuation	The process of estimating the value of financial assets or securities
Variance	The weighted average of the squared deviations from the expected value
Volatility	The probability of change
Weighted Average Cost of Capital (WACC)	The weighted average of the costs of the capital components (debt, preferred stock, and common stock)
Yield Curve	A graphical representation of the level of interest rates for securities of differing maturities at a specific point of time
Yield to Maturity	The measure of the average rate of return that will be earned on a debt security held until it matures
Zero-coupon Bond	A security that makes no interest payments; it is sold at a discount at issue and then repaid at face value at maturity



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