

$$\cos \theta = \frac{0}{(2)(2)} = 0 \Rightarrow \underline{\cos \theta = 90^\circ}$$

### EXERCISE

1. Given the two Vectors:

$$\vec{OA} = i + 5j \text{ and } \vec{OP} = 5i - j$$

Find: a) The magnitude and direction of Each Vector

b) Determine if the vectors are parallel or perpendicular.

2. A point P(4, 2) is on a Cartesian plane, Given a position vector

$$\vec{OA} = 2i + 4j \text{ and using a Scale of 1cm for 1 unit on both axes}$$

a) Draw vectors  $\vec{OA}$  and  $\vec{PB}$ , where  $\vec{OA} = \vec{PB}$

b) Write down the coordinates of B. Join the points to obtain OABP.

c) Name the figure

d) Express  $\vec{OB}$  in terms of  $i$  and  $j$  and find its magnitude and direction.

Now the angle can be found by direct measurement if the vectors are represented in the diagram to scale. OR By calculation as follows

$$V(\sqrt{3}, 1), \theta_1 = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \theta_1 = 30^\circ$$

$$W(-1, \sqrt{3}), \theta_2 = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) + 180^\circ, \theta_2 =$$

$$\theta_2 = -60^\circ + 180^\circ \Rightarrow \theta_2 = 120^\circ$$

$$\alpha = \theta_2 - \theta_1 = 120^\circ - 30^\circ \Rightarrow \alpha = 90^\circ$$

Hence  $\vec{OV}$  and  $\vec{OW}$  are perpendicular.

METHOD B: Dot Product = 0

$$V(\sqrt{3}, 1), W(-1, \sqrt{3})$$

$$V \cdot W = (\sqrt{3})(-1) + (1)(\sqrt{3}) = 0 + 0$$

$$\Rightarrow V \cdot W = 0$$

Hence the vectors are perpendicular.

The angle between two vectors can also be calculated using

$$\text{the formula } \cos \theta = \frac{W \cdot V}{|W||V|} \quad \theta = \text{Angle between the vectors}$$

Using the above example,

$$W \cdot V = V \cdot W = 0$$

$$|W| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$|V| = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$