VECTOR GEOMETRY

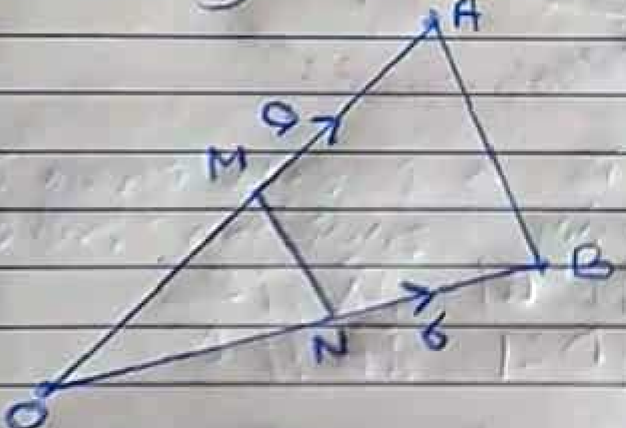
In Vector Geometry we deal only with magnitudes using small letters to represent position vectors such as $\overline{OA} = a$, $\overline{OB} = b$ --- etc.

USEFULL THEOREMS

1. Mid-Point Theorem

Suppose that the position vectors of the points A and B relative to the origin are respectively a and b such that;

$\overline{OA} = a$ and $\overline{OB} = b$, as shown in the diagram below



M is the mid-point on $\overline{OA}$
N is the mid-point on $\overline{OB}$
Such that $\overline{AB} = b - a$
and $\overline{BA} = a - b$
Hence $\overline{MN} = \frac{1}{2}(b - a)$
and $\overline{NM} = \frac{1}{2}(a - b)$

SOLUTIONS TO EXERCISES

$\vec{OA} = i + 5j$, $\vec{OB} = 5i - j$

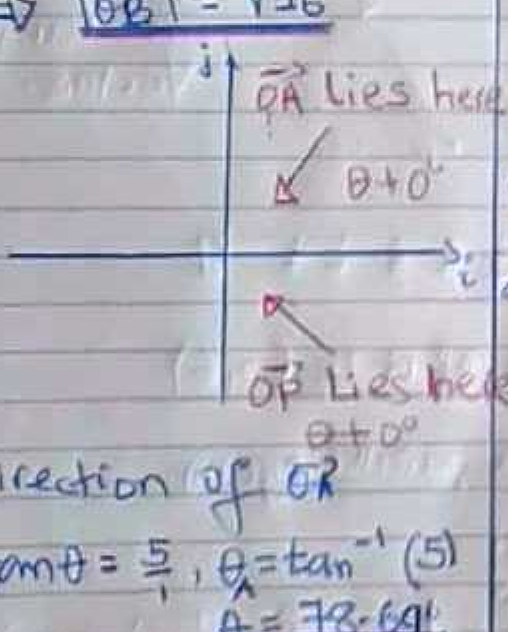
$\Rightarrow A(1,5)$ $\Rightarrow B(5,-1)$

$|\vec{OA}| = \sqrt{1^2+5^2} = \sqrt{1+25}$

$\Rightarrow |\vec{OA}| = \sqrt{26}$

$|\vec{OB}| = \sqrt{5^2+(-1)^2} = \sqrt{25+1}$

$\Rightarrow |\vec{OB}| = \sqrt{26}$



Direction of $\vec{OA}$

$\tan\theta = \frac{5}{1}$, $\theta_A = \tan^{-1}(5)$

$\theta_A = 78.69°$

Direction of $\vec{OB}$

$\tan\theta = \frac{-1}{5}$, $\theta_B = \tan^{-1}\left(\frac{-1}{5}\right)$

$\theta_B = -11.31°$

b.) By inspection if θ is the angle between $\vec{OA}$ and $\vec{OB}$

$\Rightarrow \theta = \theta_A - \theta_B$

$= 78.69° - (-11.31°)$

$= 78.69 + 11.31$

$\theta = 90°$

since the angle between the vectors is 90° it follows that the vectors $\vec{OA}$ and $\vec{OB}$ are perpendicular.

OR Find the Dot product.

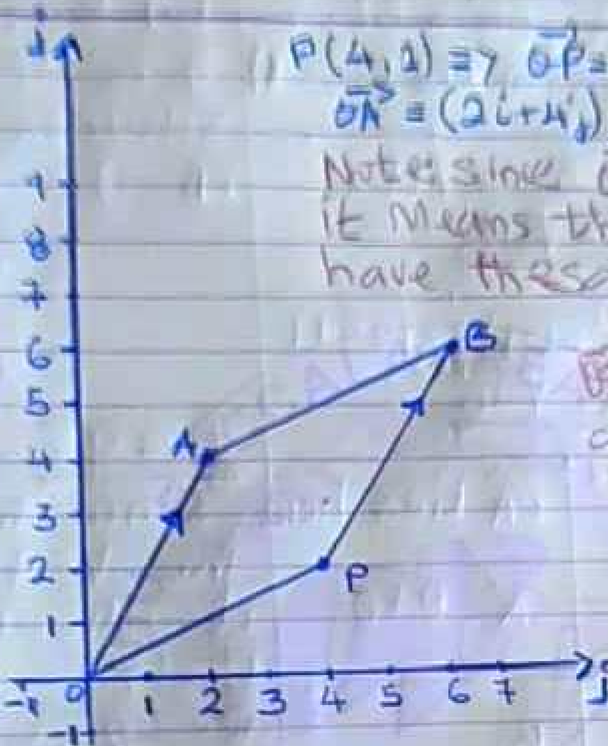
$A \cdot B = 1 \times 5 + (5) \times (-1)$

$= 5 - 5$

$A \cdot B = 0$

Hence $\vec{OA}$ and $\vec{OB}$ are perpendicular

2.
a)



$P(4,2) \Rightarrow \vec{OP} = 4i + 2j$

$\vec{OA} = (2i + 4j) \Rightarrow A(2,4)$

Note: since $\vec{OA} = \vec{PB}$ it means that both vectors have the same Direction vector, hence $\vec{OA}$ and $\vec{PB}$ are Parallel Vectors.

b.) Coordinates of B is (6,6)

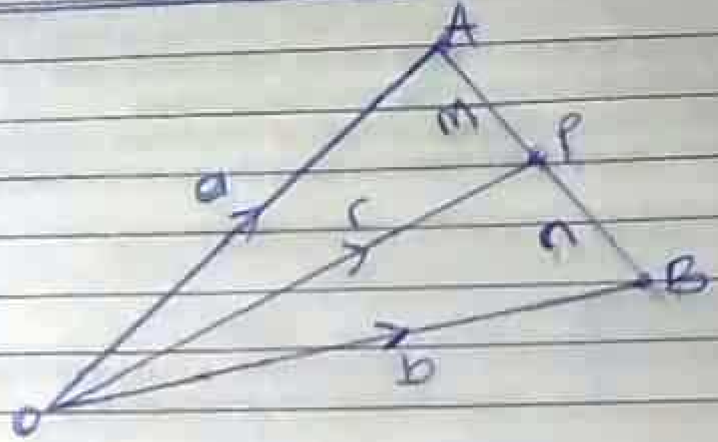
c) OABP is a parallelogram.

d) $\vec{OB} = 6i + 6j$

$|OB| = \sqrt{6^2 + 6^2} = \sqrt{36 + 36} = \sqrt{72} \Rightarrow \underline{|\vec{OB}| = 6\sqrt{2}}$ [Magnitude]

$\theta = \tan^{-1}(6/6) \Rightarrow \underline{\theta = 45^\circ}$ [Direction]

Division of Two position Vectors in a Given Ratio



If P divides the Line joining A and B in the ratio m:n then the position Vector $\vec{OP} = r$ is given by $\vec{OP} = \frac{na+mb}{m+n}$

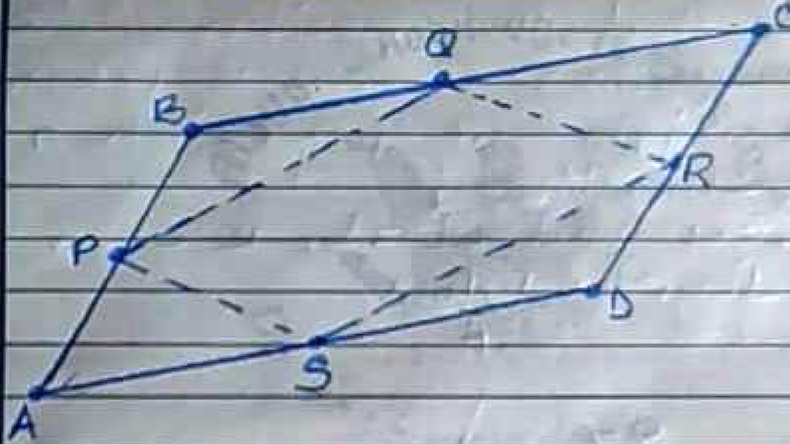
Suppose that P lies mid-way on the line joining AB, then $n = m$

$\Rightarrow \vec{OP} = \frac{1(a)+1(b)}{1+1}$ $\Rightarrow \vec{OP} = \frac{1}{2}(a+b)$

[This is the Mid-point theorem]

Other analysis; Since $\vec{MN} = \frac{1}{2}\vec{AB}$
$|MN| = \text{Half } |AB|$,
It follows that $\vec{MN}$ and $\vec{AB}$ have the same direction vectors. Hence ~~MN and~~ $\vec{MN}$ and $\vec{AB}$ are parallel vectors.
Similarly $\vec{NM}$ and $\vec{BA}$ are also parallel vectors.

THE PARALLELOGRAM THEOREM



If ABCD is a parallelogram the ~~the~~ following condition holds.
$|AD| = |BC|$, $\vec{AD} \parallel BC$
$|AB| = |DC|$ $\vec{AB} \parallel DC$

S is mid-point on AD
Q is mid-point on BC
P is mid-point on AB
R is mid-point on DC
It follows that PQRS is also a parallelogram.