PRACTICE EXERCISES

1. The position vectors of the points A, B and C relative to the origin are respectively: $2i + mj$, $i + 4j$ and $-5i + 2j$

a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$

b) $|\overrightarrow{CB}|$, the modulus of $\overrightarrow{CB}$ Leave the answer in surd form

2. Given the vectors $P = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $q = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and $r = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

a. Write as a column vector $2P$

b. Calculate $|q|$ (the magnitude of q)

c. Express $2p - r$ in the form $mi + nj$, where i and j are unit vectors along the x- and y-axes.

d. Given ~~that~~ $tp + Kr = q$, where t and K are constants. determine the values of t and K

3. Calculate the direction of the following vectors: $\overrightarrow{OP} = i + j$, $\overrightarrow{OB} = -i - \sqrt{3}j$, $\overrightarrow{ON} = -3j$, $\overrightarrow{Oq} = -\sqrt{3}i + j$.

THE END

1.) $tp + kr = q$

Now substitute the vectors p, q and r and form two equations

$\Rightarrow t\begin{pmatrix}3\\2\end{pmatrix} + k\begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}5\\4\end{pmatrix}$

r r q

Multiply the various values by the column matrix

ie $\begin{pmatrix}3t\\2t\end{pmatrix} + \begin{pmatrix}-2k\\-k\end{pmatrix} = \begin{pmatrix}5\\4\end{pmatrix}$

Add L.H.S

$\begin{bmatrix}3t + (-2k)\\2t + (-k)\end{bmatrix} = \begin{pmatrix}5\\4\end{pmatrix}$

Here you only Add corresponding terms

$\Rightarrow \begin{pmatrix}3t - 2k\\2t - k\end{pmatrix} = \begin{pmatrix}5\\4\end{pmatrix}$

Equate corresponding terms

ie $3t - 2k = 5$ --- (i)

$2t - k = 4$ --- (ii)

This is a pair of simultaneous equation to solve for t and k

$(i \times 1) - (ii \times 2)$

$3t - 2k = 5$

$-2(2t - k = 4)$

ie $3t - 2k = 5$

$- \; 4t - 2k = 8$

$-t = -3$

$\Rightarrow \underline{t = 3}$ {when multiply both sides by -1}

Substitute $t = 3$ in (ii) to solve for k

ie $2t - k = 4 \Rightarrow 2(3) - k = 4$

$6 - k = 4$, $6 - 4 = k \Rightarrow \underline{k = 2}$

3. •

$\vec{OP}$ lies here

let θ = direction

$\theta = \arctan\left(\frac{1}{1}\right)$

$\underline{\theta = 45^\circ}$

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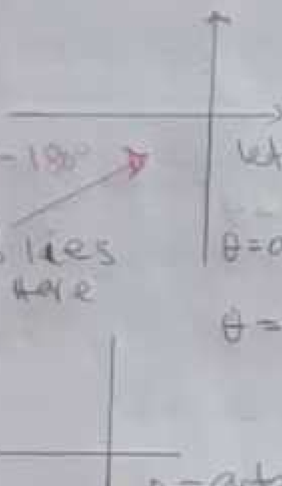

$\theta - 180^\circ$

$\vec{OB}$ lies here

let θ = direction

$\theta = \arctan\left(\frac{-\sqrt{3}}{-1}\right) - 180^\circ$

$\theta = 60^\circ - 180^\circ$

$\underline{\theta = -120^\circ}$

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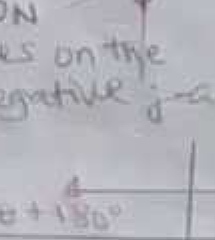

$\theta = \arctan\left(\frac{-3}{0}\right)$

$\underline{\theta = -90^\circ}$

ON lies on the negative j-axis] This is a special case

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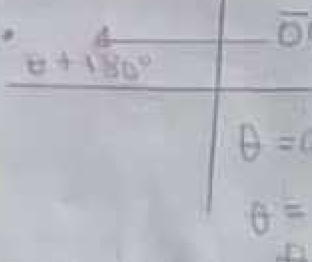

$\theta + 180^\circ$

$\vec{OQ}$ lies here

$\theta = \arctan\left(\frac{1}{-\sqrt{3}}\right) + 180^\circ$

$\theta = -30^\circ + 180^\circ$

$\underline{\theta = 150^\circ}$

SOLUTIONS TO PRACTICAL EXERCISES

1. From the given information:

$\overrightarrow{OA} = 2i + mj$, $\overrightarrow{OB} = i + 4j$ and $\overrightarrow{OC} = -5i + 3j$.

a.) To find $\overrightarrow{AB}$ means

$\overrightarrow{OA} + \overrightarrow{OB} = (2i + mj) + (i + 4j)$

Add i to i and j to j

$\Rightarrow \overrightarrow{OA} + \overrightarrow{OB} = 2i + i + mj + 4j$

$\overrightarrow{AB}$ $\Rightarrow \overrightarrow{OA} + \overrightarrow{OB} = 3i + (m+4)j$

$\overrightarrow{BC}$ means $\overrightarrow{OB} + \overrightarrow{OC}$

$\Rightarrow \overrightarrow{BC} = i + 4j + -5i + 2j$

$= i - 5i + 4j + 2j$

$\Rightarrow \overrightarrow{BC} = -4i + 6j$

b) $|\overrightarrow{CB}|$ means calculate the length of $\overrightarrow{CB}$

Note that $\overrightarrow{CB}$ and $\overrightarrow{BC}$ have the same magnitude but opposite direction

Hence $|\overrightarrow{CB}| = |\overrightarrow{BC}|$

$\Rightarrow |\overrightarrow{CB}| = \sqrt{(-4)^2 + 6^2} = \sqrt{16+36}$

$\Rightarrow |\overrightarrow{CB}| = \sqrt{52} = \sqrt{4 \times 13} \Rightarrow |\overrightarrow{CB}| = 2\sqrt{13}$

2. a) $2P$ means multiply the vector P by 2

$\Rightarrow 2P = 2\begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}2\times3\\2\times2\end{pmatrix} = \begin{pmatrix}6\\4\end{pmatrix}$

$\therefore 2P = \begin{pmatrix}6\\4\end{pmatrix}$

b) $|q| = \sqrt{5^2 + 4^2}$

$= \sqrt{25+16} = \sqrt{41}$

$|P| = \sqrt{41}$

c.) $2P - r = \begin{pmatrix}6\\4\end{pmatrix} - \begin{pmatrix}-2\\-1\end{pmatrix}$

$= \begin{pmatrix}6-(-2)\\4-(-1)\end{pmatrix}$

$= \begin{pmatrix}6+2\\4+1\end{pmatrix} \Rightarrow \begin{pmatrix}8\\5\end{pmatrix}$

$2P - r = \begin{pmatrix}8\\5\end{pmatrix}$ in the form $mi + nj$,

$m = 8,\ n = 5$

$2P - r = 8i + 5j$

$m = i$ - component

$n = j$ - component