

VECTORS OF UNIT LENGTH

They are vectors having a length of one unit. If the vector a and b has a length one unit by sign convention they are written as \hat{a} and \hat{b} respectively. Such that $|\hat{a}|=1$ and $|\hat{b}|=1$. Vectors can also be represented in terms of two unit vectors which lie along the x -axis and along the y -axis

$\hat{i} \Rightarrow$ Component along the x -axis

$\hat{j} \Rightarrow$ Component along the y -axis.

The \vec{OA} can be written in terms of \hat{i} and \hat{j} components as $\vec{OA} = P\hat{i} + q\hat{j}$ where P and q are Real Numbers.

\vec{OA} can also be represented as a Column matrix i.e. $\vec{OA} = \begin{pmatrix} P \\ q \end{pmatrix}$

In this ~~situation~~ case the \hat{i} and \hat{j} components disappear.

Vectors in Column matrix form are used in Matrix translation.

MAGNITUDE OF A VECTOR

$$\text{If } \vec{OA} = P\hat{i} + q\hat{j} \quad \text{OR} \quad \vec{OA} = \begin{pmatrix} P \\ q \end{pmatrix}$$

$$|\vec{OA}| = \sqrt{P^2 + q^2} \quad \text{Example}$$

$$\text{If } \vec{OA} = 3\hat{i} + 4\hat{j} \quad |\vec{OA}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$$

$$\underline{|\vec{OA}| = 5}$$

$$\vec{OB} = 5\vec{i} - 7\vec{j} \Rightarrow |\vec{OB}| = \sqrt{5^2 + (-7)^2} = \sqrt{25 + 49}$$

$$\Rightarrow |\vec{OB}| = \sqrt{74}$$

Since 74 has no exact square root leave the answer in surd form except that you are asked to write the answer to some number of decimal places or significant figures.

DIRECTION OF A VECTOR

This is the angle that a vector makes with the positive x-axis. The diagram below would help you easily determine directions of vectors.

For $\vec{OA} = p\vec{i} + q\vec{j}$ and $\theta =$ direction of \vec{OA} ,

Consider signs $\tan \theta = \frac{q}{p}$ $\theta = \arctan(p/q)$		Don't Consider Signs here
When \vec{OA} lies Here $\theta = \arctan(p/q) + 180^\circ$	When \vec{OA} lies here $\theta = \arctan(p/q)$	
When θ lies Here $\theta = \arctan(p/q) - 180^\circ$	When \vec{OA} lies Here $\theta = \arctan(p/q)$	Don't Consider Signs Here
Consider signs here		

Examples

$$\vec{OA} = \vec{i} + 2\vec{j}$$

$$\theta = \arctan\left(\frac{2}{1}\right)$$

$$\theta = \underline{63.4^\circ}$$

$$\vec{OA} = -\vec{i} - \sqrt{3}\vec{j}$$

$$\theta = \arctan\left(\frac{-\sqrt{3}}{-1}\right) - 180^\circ$$

$$\theta = 60^\circ - 180^\circ \Rightarrow \theta = \underline{-120^\circ}$$

$$\vec{OA} = -2\vec{i} + 2\vec{j}$$

$$\theta = \arctan\left(\frac{2}{-2}\right) + 180^\circ$$

$$\theta = -45^\circ + 180^\circ \Rightarrow \theta = \underline{135^\circ}$$

$$\vec{OB} = \frac{1}{2}\vec{i} - \frac{1}{2}\vec{j}$$

$$\theta = \arctan\left(\frac{-1/2}{1/2}\right)$$

$$\theta = \underline{45^\circ}$$

P₄

TOPIC ONE: VECTORS

Definition: They are quantities which have both magnitude and direction.

- Vectors are Represented by Line Segments
For example \vec{AB} , \vec{OA} , \vec{BA} --- etc. The arrow starts from the first letter and ends at the 2nd letter. For example for the vector \vec{AB} it starts at A and ends at B.
- Vectors can also be represented by small letters a, b, c, d, --- etc. The vectors in small letters are written as \vec{a} , \vec{b} , etc.

Position Vectors: These are vectors referred to a fixed point. Most of times the fixed point is the origin.

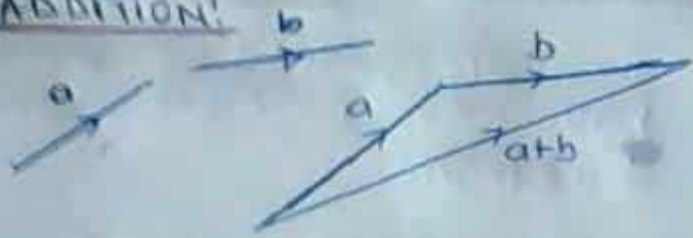
Example: The position vector of the point P, Q --- etc relative to the origin O are \vec{OP} and \vec{OQ} respectively.

VECTOR NOTATION

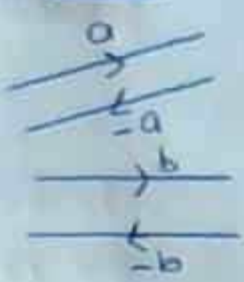
- $a = b$ means the length of a equals the length of b.
- $a \parallel b$ means a and b have the same direction.
- $|AB|$ means the size of vector AB

OPERATIONS ON VECTORS

ADDITION:

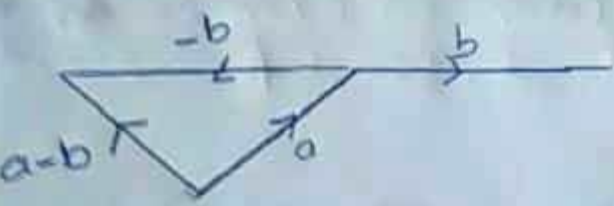


NOTE



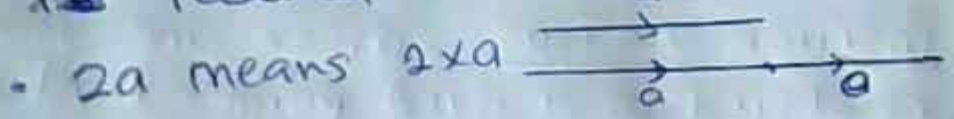
SUBTRACTION

Now $a - b$ means adding $a + (-b)$

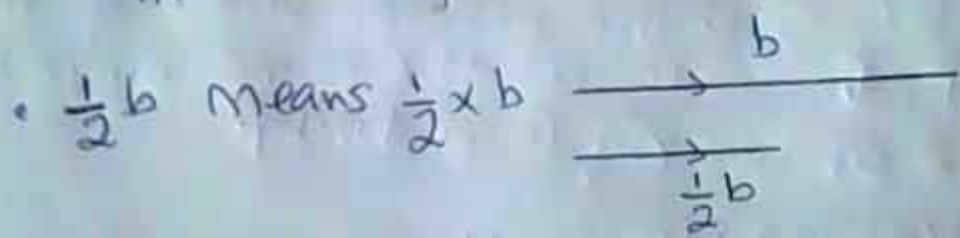


MULTIPLICATION BY A NUMBER

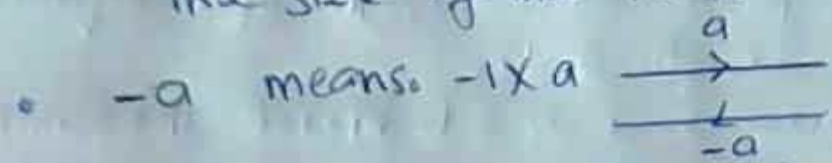
Multiplication would either increase or ~~re~~ reduced the size of a vector.



The size of the vector increases



The size of the vector b , reduces



The direction of the vector changes when multiplied by a negative number